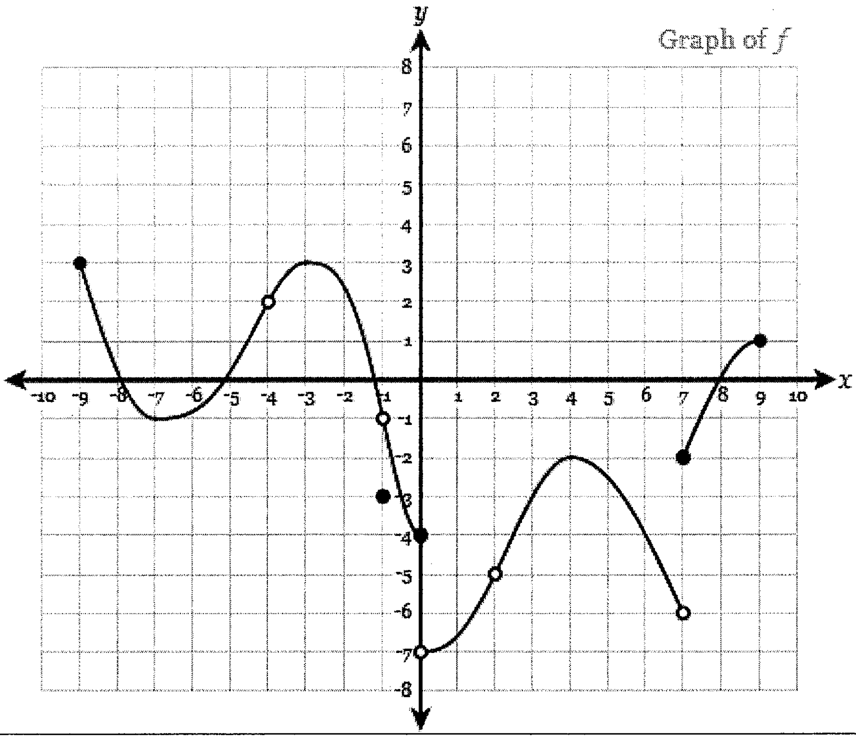
Non-AP Calculus **1.4-1.5 Continuity/IVT/Limits Classwork Problems**

Non-Removable discontinuity: point where graph is not continuous and Limit does not exist

Removable Discontinuity: point where graph is not continuous but the limit exists

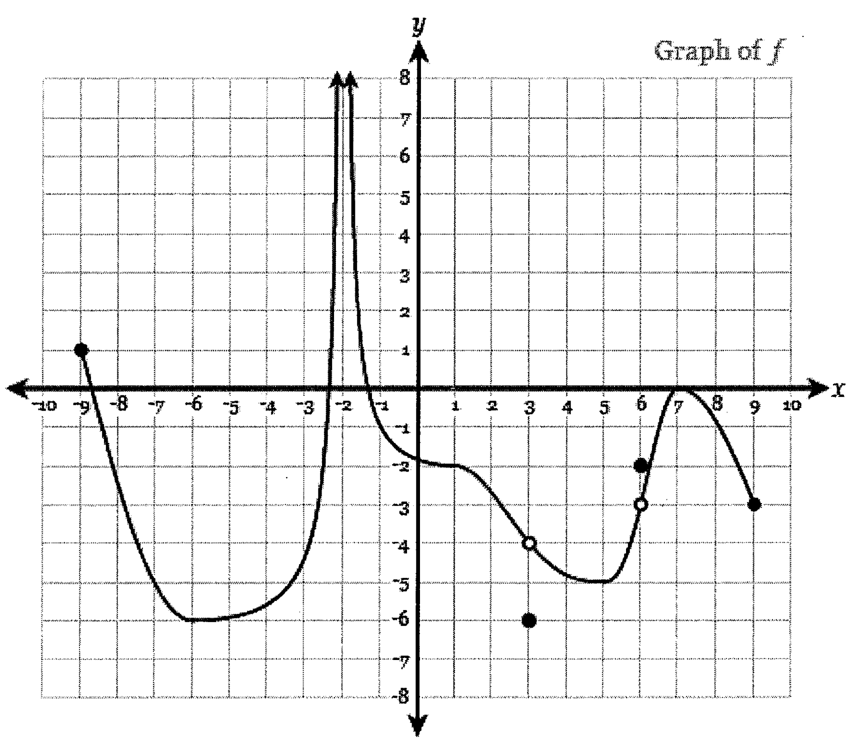
1) Identify values of x and determine the types of discontinuity for the below graph:



