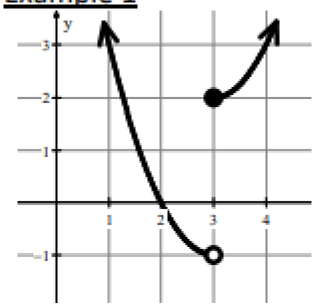
AP Calculus – 1.2 Notes – Techniques for finding Limits (Algebraic, Numeric, Graph)

What is a *one-sided* limit?

A ***one-sided limit*** is the _______ a function approaches as you approach a given __________ from either the _______ or _______ side.

Example 1



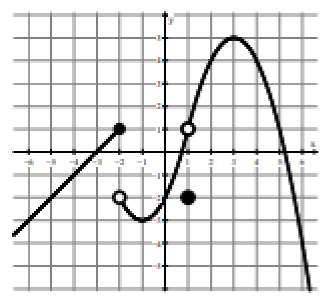
The limit of f as x approaches 3 from the left side is -1. $\lim\limits_{x\to} f(x) =$

The limit of f as x approaches 3 from the right side is 2. $\lim\limits_{x\to} f(x) =$

If the two sides are different? $\lim\limits_{x\to} f(x) =$

Example 2

a. $\lim\limits_{x\to-2^-} f(x) =$	b. $\lim\limits_{x\to-2^+} f(x) =$	c. $\lim\limits_{x\to-2} f(x) =$
d. $\lim\limits_{x\to1} f(x) =$	e. $\lim\limits_{x\to0} f(x) =$	f. $\lim\limits_{x\to3^-} f(x) =$
g. $\lim\limits_{x\to-1} f(x) =$	h. $f(1) =$	i. $f(-2) =$



Example 3

Sketch a graph of a function g that satisfies all of the following conditions.

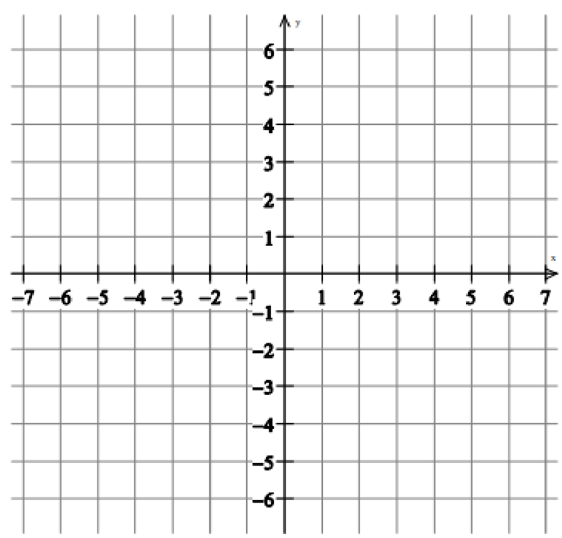
a. $g(3) = -1$

b. $\lim\limits_{x\to3} g(x) = 4$

c. $\lim\limits_{x\to-2^+} g(x) = 1$

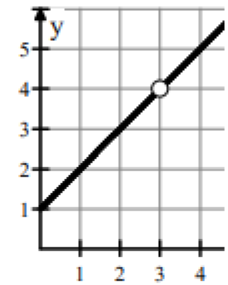
d. g is increasing on $-2 < x < 3$

e. $\lim\limits_{x\to-2^-} g(x) > \lim\limits_{x\to-2^+} g(x)$



Finding Limits from tables:

If we have the graph, it is easy to see the value of $\lim_{x \to 3} f(x) =$



Without the graph, we could use a table of values.

x	2.9	2.99	3.01	3.1
$f(x)$	3.9	3.99	4.01	4.1

The function f is continuous and increasing for $x \geq 1$. The table gives values of f at selected values of x. Approximate the value of $\lim_{x \to 2} \cos(f(x))$.

x	1.99	1.999	2.001	2.01
$f(x)$	4.85	4.999	5.001	5.15

$$\lim_{x \to 2} \cos(f(x)) =$$

Finding Limits Using Algebraic Methods:

Direct Substitution		Factor and Cancel	
1. $\lim_{x \to -1} (x^2 + 2x - 4)$	2. $\lim_{x \to 2} 6$	3. $\lim_{x \to 0} \frac{4x^2 - 5x}{x}$	4. $\lim_{x \to -7} \frac{2x^2 + 13x - 7}{x + 7}$

Limit Does Not Exist
5. $\lim_{x \to -6} \frac{x^2 + 4x + 3}{x + 6}$

Rationalize Fractions with Radicals

1. $\lim_{x\to 5} \frac{\sqrt{x+4}-3}{x-5}$

2. $\lim_{x\to 10} \frac{x-10}{3-\sqrt{x-1}}$

Complex Fractions

3. $\lim_{x\to 0} \frac{x}{\frac{1}{x-4}+\frac{1}{4}}$

4. $\lim_{x\to 0} \frac{\frac{1}{(x+3)^2}-\frac{1}{9}}{x}$

10. $\lim_{x\to 1} \frac{\frac{1}{3}-\frac{1}{3x}}{x-1}$

11. $\lim_{x\to 0} \frac{\sqrt{x+11}-\sqrt{11}}{x}$

12. $\lim_{x\to 3} \frac{\sqrt{2x-6}}{x}$

13. $\lim_{x\to 0} \frac{\frac{1}{(x+2)^2}-\frac{1}{4}}{x}$

14. $\lim_{x \to b} \frac{b-x}{\sqrt{x}-\sqrt{b}}$ is

(A) $-2\sqrt{b}$ (B) $-\sqrt{b}$ (C) $2b$ (D) $\sqrt{b}$ (E) $2\sqrt{b}$

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Use the piecewise functions to find the given values.

15)

. $g(x) = \begin{cases} \sqrt{5-x}, & x < -4 \\ x^2 - 5, & -4 \leq x < 2 \\ x - 3, & x \geq 2 \end{cases}$

a. $\lim_{x \to 2^-} g(x) =$ b. $\lim_{x \to -4^+} g(x) =$

c. $g(2) =$ d. $\lim_{x \to -4^-} g(x) =$

e. $\lim_{x \to 2^+} g(x) =$ f. $\lim_{x \to 2} g(x) =$

g. $\lim_{x \to -4} g(x) =$ h. $g(-4) =$

16) **Limits of absolute value functions**

$\lim_{x \to 2^-} \frac{|x-2|}{x-2}$