Ch. 1.3 Evaluating Limits Algebraically Classwork Problems

Algebraic Steps: **1)** Plug in x-value first **2)** If result is a real number value, the value is the limit. **3)** If the result is $\frac{0}{0}$ (indeterminate form) then reduce by **i)** factoring **ii)** conjugate method **iii)** simplify complex fraction **4)** Re-evaluate the reduced Expression

1)

$$\lim_{x\to 1} (2x^3 - 6x + 5)$$

2)

$$\lim_{x\to 3} \frac{\sqrt{x+6}}{x+2}$$

3)

$$\lim_{x\to 0} \frac{x^2+3x}{x}$$

4)

$$\lim_{x\to 0} \frac{x^4-5x^2}{x^2}$$

5)

$$\lim_{x\to -1} \frac{x^2-1}{x+1}$$

6)

$$\lim_{x\to -2} \frac{3x^2+5x-2}{x+2}$$

Difference of cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ Sum of cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

7)

$$\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$$

8)

$$\lim_{x \to -1} \frac{x^3 + 1}{x + 1}$$

9)

$$\lim_{x \to 4} \frac{x - 4}{x^2 - 16}$$

10)

$$\lim_{x \to 5} \frac{5 - x}{x^2 - 25}$$

11)

$$\lim_{x \to -3} \frac{x^2 + x - 6}{x^2 - 9}$$

12)

$$\lim_{x \to 2} \frac{x^2 + 2x - 8}{x^2 - x - 2}$$

13) *conjugate method

$$\lim_{x \to 4} \frac{\sqrt{x + 5} - 3}{x - 4}$$

14) *conjugate method

$$\lim_{x \to 3} \frac{\sqrt{x + 1} - 2}{x - 3}$$

15)

$$\lim_{x\to 0} \frac{\sqrt{x+5}-\sqrt{5}}{x}$$

16)

$$\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x}$$

17) *Simplify Complex Fraction

$$\lim_{x\to 0} \frac{\frac{1}{3+x}-\frac{1}{3}}{x}$$

18) *Simplify Complex Fraction

$$\lim_{x\to 0} \frac{\frac{1}{4}-\frac{1}{4+x}}{x}$$

19)

$$\lim_{x\to 16} \frac{4-\sqrt{x}}{x-16}$$

20)

$$\lim_{x\to 0} \frac{\sqrt{x+2}-\sqrt{2}}{x}$$

Algebraic Steps: **1)** Plug in x-value first **2)** If result is a real number value, the value is the limit. **3)** If the result is $\frac{0}{0}$ (indeterminate form) then reduce by **i)** factoring **ii)** conjugate method **iii)** simplify complex fraction **4)** Re-evaluate Expression

21)

$$\lim_{x \to 1} \frac{x^2 - 4}{x^2 - 3x + 2}$$

22)

$$\lim_{x \to 1} \frac{x^2 - 2x + 1}{x^3 - x}$$

23)

$$\lim_{x \to 4} \frac{x^2 + 7x - 44}{x^2 - 6x + 8}$$

24)

$$\lim_{x \to 0} \frac{\sqrt{1 + 2x} - 1}{3x}$$

25)

$$\lim_{x \to 5} \frac{\sqrt{x - 1} - 2}{x^2 - 25}$$

26)

$$\lim_{x \to 0} \frac{\frac{1}{4} + \frac{1}{x - 4}}{x}$$

Ch. 1.3 Evaluating Limits Algebraically Classwork Problems

Key

Algebraic Steps: **1)** Plug in x-value first **2)** If result is a real number value, the value is the limit. **3)** If the result is $\frac{0}{0}$ (indeterminate form) then reduce by **i)** factoring **ii)** conjugate method **iii)** simplify complex fraction **4)** Re-evaluate Expression/Limit

1)

$$\lim_{x\to1}(2x^3 - 6x + 5)$$

$2(1)^3 - 6(1) + 5$

$= 2 - 6 + 5 = -4 + 5$

$= \boxed{1}$

2)

$$\lim_{x\to3}\frac{\sqrt{x+6}}{x+2} \to \frac{\sqrt{3+6}}{3+2} = \frac{\sqrt{9}}{5} = \boxed{\frac{3}{5}}$$

