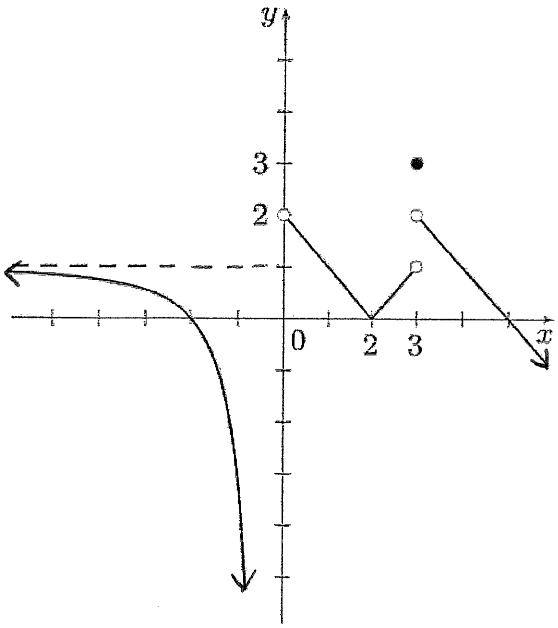
Calculus AB 1.4-3.5 Morning Quiz Review

1. Use the graph of the function $f(x)$ to answer each question.
   Use ∞, $-\infty$ or DNE where appropriate.



(a) $f(0) =$

(b) $f(2) =$

(c) $f(3) =$

(d) $\lim_{x \to 0^-} f(x) =$

(e) $\lim_{x \to 0} f(x) =$

(f) $\lim_{x \to 3^+} f(x) =$

(g) $\lim_{x \to 3} f(x) =$

(h) $\lim_{x \to -\infty} f(x) =$

1b) Step through continuity conditions to determine if f(x) is continuous at x = 3. If discontinuous, determine type of discontinuity

2. Use the graph of the function $f(x)$ to answer each question.
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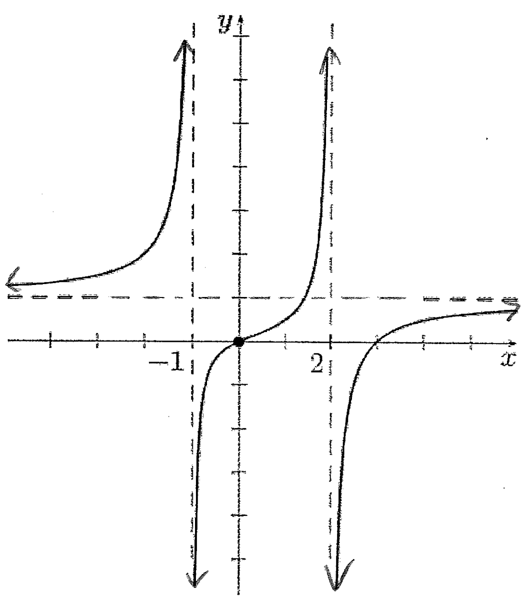

(a) $f(0) =$

(b) $f(2) =$

(c) $f(3) =$

(d) $\lim_{x \to -1^+} f(x) =$

(e) $\lim_{x \to 0} f(x) =$

(f) $\lim_{x \to 2^+} f(x) =$

(g) $\lim_{x \to \infty} f(x) =$

3. Determine the horizontal asymptote(s) of $y = \dfrac{-4x^{\frac{2}{3}}+15}{\sqrt[3]{5x^2-23}}$

4. Step through continuity conditions. If discontinuous, Show work to determine type of discontinuity

a) $f(x) = \begin{cases} -2x+3 & x > -4 \\ 4x-1 & x = -4 \\ \dfrac{5x-1}{2x} & x < -4 \end{cases}$ at x = -4

b) $g(x) = \begin{cases} \dfrac{x^2-4}{x+2} & x \neq -2 \\ 3x+2 & x = -2 \end{cases}$

Answer the following questions for the piecewise defined function $f(t)$ described on the right hand side.