Non-Removable Discontinuity:

Removable Discontinuity:

2) Identify values of x and determine the types of discontinuity for the below graph:



Non-Removable Discontinuity:

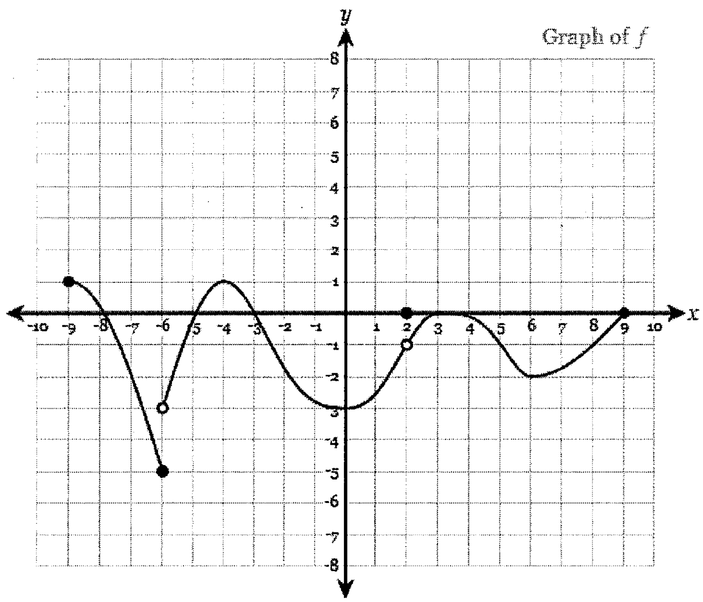
Removable Discontinuity:

Continuity Conditions

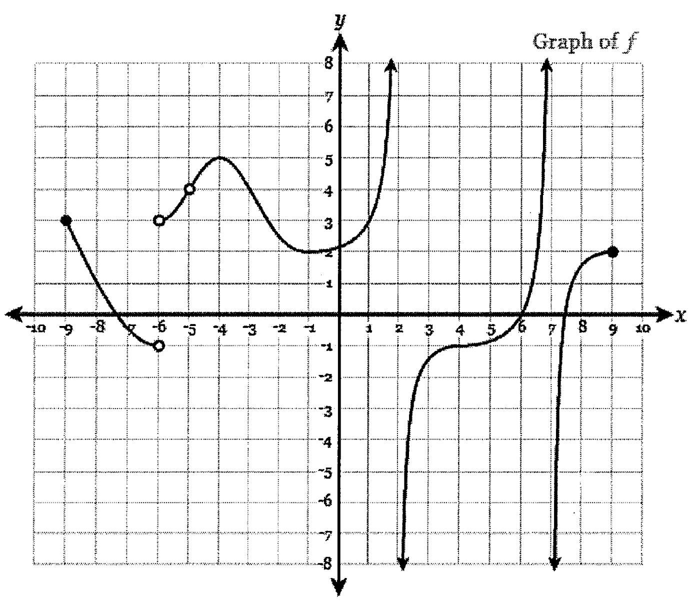
i) f(c) is defined (point exists on the graph)

ii) The $\lim_{x \to c} f(x)$ exists $\left[\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x) \right]$

iii) f(c) = $\lim_{x \to c} f(x)$



3) Use the definition of continuity to determine whether the function f(x) graphed below is continuous at x= 2.



4) Use the definition of continuity to determine whether the function f(x) graphed below is continuous at x= -6.