3)

$$\lim_{x\to0}\frac{x^2+3x}{x} \to \frac{0^2+3(0)}{0} = \frac{0}{0}$$

$$\lim_{x\to0}\frac{\cancel{x}(x+3)}{\cancel{x}} = 0+3 = \boxed{3}$$

4)

$$\lim_{x\to0}\frac{x^4-5x^2}{x^2} \to \frac{0^4-5(0)^2}{0^2} = \frac{0}{0}$$

$$\lim_{x\to0}\frac{\cancel{x^2}(x^2-5)}{\cancel{x^2}} \to 0^2-5 = \boxed{-5}$$

5)

$$\lim_{x\to-1}\frac{x^2-1}{x+1} \to \frac{(-1)^2-1}{-1+1} = \frac{0}{0}$$

$$\lim_{x\to-1}\frac{(x-1)\cancel{(x+1)}}{\cancel{(x+1)}} \to -1-1 = \boxed{-2}$$

6)

$$\lim_{x\to-2}\frac{3x^2+5x-2}{x+2} \to \frac{3(-2)^2+5(-2)-2}{2-2} = \frac{0}{0}$$

a·c

$6 \;\; -6 \;\; -1$

$3 \;\; 5 \;\; 3$

b

$$\lim_{x\to-2}\frac{\cancel{(x+2)}(3x-1)}{\cancel{(x+2)}} = 3(-2)-1 = \boxed{-7}$$

Difference of cubes: $a^3 - b^3 = (a-b)(a^2+ab+b^2)$ Sum of cubes: $a^3 + b^3 = (a+b)(a^2-ab+b^2)$

7)

$x^3 - 2^3 = (x-2)(x^2+2x+4)$

$$\lim_{x\to 2} \frac{x^3-8}{x-2} \to \frac{2^3-8}{2-2} = \frac{0}{0}$$

$$\lim_{x\to 2} \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}} = 2^2+2(2)+4$$

$$= \boxed{12}$$

8)

$$\lim_{x\to -1} \frac{x^3+1}{x+1} \to \frac{(-1)^3+1}{-1+1} = \frac{0}{0}$$

$$\lim_{x\to -1} \frac{\cancel{(x+1)}(x^2-1x+1)}{\cancel{(x+1)}} = (-1)^2 - 1(-1)+1 =$$

$$\boxed{3}$$

9)

$$\lim_{x\to 4} \frac{x-4}{x^2-16} \to \frac{0}{0}$$

$$\lim_{x\to 4} \frac{x-4}{(x-4)(x+4)} \to \lim_{x\to 4} \frac{1}{x+4} = \boxed{\frac{1}{8}}$$

10)

$$\lim_{x\to 5} \frac{5-x}{x^2-25} \to \frac{0}{0}$$

$$\lim_{x\to 5} \frac{-\cancel{(-5+x)}}{(x+5)\cancel{(x-5)}} \to \lim_{x\to 5} \frac{-1}{x+5} = \boxed{\frac{-1}{10}}$$

11)

$$\lim_{x\to -3} \frac{x^2+x-6}{x^2-9} \to \lim_{x\to -3} \frac{\cancel{(x+3)}(x-2)}{(x-3)\cancel{(x+3)}}$$

$$\lim_{x\to -3} \frac{x-2}{x-3} = \frac{-5}{-6} = \boxed{\frac{5}{6}}$$

12)

$$\lim_{x\to 2} \frac{x^2+2x-8}{x^2-x-2} \to \frac{0}{0}$$

$$\lim_{x\to 2} \frac{(x+4)(x-2)}{(x-2)(x+1)} = \frac{6}{3} = \boxed{2}$$

13) *conjugate method

$$\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4} \to \frac{0}{0}$$

$$\lim_{x\to 4} \frac{(\sqrt{x+5}-3)}{x-4} \cdot \frac{(\sqrt{x+5}+3)}{(\sqrt{x+5}+3)}$$

$$\lim_{x\to 4} \frac{x+5-9}{(x-4)(\sqrt{x+5}+3)} \to \lim_{x\to 4} \frac{(x-4)}{(x-4)(\sqrt{x+5}+3)}$$

$$\lim_{x\to 4} \frac{1}{\sqrt{x+5}+3} = \frac{1}{\sqrt{9}+3} = \boxed{\frac{1}{6}}$$