(a) $f(-3/2) =$

(b) $f(2) =$

(c) $f(1) =$

(d) $\lim\limits_{t \to -2} f(t) =$

(e) $\lim\limits_{t \to -1^+} f(t) =$

(f) $\lim\limits_{t \to 2} f(t) =$

(g) $\lim\limits_{t \to 0} f(t) =$

(h) $\lim\limits_{t \to 0^-} f(t) =$

$$f(t) = \begin{cases} t^2 & \text{for } t < -2 \\ \dfrac{t+6}{t^2-t} & \text{for } -1 < t < 2 \\ 3t-2 & \text{for } t \geq 2 \end{cases}$$

Key

Calculus AB 1.4-3.5 Morning Quiz Review

1. Use the graph of the function $f(x)$ to answer each question.
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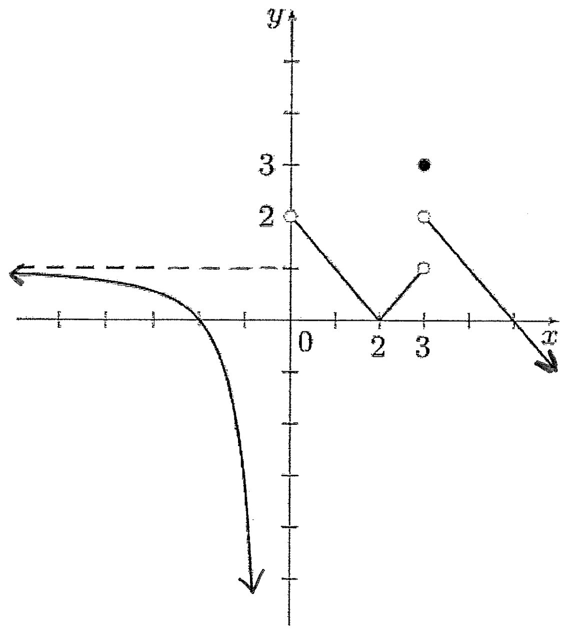


(a) $f(0) = DNE$

(b) $f(2) = 0$

(c) $f(3) = 3$

(d) $\lim_{x\to 0^-} f(x) = -\infty$

(e) $\lim_{x\to 0} f(x) = DNE$

(f) $\lim_{x\to 3+} f(x) = 2$

(g) $\lim_{x\to 3} f(x) = DNE$

(h) $\lim_{x\to -\infty} f(x) = 1$

1b) Step through continuity conditions to determine if f(x) is continuous at x = 3. If discontinuous, determine type of discontinuity

i) $f(3)=3$

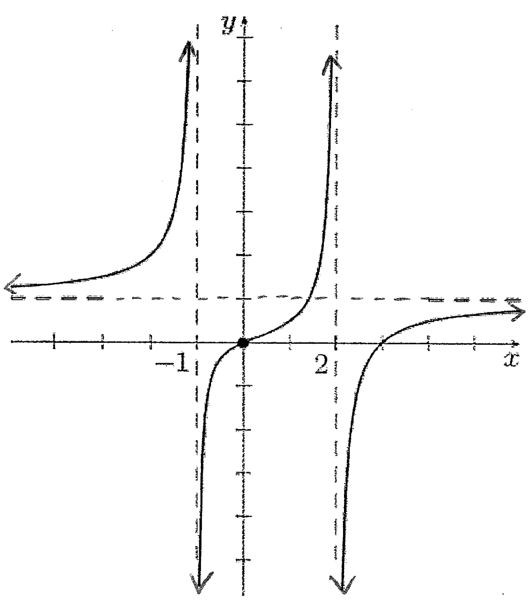
ii) $\lim_{x\to 3^-} f(x) = 1$

$\lim_{x\to 3^+} f(x) = 2$

since $\lim_{x\to 3^-} f(x) \neq \lim_{x\to 3^+} f(x)$, $\lim_{x\to 3} f(x) = DNE$