Continuity Conditions

i) f(c) is defined (point exists on the graph)

ii) The $\lim_{x \to c} f(x)$ exists $\left[\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x)\right]$

iii) f(c) = $\lim_{x \to c} f(x)$

Use Continuity Conditions to show that f(x) is discontinuous at a point and state reason for discontinuity. Then determine if the discontinuity is removable or non-removable and state why.

5)

$$f(x) = \begin{cases} x^2 - 4x + 6, & x < 3 \\ -x^2 + 4x - 2, & x \geq 3 \end{cases}$$

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6)

$$f(x) = \begin{cases} x, & x \leq 1 \\ 1 - x, & x > 1 \end{cases}$$

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7)

$$f(x) = \begin{cases} \dfrac{x + 2}{2}, & x \leq 3 \\ \dfrac{12 - 2x}{3}, & x > 3 \end{cases}$$

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8)

$$f(x) = \begin{cases} x^2 - 4x + 6, & x < 3 \\ -x^2 + 4x - 2, & x \geq 3 \end{cases}$$

Using the Intermediate Value Theorem **In Exercises 95–98, verify that the Intermediate Value Theorem applies to the indicated interval and find the value of *c* guaranteed by the theorem.**

95. $f(x) = x^2 + x - 1, \quad [0, 5], \quad f(c) = 11$

96. $f(x) = x^2 - 6x + 8, \quad [0, 3], \quad f(c) = 0$

97. $f(x) = x^3 - x^2 + x - 2, \quad [0, 3], \quad f(c) = 4$

Find the following:

1) $\lim_{x\to -3^-} \frac{9-x^2}{x-4} =$

2) $\lim_{x\to 0^-} \frac{5x-x^2}{x^2-x} =$

3) $\lim_{x\to -2^-} \frac{x^2+1}{x+2} =$

4) $\lim_{x\to 5} \frac{3x^2-1}{25-x^2} =$

5) $\lim_{x\to -3^+} \frac{2x^2+3x-9}{x+3}$

6) $\lim_{x\to -4^+} \frac{2x^2-1}{x^2-16}$

7) $\lim_{x\to 1^+} \frac{x^2-2}{x^2+2x+1} =$

8) $\lim_{x\to 3^+} \frac{4x^2-14x+6}{x-3} =$

Finding a One-Sided Limit In Exercises 33–48, find the one-sided limit (if it exists).

