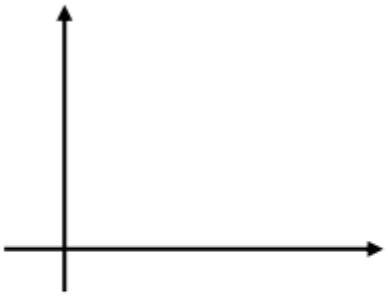
Intermediate Value Theorem (for continuous functions) - IVT



Justification with the IVT.

1. The function $f(x)$ is continuous on an interval [ ].
2. $f(\ ) < f(\ )$ or $f(\ ) > f(\ )$.
3. $f(\ )$ is between $f(\ )$ and $f(\ )$.

Conclusion: "According to the IVT, there is a value such that $f(\ ) =$ ___ and ≤ ≤ ."

Below is a table of values for a continuous function f.

x	0	3	4	8	9
$f(x)$	1	−5	3	7	−1

1. On the interval $0 \le x \le 9$ what is the minimum number of zeros?

2. On the interval $4 \le x \le 9$, what is the fewest possible times $f(x) = 1$?

3. On the interval $0 \le x \le 4$, ***must*** there be a value of x for which $f(x) = 2$? Explain.

4. On the interval $4 \le x \le 8$, ***could*** there be a value of x for which $f(x) = -2$? Explain.

5. Will the function $f(x) = x^2 - x + 1$ ever equal 8 on the interval $[-1, 5]$? Explain.

Use the Intermediate Value Theorem to answer each problem.

16. If $f(x) = 3 - x^2$, will $f(x) = 0$ on the interval $[-2, 1]$? Explain.

17. If $g(x) = \frac{1}{x}$, will $g(x) = -1$ on the interval $[2, 5]$? Explain.

1.4 – Squeeze Theorem

Squeeze Theorem: a.k.a. "Sandwich Theorem" or "Pinching Theorem"

If $(x) \leq (x) \leq (x)$

and if $\lim g(x) =$ and $\lim h(x) =$

then $\lim f(x) =$

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1. Find $\lim_{x\to 0} x^2 \cos\left(\frac{1}{x^2}\right)$

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2. Let g and h be the functions defined by $g(x) = -x^2 + 2x - 3$ and $h(x) = 2x + 1$. If f is a function that satisfies $g(x) \leq f(x) \leq h(x)$ for all x, what is $\lim_{x\to 2} f(x)$?

3. Let g and h be the functions defined by $g(x) = \cos\left(\frac{\pi}{2}x\right) + 2$ and $h(x) = x^2 + 3$. If f is a function that satisfies $g(x) \leq f(x) \leq h(x)$ for $-1 \leq x \leq 5$, what is $\lim_{x\to 0} f(x)$?

4. Let g and h be the functions defined by $g(x) = x^2 - 3x$ and $h(x) = 2 - 2x$. If f is a function that satisfies $g(x) \leq f(x) \leq h(x)$ for all x, what is $\lim_{x\to 2} f(x)$?

5. Let g and h be the functions defined by $g(x) = \cos(\pi(x+2)) - 3$ and $h(x) = \frac{x^2}{2} + x - \frac{7}{2}$. If f is a function that satisfies $g(x) \leq f(x) \leq h(x)$ for $-2 \leq x \leq 0$, what is $\lim_{x\to -1} f(x)$?