1.15 Evaluating Points not on Unit Circle Name: ________________

We now know how to answer trigonometric problems that use the special angles found on the unit circle. Problems like:

1) $\sin 315°$: Because this is a 45° family, the answer is either $\frac{\sqrt{2}}{2}$ or $-\frac{\sqrt{2}}{2}$. The angle 315° is found in the 4th quadrant, and sine is based on y which is negative in the 4th quadrant. So, the answer is the negative y-value: $\sin 315° = -\frac{\sqrt{2}}{2}$

2) $\tan\frac{7\pi}{6}$: Because this is the "over 6" family (30° family) but in the 3rd quadrant, the coordinates are $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$. The formula for tangent is $\frac{y}{x}$, here that means $\frac{-1/2}{-\sqrt{3}/2}$. Both the negatives and the denominators of 2 cancel out, leaving $\frac{1}{\sqrt{3}}$. This has to be rationalized. So, the answer is: $\tan\frac{7\pi}{6} = \frac{\sqrt{3}}{3}$

What if the angle is bigger than 1 revolution around the circle?

3) $\cos 840°$: As before, if values are too large to know, use coterminal angles to bring them down to a value we already know. Subtract 360° enough times so that the angle is now between 0° and 360°: $840° - 360° - 360° = 120°$. The coterminal angle will have the same cosine value, so consider it as $\cos 120°$. This angle is in the 60° family in Quadrant 2 with coordinates $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. Cosine uses the x-value of the coordinates.

So, the answer is: $\cos 840° = \cos 120° = -\frac{1}{2}$

What if the angle is negative?

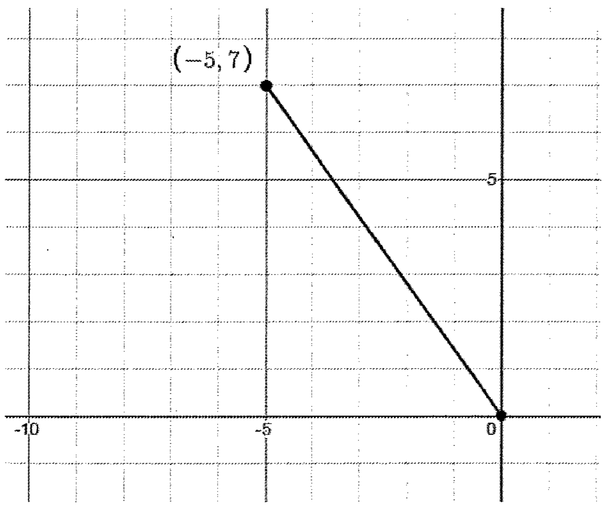
4) $\csc\left(-\frac{4\pi}{3}\right)$: The angle is from the "over 3" family (60° family). Cosecant is the flip of sine, and sine is the y-value. The y-values for all "over 3" angles are $\frac{\sqrt{3}}{2}$ or $-\frac{\sqrt{3}}{2}$. To determine the quadrant, find the coterminal angle by adding 2π:

$-\frac{4\pi}{3} + \frac{6\pi}{3} = \frac{2\pi}{3}$. The y-value in Quadrant 2 will be positive, $y = \frac{\sqrt{3}}{2}$. The cosecant value requires the reciprocal. So $\csc\left(-\frac{4\pi}{3}\right) = \csc\frac{2\pi}{3} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

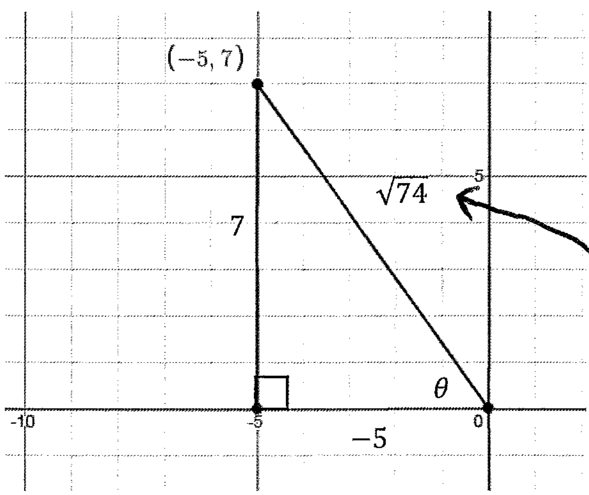
What if the location is not even *on* the unit circle? Sometimes we have to fall back on **right triangle trigonometry** rather than using the unit circle.

5) Find sec θ where the terminal side of θ passes through (-5, 7).

Step 1: Plot the point and connect it to the origin.