33. $\lim_{x \to -1^+} \frac{1}{x + 1}$

34. $\lim_{x \to 1^-} \frac{-1}{(x - 1)^2}$

35. $\lim_{x \to 2^+} \frac{x}{x - 2}$

36. $\lim_{x \to 2^-} \frac{x^2}{x^2 + 4}$

37. $\lim_{x \to -3^-} \frac{x + 3}{x^2 + x - 6}$

50. $f(x) = \frac{x^3 - 1}{x^2 + x + 1}$

$\lim_{x \to 1^-} f(x)$

51. $f(x) = \frac{1}{x^2 - 25}$

$\lim_{x \to 5^-} f(x)$

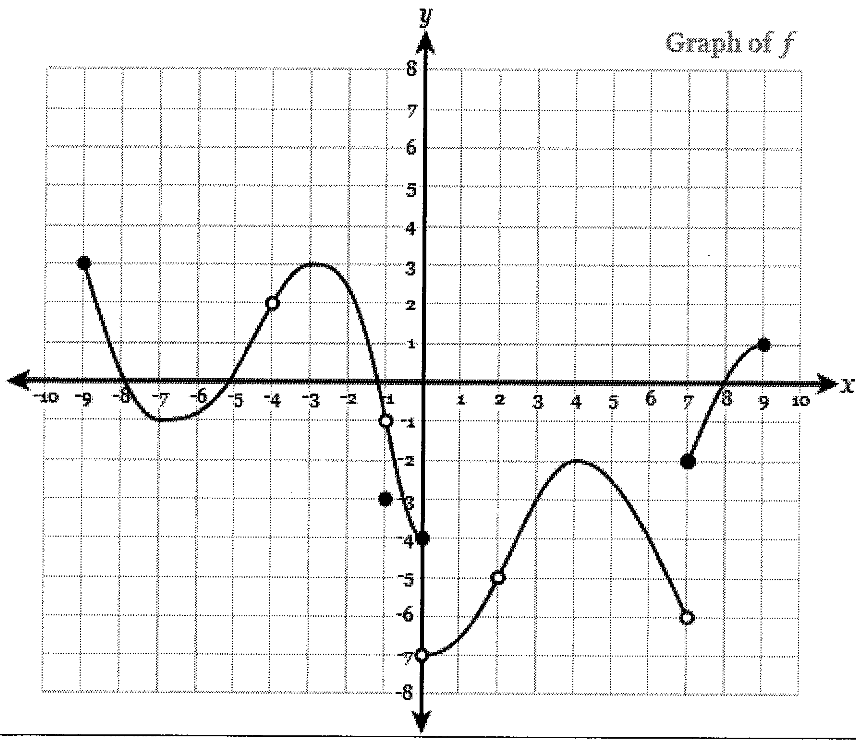
Key

Non-AP Calculus 1.4-1.5 Continuity/IVT/Limits Classwork Problems

Non-Removable discontinuity: point where graph is not continuous and Limit does not exist

Removable Discontinuity: point where graph is not continuous but the limit exists

1) Identify values of x and determine the types of discontinuity for the below graph:



Non-Removable Discontinuity:

$x = 0$

$x = 7$

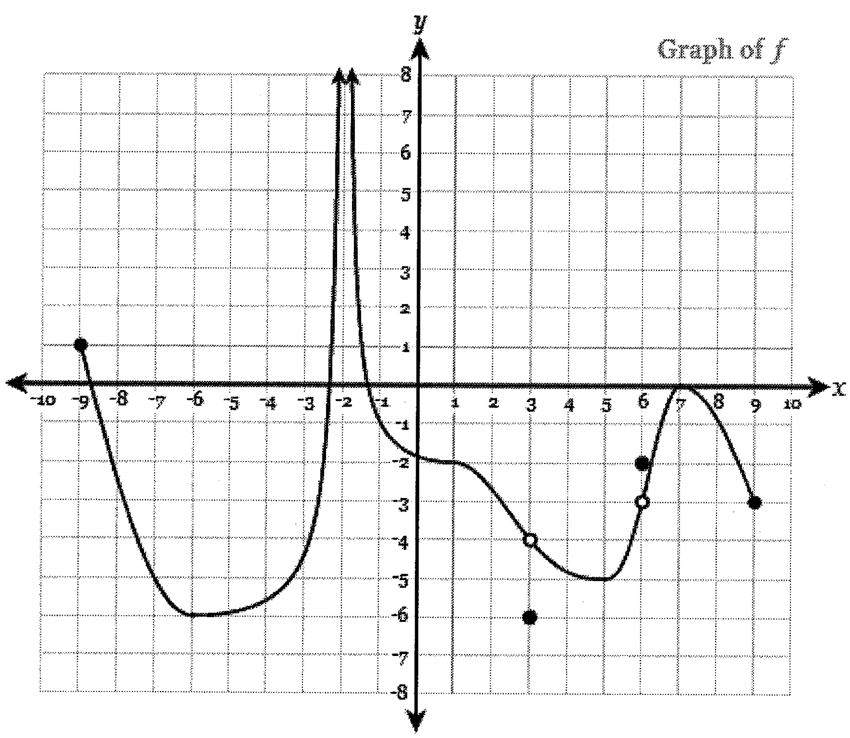
Removable Discontinuity:

$x = -4$

$x = -1$

$x = 2$

2) Identify values of x and determine the types of discontinuity for the below graph:



Non-Removable Discontinuity:

$x = -2$

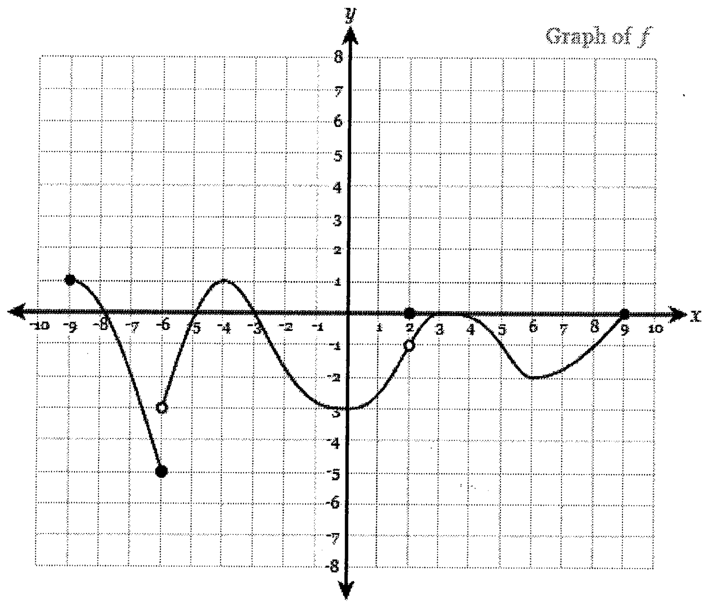
Removable Discontinuity:

$x = 3$

$x = 6$

Continuity Conditions

1. f(c) is defined (point exists on the graph)
2. The $\lim_{x\to c} f(x)$ exists $[\lim_{x\to c^+} f(x) = \lim_{x\to c^-} f(x)]$
3. $f(c) = \lim_{x\to c} f(x)$



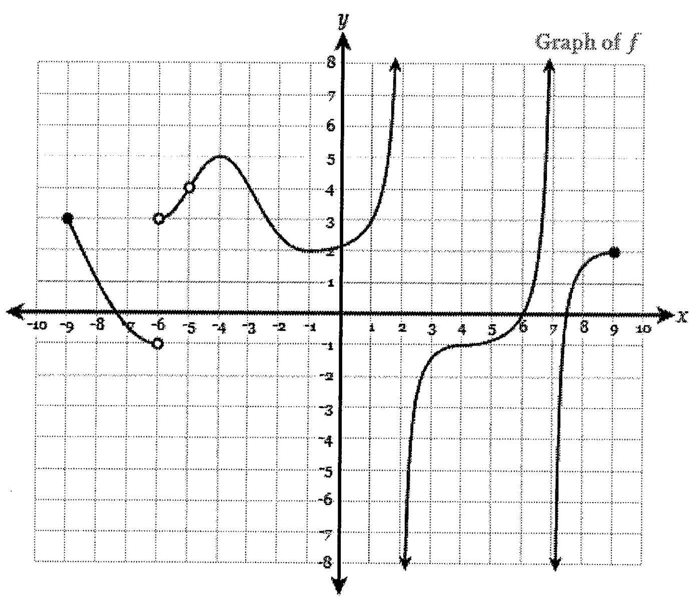
3) Use the definition of continuity to determine whether the function f(x) graphed below is continuous at x= 2.

i) $f(2)=0$

ii) $\lim_{x\to 2} f(x) = -1$

iii) $f(2) \neq \lim_{x\to 2} f(x)$

Removable discontinuity at $x=2$



4) Use the definition of continuity to determine whether the function f(x) graphed below is continuous at x= -6.

i) $f(-6)$ = undefined

ii) $\lim_{x\to -6^-} f(x) = -1$ $\lim_{x\to -6^+} f(x) = 3$

$\lim_{x\to -6} f(x)$ does not exist

Nonremovable discontinuity at $x=-6$

Continuity Conditions

i) f(c) is defined (point exists on the graph)
ii) The $\lim_{x \to c} f(x)$ exists [$\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x)$]
iii) f(c) = $\lim_{x \to c} f(x)$

Use Continuity Conditions to show that f(x) is discontinuous at a point and state reason for discontinuity. Then determine if the discontinuity is removable or non-removable and state why.