$\boxed{\text{nonremovable discontinuity at } x=3}$

2. Use the graph of the function $f(x)$ to answer each question.
Use ∞, $-\infty$ or DNE where appropriate.



(a) $f(0) = 0$

(b) $f(2) = DNE$

(c) $f(3) = 0$

(d) $\lim_{x\to -1^+} f(x) = -\infty$

(e) $\lim_{x\to 0} f(x) = 0$

(f) $\lim_{x\to 2+} f(x) = -\infty$

(g) $\lim_{x\to \infty} f(x) = 1$

3. Determine the horizontal asymptote(s) of $y = \frac{-4x^{\frac{2}{3}}+\cancel{15}}{\sqrt[3]{5x^2\cancel{-23}}}$

$$\lim_{x\to\infty} \frac{-4x^{2/3}}{\sqrt[3]{5}\,x^{2/3}} = \frac{-4}{\sqrt[3]{5}}$$

$$\boxed{\text{H.A. at } y = \frac{-4}{\sqrt[3]{5}},\ y = \frac{4}{\sqrt[3]{5}}}$$

$$\lim_{x\to-\infty} \frac{-4x^{2/3}}{\sqrt[3]{5}x^{2/3}} = -\left(\frac{-4x^{2/3}}{\sqrt[3]{5}x^{2/3}}\right) = \frac{4}{\sqrt[3]{5}}$$

4. Step through continuity conditions. If discontinuous, Show work to determine type of discontinuity

a) $f(x) = \begin{cases} -2x+3 & x > -4 \\ 4x-1 & x = -4 \\ \frac{5x-1}{2x} & x < -4 \end{cases}$ at x = -4

i) $f(-4) = -16-1 = -17$

ii) $\lim_{x\to-4^-} \frac{5x-1}{2x} = \frac{-21}{-8} = 21/8$

$\lim_{x\to-4^+} -2x+3 = -5$

$\boxed{\text{Since } \lim_{x\to-4^+} f(x) \neq \lim_{x\to-4^-} f(x),\ \lim_{x\to-4} f(x) = \text{DNE, nonremovable discontinuity at } x=-4}$

b) $g(x) = \begin{cases} \frac{x^2-4}{x+2} & x \neq -2 \\ 3x+2 & x = -2 \end{cases}$ at $x=-2$

i) $g(-2) = -4$

ii) $\lim_{x\to-2} \frac{x^2-4}{x+2} = \lim_{x\to-2} \frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}} = -4$

iii) $g(-2) = \lim_{x\to-2} g(x) = -4$

$\boxed{\text{All 3 conditions pass, } g(x) \text{ continuous at } x=-2}$

Answer the following questions for the piecewise defined function $f(t)$ described on the right hand side.

$$f(t) = \begin{cases} t^2 & \text{for } t < -2 \\ \frac{t+6}{t^2-t} & \text{for } -1 < t < 2 \\ 3t-2 & \text{for } t \geq 2 \end{cases}$$

(a) $f(-3/2) =$ DNE

(b) $f(2) = 4 \qquad 3(2)-2 = 4$

(c) $f(1) =$ DNE $\qquad$ c) $\frac{1+6}{1^2-1} = \frac{7}{0}$

(d) $\lim_{t\to-2} f(t) =$ DNE

(e) $\lim_{t\to-1^+} f(t) = 5/2$

(f) $\lim_{t\to2} f(t) = 4$

f) $\lim_{x\to2^-} \frac{t+6}{t^2-t} = \frac{8}{2} = 4$, $\lim_{x\to2^+} 3t-2 = 4$ $\Big\}\ \lim_{x\to2} f(x) = 4$

(g) $\lim_{t\to0} f(t) =$ DNE

(h) $\lim_{t\to0^-} f(t) = +\infty$

$\lim_{t\to0^-} \frac{t+6}{t^2-t} = \frac{6}{0} \nearrow +\infty$ or $\searrow -\infty$

$\lim_{t\to0^-} \frac{-0.1+6}{(-0.1)^2-(-0.1)} = \frac{+}{+} = \boxed{+\infty}$