Ch. 1.3b More Evaluating Limits Notes

Key

A. Simplify using conjugate expression

* If there is a sum or difference of 2 terms in numerator, multiply numerator and denominator by the conjugate.

$\boxed{\text{Ex. 1}}$ $\lim_{x\to4} \dfrac{6-\sqrt{x+32}}{x-4}$ $\quad \dfrac{0}{0}$

$$\lim_{x\to4} \frac{(6-\sqrt{x+32})}{(x-4)} \cdot \frac{(6+\sqrt{x+32})}{(6+\sqrt{x+32})} = \lim_{x\to4} \frac{\overset{36-x-32}{36-(x+32)}}{(x-4)(6+\sqrt{x+32})}$$

$$\lim_{x\to4} \frac{(-x+4)}{(x-4)(6+\sqrt{x+32})} = \lim_{x\to4} \frac{-\cancel{(x-4)}}{\cancel{(x-4)}(6+\sqrt{x+32})} = \lim_{x\to4} \frac{-1}{6+\sqrt{x+32}} = \boxed{\frac{-1}{12}}$$

B. Simplify by finding common denominator

$\boxed{\text{Ex. 2}}$ $\lim_{x\to0} \dfrac{\boxed{\frac{1}{x+4} - \frac{1}{4}}}{x} = \dfrac{0}{0}$

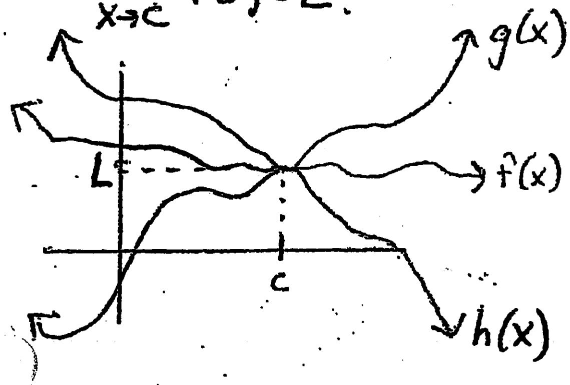
$$\lim_{x\to0} \frac{\;\frac{4-(x+4)}{4(x+4)} \cdot \frac{1}{x}\;}{\cancel{x} \cdot \frac{1}{\cancel{x}}}$$

$$\lim_{x\to0} \frac{4-(x+4)}{4(x+4)} \cdot \frac{1}{x}$$

$$\lim_{x\to0} \frac{\cancel{4}-x-\cancel{4}}{4(x+4)\cdot \cancel{x}}$$

$$\lim_{x\to0} \frac{-1}{4(x+4)} = \boxed{-\frac{1}{16}}$$

C. Squeeze Theorem: If $\boxed{h(x) \le f(x) \le g(x)}$ for all x in an open interval containing c, and if $\lim_{x\to c} h(x) = L = \lim_{x\to c} g(x)$, then $\lim_{x\to c} f(x) = L$.



$\boxed{\text{Ex. 3}}$ Let $h(x) = 1$, $g(x) = x^2+1$.
If $h(x) \le f(x) \le g(x)$, find $\boxed{\lim_{x\to0} f(x)}$

$$\lim_{x\to0} 1 = 1$$

$$\lim_{x\to0} x^2+1 = 1$$

By squeeze theorem $\lim_{x\to0} f(x) = 1$

$\boxed{\text{Ex. 4}}$ $\lim\limits_{x \to 5} \dfrac{\frac{2}{x+3} - \frac{1}{4}}{x-5}$

$\lim\limits_{x \to 5} \dfrac{\frac{2}{x+3} - \frac{1}{4}}{x-5} = \dfrac{0}{0}$ $\quad \lim\limits_{x \to 5} \dfrac{\frac{8-(x+3)}{4(x+3)} \cdot \frac{1}{x-5}}{\cancel{x-5} \cdot \frac{1}{\cancel{x-5}}}$ $\quad \lim\limits_{x \to 5} \dfrac{8-x-3}{4(x+3)} \cdot \dfrac{1}{x-5}$

$\lim\limits_{x \to 5} \dfrac{(5-x)}{4(x+3)(x-5)} = \lim\limits_{x \to 5} \dfrac{-1(\cancel{x-5})}{4(x+3)(\cancel{x-5})} = \lim\limits_{x \to 5} \dfrac{-1}{4(x+3)} = \boxed{\dfrac{-1}{32}}$

$\boxed{\text{Ex. 5}}$ $\lim\limits_{x \to 3} \dfrac{\sqrt{x+6} - 3}{x-3} =$

$\lim\limits_{x \to 3} \dfrac{\sqrt{x+6} - 3}{x-3} = \dfrac{0}{0}$

$\lim\limits_{x \to 3} \dfrac{(\sqrt{x+6} - 3)}{(x-3)} \cdot \dfrac{(\sqrt{x+6} + 3)}{(\sqrt{x+6} + 3)}$

$\lim\limits_{x \to 3} \dfrac{x+6-9}{(x-3)(\sqrt{x+6}+3)} = \lim\limits_{x \to 3} \dfrac{(\cancel{x-3})}{(\cancel{x-3})(\sqrt{x+6}+3)} = \boxed{\dfrac{1}{6}}$

$\lim\limits_{x \to 3} \dfrac{1}{\sqrt{x+6}+3} = \dfrac{1}{\sqrt{9}+3} = \dfrac{1}{3+3} = \boxed{\dfrac{1}{6}}$