14) *conjugate method

$$\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} \to \frac{0}{0}$$

$$\lim_{x\to 3} \frac{(\sqrt{x+1}-2)}{(x-3)} \cdot \frac{(\sqrt{x+1}+2)}{\sqrt{x+1}+2}$$

$$\lim_{x\to 3} \frac{x+1-4}{(x-3)(\sqrt{x+1}+2)} \to \lim_{x\to 3} \frac{\cancel{(x-3)}}{\cancel{(x-3)}(\sqrt{x+1}+2)}$$

$$\lim_{x\to 3} \frac{1}{(\sqrt{x+1}+2)} = \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$$

15)

$$\lim_{x\to0}\frac{\sqrt{x+5}-\sqrt{5}}{x} \to \frac{0}{0}$$

$$\lim_{x\to0}\frac{\sqrt{x+5}-\sqrt{5}}{x}\cdot\frac{(\sqrt{x+5}+\sqrt{5})}{(\sqrt{x+5}+\sqrt{5})}$$

$$\lim_{x\to0}\frac{x+5-5}{x(\sqrt{x+5}+\sqrt{5})} \to \lim_{x\to0}\frac{1}{\sqrt{x+5}+\sqrt{5}} = \frac{1}{\sqrt{5}+\sqrt{5}}$$

$$\boxed{\frac{1}{2\sqrt{5}}}$$

16)

$$\lim_{x\to0}\frac{\sqrt{2+x}-\sqrt{2}}{x} \to \frac{0}{0}$$

$$\lim_{x\to0}\frac{(\sqrt{2+x}-\sqrt{2})(\sqrt{2+x}+\sqrt{2})}{x\cdot(\sqrt{2+x}+\sqrt{2})}$$

$$\lim_{x\to0}\frac{2+x-2}{x(\sqrt{2+x}+\sqrt{2})} \to \lim_{x\to0}\frac{1}{(\sqrt{2+x}+\sqrt{2})} = \frac{1}{\sqrt{2}+\sqrt{2}}$$

$$= \boxed{\frac{1}{2\sqrt{2}}}$$

17) *Simplify Complex Fraction

$$\lim_{x\to0}\frac{\frac{1}{3+x}-\frac{1}{3}}{x} \to \frac{0}{0}$$

$$\lim_{x\to0}\frac{\frac{1}{3+x}-\frac{1}{3}}{x}\cdot\frac{3(x+3)}{3(x+3)}$$

$$\lim_{x\to0}\frac{3-(x+3)}{x(3)(x+3)} \to \lim_{x\to0}\frac{3-x-3}{3(x)(x+3)}$$

$$\lim_{x\to0}\frac{-1}{3(x+3)} = \boxed{\frac{-1}{9}}$$

18) *Simplify Complex Fraction

$$\lim_{x\to0}\frac{\frac{1}{4}-\frac{1}{4+x}}{x} \to \frac{0}{0}$$

$$\lim_{x\to0}\frac{\frac{1}{4}-\frac{1}{4+x}}{x}\cdot\frac{4(4+x)}{4(4+x)}$$

$$\lim_{x\to0}\frac{4+x-4}{x\cdot4(4+x)} \to \lim_{x\to0}\frac{1}{4(4+x)} = \frac{1}{4(4)} = \boxed{\frac{1}{16}}$$