Step 2: Connect the point perpendicularly to the closest x-axis, making a reference triangle. Label θ as the reference angle. Use Pythagorean Theorem to determine the length of the hypotenuse. Label all 3 side lengths.



$(-5)^2+(7)^2=c^2$

$25+49=74$

$c=\sqrt{74}$

Step 3: Use SohCahToa and the reciprocal relations to complete the problem.

$$\sec\theta=\frac{hypotenuse}{adjacent}=\frac{\sqrt{74}}{-5}$$

$\sec\theta=\dfrac{\sqrt{74}}{-5}$

1.15 Practice

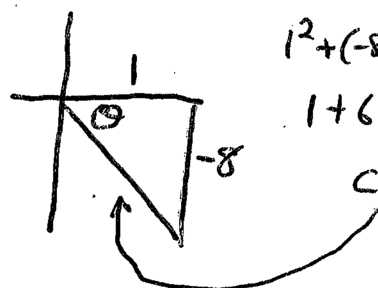
Page 251: #5-31 odd and #43-51 odd

The given point lies on the terminal side of an angle θ in standard position. Find the values of the six trigonometric functions of θ. (Example 1)

5. (1, −8)

7. (−8, 15)

5)



$1^2+(-8)^2=c^2$

$1+64=c^2$

$c=\sqrt{65}$

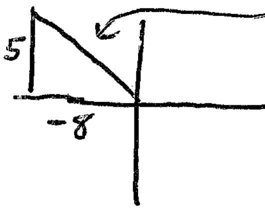
$\sin\theta = \frac{-8}{\sqrt{65}}, \frac{-8\sqrt{65}}{65}$ $\csc\theta = \sqrt{65}/8$

$\cos\theta = \sqrt{65}/65$ $\sec\theta = \sqrt{65}$

$\tan\theta = -8$ $\cot\theta = -1/8$

7) $(-8)^2+15^2=c^2$

$c=17$



$\sin\theta = 15/17$ $\csc\theta = \frac{17}{15}$

$\cos\theta = \frac{-8}{17}$ $\sec\theta = \frac{-17}{8}$

$\tan\theta = \frac{-15}{8}$ $\cot\theta = \frac{-8}{15}$

Find the exact value of each trigonometric function, if defined. If not defined, write *undefined*. (Example 2)

9. $\sin \frac{\pi}{2}$ $= 1$

11. $\cot(-180°)$ und.

13. $\cos(-270°)$ 0

15. $\tan \pi$ 0

$-405+360$

Sketch each angle. Then find its reference angle. (Example 3)

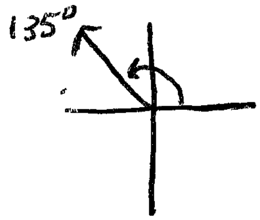
17. 135°

19. $\frac{7\pi}{12}$

21. −405°

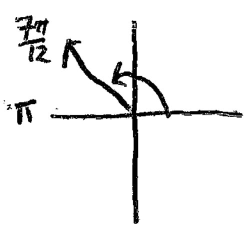
23. $\frac{5\pi}{6}$

17) 135°



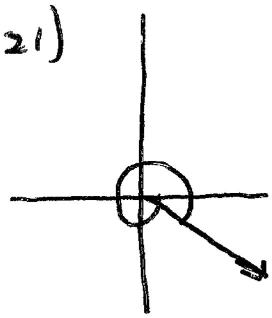
ref angle: 45°

19) $\frac{7\pi}{12}$



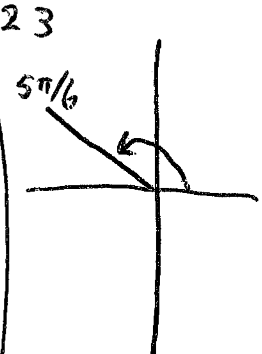
ref: $\frac{5\pi}{12}$

21)



ref angle: 45°

23



ref angle $= \frac{\pi}{6}$

Find the exact value of each expression. (Example 4)

25. $\cos \frac{4\pi}{3}$ $= -\frac{1}{2}$

27. $\sin \frac{3\pi}{4}$ $= \sqrt{2}/2$

29. $\csc 390°$ $= 2$

31. $\tan \frac{11\pi}{6}$ $= -\frac{\sqrt{3}}{3}$

Find the exact value of each expression. If undefined, write *undefined*. (Examples 7 and 8)

43. $\sec 120°$ $= -2$

45. $\cos \frac{11\pi}{3}$ $= \frac{1}{2}$

47. $\csc 390°$ $= 2$

49. $\csc 5400°$ $=$ undefined

51. $\cot\left(-\frac{5\pi}{6}\right)$ $= \sqrt{3}$