

1.3b Continuity Conditions Practice Worksheet with Piecewise Functions

Key

Continuity Conditions

i. $f(c)$ is defined

ii. $\lim_{x \rightarrow c} f(x)$ exists
 $(\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x))$

iii. $\lim_{x \rightarrow c} f(x) = f(c)$

Use the definition of continuity to determine whether f is continuous at a . Determine type of discontinuity if function is not continuous at a

$$1) f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ 5 & \text{if } x = 2 \end{cases} \rightarrow \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x < 2 \\ 5 & \text{if } x = 2 \\ \frac{x^2 - 4}{x - 2} & \text{if } x > 2 \end{cases}$$

$a = 2$

i) $f(2) = 5$

ii) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} \rightarrow \frac{0}{0} \rightarrow \lim_{x \rightarrow 2} \frac{(x+2)(x-2)}{(x-2)} \rightarrow 2+2 = \boxed{4}$

iii) $f(2) \neq \lim_{x \rightarrow 2} f(x)$, Removable Discontinuity at $x = 2$

$$2) f(x) = \begin{cases} x - 4 & \text{if } x \leq 0 \\ x^2 + x - 4 & \text{if } x > 0 \end{cases}$$

$a = 0$

i) $f(0) = 0 - 4 = -4$

ii) $\lim_{x \rightarrow 0^-} x - 4 = -4$ $\lim_{x \rightarrow 0^+} x^2 + x - 4 = -4 \rightarrow \lim_{x \rightarrow 0} f(x) = -4$

iii) $f(0) = \lim_{x \rightarrow 0} f(x)$

$f(x)$ is continuous at $x = 0$

$$3) f(x) = \begin{cases} 2 - x & \text{if } x < 1 \\ 1 & \text{if } x = 1 \\ x^2 & \text{if } x > 1 \end{cases}$$

$a = 1$

i) $f(1) = 1$

ii) $\lim_{x \rightarrow 1^-} 2 - x = 1$ $\lim_{x \rightarrow 1^+} x^2 = 1 \rightarrow \lim_{x \rightarrow 1} f(x) = 1$

iii) $f(1) = \lim_{x \rightarrow 1} f(x)$

$f(x)$ is continuous at $x = 1$

Find the value of "a" and/or "b" for which the function is continuous.

$$4) f(x) = \begin{cases} 7x-2 & \text{if } x \leq 1 \\ ax^2 & \text{if } x > 1 \end{cases}$$

$$i) f(1) = 7(1) - 2 = 5$$

$$ii) \lim_{x \rightarrow 1^-} 7x - 2 = 5 \quad \lim_{x \rightarrow 1^+} ax^2 = a(1)^2 = a$$

$$\boxed{5 = a}$$

$$5) f(x) = \begin{cases} ax^2 & \text{if } x \leq 2 \\ 2x + a & \text{if } x > 2 \end{cases}$$

$$i) f(2) = a(2)^2 = 4a$$

$$ii) \lim_{x \rightarrow 2^-} ax^2 = a(2)^2 = 4a \quad \lim_{x \rightarrow 2^+} 2x + a = 4 + a$$

$$4a = 4 + a$$

$$3a = 4$$

$$\boxed{a = 4/3}$$

$$6. h(x) = \begin{cases} x^3; x \leq 2 \\ ax^2; x > 2 \end{cases}$$

$$i) h(2) = 2^3 = 8$$

$$ii) \lim_{x \rightarrow 2^-} x^3 = 2^3 = 8 \quad \lim_{x \rightarrow 2^+} ax^2 = a(2)^2 = 4a$$

$$4a = 8$$

$$\boxed{a = 2}$$

$$7. g(x) = \begin{cases} \frac{x^2+3x+2}{x+1}; x \neq -1 \\ a; x = -1 \end{cases}$$

$$i) g(-1) = a$$

$$ii) \lim_{x \rightarrow -1} \frac{x^2+3x+2}{x+1} \rightarrow \frac{0}{0} \rightarrow \lim_{x \rightarrow -1} \frac{(x+2)(x+1)}{(x+1)} \rightarrow 1$$

$$iii) g(-1) = \lim_{x \rightarrow -1} g(x) \rightarrow \boxed{a = 1}$$