19)

$$\lim_{x\to16}\frac{4-\sqrt{x}}{x-16}\cdot\frac{0}{0}$$

$$\lim_{x\to16}\frac{(4-\sqrt{x})(4+\sqrt{x})}{x-16\;(4+\sqrt{x})} \to \lim_{x\to16}\frac{16-x}{(x-16)(4+\sqrt{x})}$$

$$\lim_{x\to16}\frac{-(-16+x)}{(x-16)(4+\sqrt{x})} \to \lim_{x\to16}\frac{-1}{4+\sqrt{x}}$$

$$\lim_{x\to16}\frac{-1}{4+\sqrt{16}} = \frac{-1}{4+4} = \boxed{\frac{-1}{8}}$$

20)

$$\lim_{x\to0}\frac{\sqrt{x+2}-\sqrt{2}}{x} \to \frac{0}{0}$$

$$\lim_{x\to0}\frac{\sqrt{x+2}-\sqrt{2}}{x}\cdot\frac{(\sqrt{x+2}+\sqrt{2})}{(\sqrt{x+2}+\sqrt{2})}$$

$$\lim_{x\to0}\frac{x+2-2}{x(\sqrt{x+2}+\sqrt{2})} \to \lim_{x\to0}\frac{1}{\sqrt{x+2}+\sqrt{2}}$$

$$\frac{1}{\sqrt{2}+\sqrt{2}} = \boxed{\frac{1}{2\sqrt{2}}}$$

Algebraic Steps: **1)** Plug in x-value first **2)** If result is a real number value, the value is the limit. **3)** If the result is $\frac{0}{0}$ (indeterminate form) then reduce by **i)** factoring **ii)** conjugate method **iii)** simplify complex fraction **4)** Re-evaluate Expression

21)

$$\lim_{x\to1}\frac{x^2-4}{x^2-3x+2} \rightarrow \frac{-3}{0}$$

$\boxed{\text{undefined, limit does not exist}}$

22)

$$\lim_{x\to1}\frac{x^2-2x+1}{x^3-x} \rightarrow \frac{0}{0}$$

$$\lim_{x\to1}\frac{(x-1)(x-1)}{x(x^2-1)} \rightarrow \lim_{x\to1}\frac{\cancel{(x-1)}(x-1)}{x\cancel{(x-1)}(x+1)}$$

$$\frac{0}{1(2)} = \boxed{0}$$

23)

$$\lim_{x\to4}\frac{x^2+7x-44}{x^2-6x+8} \rightarrow \frac{0}{0}$$

$$\lim_{x\to4}\frac{(x+11)\cancel{(x-4)}}{\cancel{(x-4)}(x-2)} \rightarrow \boxed{\frac{15}{2}}$$

24)

$$\lim_{x\to0}\frac{\sqrt{1+2x}-1}{3x} \rightarrow \frac{0}{0}$$

$$\lim_{x\to0}\frac{\sqrt{1+2x}-1}{3x}\cdot\frac{(\sqrt{1+2x}+1)}{(\sqrt{1+2x}+1)}$$

$$\lim_{x\to0}\frac{1+2x-1}{3x(\sqrt{1+2x}+1)} = \lim_{x\to0}\frac{2}{3(\sqrt{1+2x}+1)} = \frac{2}{3(2)} = \boxed{\frac{1}{3}}$$

25)

$$\lim_{x\to5}\frac{\sqrt{x-1}-2}{x^2-25} \rightarrow \frac{0}{0} \qquad \frac{(\sqrt{x-1}+2)}{(\sqrt{x-1}+2)}$$

$$\lim_{x\to5}\frac{x-1-4}{(x+5)(x-5)(\sqrt{x-1}+2)}$$

$$\lim_{x\to5}\frac{1}{(x+5)(\sqrt{x-1}+2)} = \frac{1}{(10)(\sqrt{4}+2)}$$

$$= \frac{1}{10(4)} = \boxed{\frac{1}{40}}$$

26)

$$\frac{0}{0}$$

$$\lim_{x\to0}\frac{\frac{1}{4}+\frac{1}{x-4}}{x}\cdot\frac{4(x-4)}{4(x-4)}$$

$$\lim_{x\to0}\frac{x-4+4}{x(4)(x-4)}$$

$$\lim_{x\to0}\frac{1}{4(x-4)} = \frac{1}{4(-4)} = \boxed{\frac{-1}{16}}$$