Ch. 1.3a **Evaluating Limits Algebraically**

Key

Rules:

1) a) $\lim_{x \to c} b = b$

2) Suppose $\lim_{x \to c} f(x) = L$ then $\lim_{x \to c} \boldsymbol{b}f(x) = \boldsymbol{b}L$

I. **Direct Substitution Method:** To find limits for a function, first try to evaluate the argument in the expression (plug in the value). If the resulting value is a Real Number, then the value is the limit (answer).

Example 1:

Extension Question: Why do the limits for these problems all produce same value as the function value?

Answer: All are continuous functions: limit and function value are equal

a) $\lim_{x \to 2} x^2 + 3x = 2^2 + 3(2) = \boxed{10}$

b) $\lim_{x \to 2} 5 = \boxed{5}$

c) $\lim_{x \to -1} 3x^5 - 2x^2 + 7x + 4 = 3(-1)^5 - 2(-1)^2 + 7(-1) + 4$
$= -3 - 2 - 7 + 4 = \boxed{-8}$

d) $\lim_{x \to \pi} x\cos x = \pi\cos(\pi) = \pi(-1) = \boxed{-\pi}$

II. **Simplify/Reduction Method** (Factor/Simplify/Substitute) Steps:

1) Evaluate argument First! (plug in value into expression)
2) If direct substitution produces $\frac{0}{0}$ (indeterminate form), we need to evaluate further

*Note: $\frac{0}{0}$ does not mean the Limit is Undefined. $\frac{0}{0}$ just means our problem is incomplete and unfinished. (It's true that the function value is undefined because there's a hole in the graph, but the limit most times does still exist)

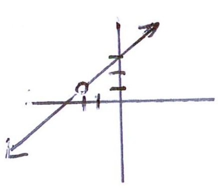
3) Factor/Reduce/Simplify: Try finding common factors in order to reduce expression
4) Using the Reduced expression, re-evaluate the limit
5) Confirm resulting value is now a Real Number, therefore the limit (answer).

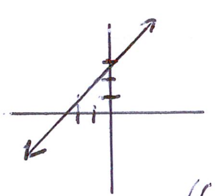
Example 2: Evaluate first!

function value is undefined, but the limit exists! (Graph is a hole)

a) $\lim_{x \to -2} \frac{x^2 + 5x + 6}{x + 2} = \frac{(-2)^2 + 5(-2) + 6}{-2 + 2} \to \frac{0}{0}$

$\lim_{x \to -2} \frac{(x+3)\cancel{(x+2)}}{\cancel{x+2}} \to -2 + 3 = \boxed{1}$





$y = x + 3$ (clone version of original function)

b) $\lim_{x \to 1} \frac{x^2 + 5x + 6}{x - 1} = \frac{1^2 + 5(1) + 6}{1 - 1} \to \frac{12}{0}$

$\boxed{\text{does not exist (d.n.e.)}}$

vertical Asymptote at $x = 1$, so therefore limit does not exist.

Distinguishing variations of zeros: limit still hidden from view

i) $\frac{0}{0} \rightarrow$ indeterminate form $\rightarrow$ hole in graph $\rightarrow$ keep going! to find limit

ii) $\frac{12}{0} \rightarrow$ vertical asymptote $\rightarrow$ limit does not exist

iii) $\frac{0}{4} \rightarrow$ Real Number $\rightarrow \boxed{0}$

Simplify/Reduction Method (Factor/Simplify/Substitute) Steps:

1) Evaluate argument First! (plug in value into expression)
2) If direct substitution produces $\frac{0}{0}$ (indeterminate form), we need to evaluate further
3) Factor/Reduce/Simplify: Try finding common factors in order to reduce expression
4) Using the Reduced expression, re-evaluate the limit
5) Confirm resulting value is now a Real Number, therefore the limit (answer)

Example 2 (continued):

c) $\lim\limits_{x \to 2} \frac{x^2+5x+6}{x+2} = \frac{2^2+5(2)+6}{2+2}$

$\frac{4+10+6}{4} = \frac{20}{4} = \boxed{5}$

d) $\lim\limits_{x \to 2} \frac{x^2-4}{x+2} = \frac{2^2-4}{2+2} \rightarrow \frac{0}{4} = \boxed{0}$

Practice Problems:

1) $\lim\limits_{x \to 1} \frac{2x^2-x-3}{x-1}$ $\quad \frac{2-1-3}{1-1} \rightarrow \frac{-2}{0}$

$\boxed{\text{does not exist}}$

2) $\lim\limits_{x \to 3} \frac{4x^2-7x-2}{x-2} \rightarrow \frac{4(3)^2-7(3)-2}{3-2} = \frac{13}{1}$

$= \boxed{13}$

3) $\lim\limits_{x \to -1} \frac{x^2-1}{x+1} \rightarrow \frac{0}{0}$

$\lim\limits_{x \to -1} \frac{(x-1)\cancel{(x+1)}}{\cancel{(x+1)}} = \boxed{-2}$

4) $\lim\limits_{x \to 1} \frac{x^2-4}{x^2-3x+2}$ $\quad \frac{1-4}{1-3+2} = \frac{-3}{0}$ $\quad$ V.A. at $x=1$

limit $\boxed{\text{dne}}$

(does not exist)

5) $\lim\limits_{x \to -3} \frac{x^2+x-6}{x^2-9}$ $\quad \frac{(-3)^2-3-6}{(-3)^2-9}$

$\frac{9-3-6}{9-9} \rightarrow \frac{0}{0}$ $\quad \lim\limits_{x \to -3} \frac{\cancel{(x+3)}(x-2)}{\cancel{(x+3)}(x-3)}$

$\frac{-3-2}{-3-3} = \frac{-5}{-6} = \boxed{\frac{5}{6}}$

6) $\lim\limits_{x \to 5} \frac{5-x}{x^2-25}$ $\quad \frac{0}{25-25} \rightarrow \frac{0}{0}$

$\lim\limits_{x \to 5} \frac{(5-x)}{(x-5)(x+5)} \rightarrow \lim\limits_{x \to 5} \frac{-1\cancel{(x-5)}}{\cancel{(x-5)}(x+5)}$

$\lim\limits_{x \to 5} \frac{-1}{x+5} \rightarrow \frac{-1}{5+5} = \boxed{\frac{-1}{10}}$