Ch. 1.3b (More) Evaluating Limits Algebraically

Recap Steps: **Simplify/Reduction Method** (Factor/Simplify/Substitute) Steps:

1) Evaluate argument First! (plug in value into expression)
2) If direct substitution produces $\frac{0}{0}$ (indeterminate form), we need to evaluate further

*Note: $\frac{0}{0}$ does not mean the Limit is Undefined. $\frac{0}{0}$ just means our problem is incomplete and unfinished. (It's true that the function value is undefined because there's a hole in the graph, but the limit most times does still exist)

3) **Factor/Reduce/Simplify: Try finding common factors in order to reduce expression**
4) Using the Reduced expression, re-evaluate the limit
5) Confirm resulting value is now a Real Number, therefore the limit (answer)

I. Simplify using conjugate method

- If there is a sum or difference of 2 terms in the numerator, then multiply the numerator and denominator by the **conjugate** term.

Example 1: $\lim_{x \to 4} \frac{6-\sqrt{x+32}}{x-4}$

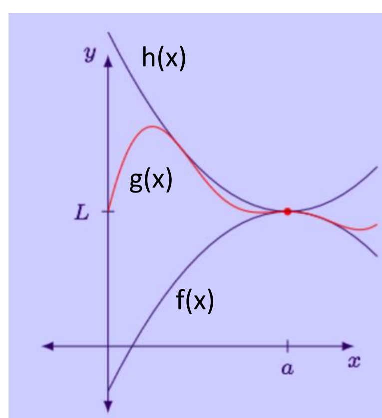
II. Simplify by finding Common Denominator

Example 2: $\lim_{x \to 0} \frac{\frac{1}{x+4}-\frac{1}{4}}{x}$

III. Squeeze Theorem

In the graph below, the lower and upper functions have the same limit value at x=a. The middle function has the same limit value because it is trapped between the two outer functions.

The middle function is "squeezed" to Limit **L** as x approaches **a**



Definition: $Suppose\ f(x) \le g(x) \le h(x)\ for\ all\ x\ in\ an\ open\ interval$

$Suppose\ that\ \lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L\quad Then,\ \lim_{x \to a} g(x) = L$

Example 3: Let $h(x) = 1\ ,\ f(x) = x^2 + 1.\ If\ f(x) \le g(x) \le h(x)$ $find\ \lim_{x \to 0} g(x)$

1.3b Practice Problems:

Simplify/Reduction Method (Factor/Simplify/Substitute) Steps:

a) Evaluate argument First! (plug in value into expression)
b) If direct substitution produces $\frac{0}{0}$ (indeterminate form), we need to evaluate further
c) Factor/Reduce/Simplify: Try finding common factors in order to reduce expression
d) Using the Reduced expression, re-evaluate the limit
e) Confirm resulting value is now a Real Number, therefore the limit (answer)

1)

$$\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$$

2)

$$\lim_{x \to 0} \frac{\frac{1}{3+x} - \frac{1}{3}}{x}$$

3)

$$\lim_{x \to 0} \frac{x^4 - 5x^2}{x^2}$$

4)

$$\lim_{x \to 16} \frac{4 - \sqrt{x}}{x - 16}$$

5)

$$\lim_{x \to 4} \frac{x^2 + 7x - 44}{x^2 - 6x + 8}.$$

6)

$$\lim_{x \to 0} \frac{\sqrt{1 + 2x} - 1}{3x}$$

7)

$$\lim_{x \to 0} \frac{\frac{1}{4} + \frac{1}{x - 4}}{x}$$

8)

$$\lim_{x \to 5} \frac{\sqrt{x - 1} - 2}{x^2 - 25}$$