1.3a Algebraic Limits p.67-68 #5, 9, 13, 17, 21 37-51 odds

39) $\lim_{x\to c} f(x) = 4$

a) $\lim_{x\to c}[f(x)]^3 = \left[\lim_{x\to c} f(x)\right]^3 = 4^3 = \boxed{64}$

b) $\lim_{x\to c}\sqrt{f(x)} = \sqrt{\lim_{x\to c} f(x)} = \sqrt{4} = \boxed{2}$

c) $\lim_{x\to c}[3f(x)] = 3\lim_{x\to c} f(x) = 3(4) = \boxed{12}$

d) $\lim_{x\to c}[f(x)]^{3/2} = \left[\lim_{x\to c} f(x)\right]^{3/2} = 4^{3/2} = 2^3 = \boxed{8}$

1.3b p.67-69

57) $$\lim_{x\to 0}\frac{\frac{1}{3+x}-\frac{1}{3}}{x} = \lim_{x\to 0}\frac{\frac{3-(3+x)}{3(3+x)}}{x} = \lim_{x\to 0}\frac{\frac{-x}{3(3+x)}}{x}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}$$

$$= \lim_{x\to 0}\frac{-x}{3(3+x)}\cdot\frac{1}{x} = \lim_{x\to 0}\frac{-1}{3(3+x)} = \frac{-1}{3(3+0)} = \frac{-1}{3(3)} = \boxed{\frac{-1}{9}}$$

61) $$\lim_{\Delta x\to 0}\frac{(x+\Delta x)^2-2(x+\Delta x)+1-(x^2-2x+1)}{\Delta x} = \lim_{\Delta x\to 0}\frac{x^2+2x\Delta x+\Delta x^2-2x-2\Delta x+1-x^2+2x-1}{\Delta x}$$

$$\lim_{\Delta x\to 0}\frac{\Delta x(2x+\Delta x-2)}{\Delta x} = 2x+0-2 = \boxed{2x-2}$$

1.3b

84) Find $\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$

$f(x) = -6x + 3$

$f(x+\Delta x) = -6(x+\Delta x) + 3$

$f(x) = -6x + 3$

$$\lim_{\Delta x \to 0} \frac{-6(x+\Delta x)+3 - (-6x+3)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{-6x - 6\Delta x + 3 + 6x - 3}{\Delta x} = \lim_{\Delta x \to 0} \frac{-6\Delta x}{\Delta x} = \boxed{-6}$$

87) $f(x) = \frac{1}{x+3}$ $\quad f(x+\Delta x) = \frac{1}{(x+\Delta x)+3}$

$$\lim_{\Delta x \to 0} \frac{\frac{1}{x+\Delta x+3} - \frac{1}{x+3}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{x+3-(x+\Delta x+3)}{(x+\Delta x+3)(x+3)}}{\Delta x}$$

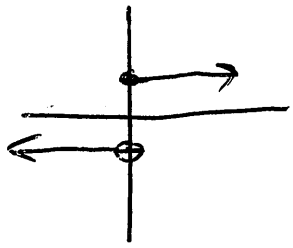
$$\lim_{\Delta x \to 0} \frac{\frac{x+3-x-\Delta x-3}{(x+\Delta x+3)(x+3)} \cdot \frac{1}{\Delta x}}{\Delta x \cdot \frac{1}{\Delta x}} = \lim_{\Delta x \to 0} \frac{-1}{(x+\Delta x+3)(x+3)} = \frac{-1}{(x+0+3)(x+3)} = \boxed{\frac{-1}{(x+3)^2}}$$

97) Squeeze Theorem: If $h(x) \le f(x) \le g(x)$ and $\lim_{x \to c} h(x) = \lim_{x \to c} g(x) = L$, then $\lim_{x \to c} f(x) = L$.

If the outer functions approach the same limit, then the middle "squeezed" function also approaches the same limit (value)

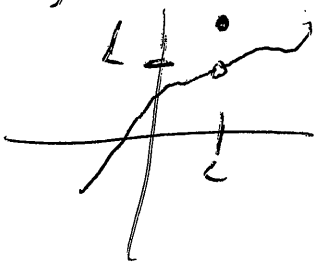
115) $\lim_{x\to 0} \frac{|x|}{x} = 1$ True or False?

*since $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$ then $\frac{|x|}{x} = \begin{cases} \frac{x}{x} = 1, & x \geq 0 \\ \frac{-x}{x} = -1, & x < 0 \end{cases}$



since $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$, limit DNE.

118) If $\lim_{x\to c} f(x) = L$, then $f(c) = L$ not always True



119) $\lim_{x\to 2} f(x) = 3$ $f(x) = \begin{cases} 0 & x > 2 \\ 3 & x \leq 2 \end{cases}$

False