5)

$$f(x) = \begin{cases} x^2 - 4x + 6, & x < 3 \\ -x^2 + 4x - 2, & x \geq 3 \end{cases}$$

i) $f(3) = -3^2 + 4(3) - 2 = 1$

ii) $\lim_{x \to 3^-} x^2 - 4x + 6 = 3$ $\lim_{x \to 3^+} -x^2 + 4x - 2 = 1$

$\lim_{x \to 3} f(x)$ does not exist

Nonremovable discontinuity at $x = 3$

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6)

$$f(x) = \begin{cases} x, & x \leq 1 \\ 1 - x, & x > 1 \end{cases}$$

i) $f(1) = 1$

ii) $\lim_{x \to 1^-} x = 1$ $\lim_{x \to 1^+} 1 - x = 0$, $\lim_{x \to 1} f(x)$ does not exist

Nonremovable discontinuity at $x = 1$

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7)

$$f(x) = \begin{cases} \frac{x + 2}{2}, & x \leq 3 \\ \frac{12 - 2x}{3}, & x > 3 \end{cases}$$

i) $f(3) = \frac{3+2}{2} = \frac{5}{2}$

ii) $\lim_{x \to 3^-} \frac{x+2}{2} = \frac{5}{2}$ $\lim_{x \to 3^+} \frac{12 - 2(3)}{3} = \frac{6}{3} = 2$

$\lim_{x \to 3} f(x)$ does not exist

Nonremovable discontinuity at $x = 3$

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8)

$$f(x) = \begin{cases} x^2 - 4x + 6, & x < 3 \\ -x^2 + 4x - 2, & x \geq 3 \end{cases}$$

i) $f(3) = 1$

ii) $\lim_{x \to 3} f(x)$ dne

Nonremovable discontinuity at $x = 3$

Using the Intermediate Value Theorem **In Exercises 95–98, verify that the Intermediate Value Theorem applies to the indicated interval and find the value of c guaranteed by the theorem.**

95. $f(x) = x^2 + x - 1, \quad [0, 5], \quad f(c) = 11$

$f(x)$ continuous $[0,5]$

$f(0) = -1$

$f(5) = 29$

By IVT, since $f(0) < 11 < f(5)$
$f(c) = 11$ in interval $[0,5]$

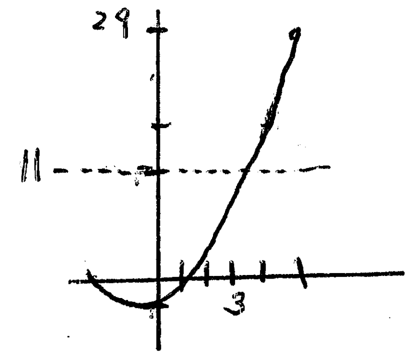
$x^2 + x - 1 = 11$

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$\cancel{x=-4}, \ x = 3$

$\boxed{c = 3}$



96. $f(x) = x^2 - 6x + 8, \quad [0, 3], \quad f(c) = 0$

$f(x)$ continuous $[0,3]$

$f(0) = 8$

$f(3) = -1$

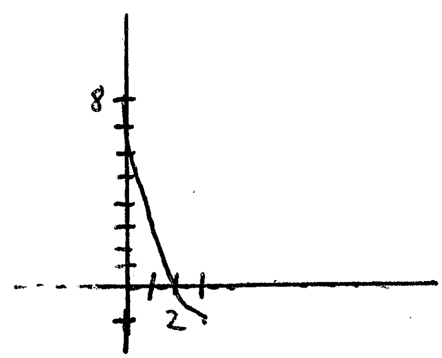
By IVT, since $f(3) < 0 < f(0)$,
$f(c) = 0$ in interval $[0,3]$

$x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

$\cancel{x=4}, \ x = 2$

$\boxed{c = 2}$



97. $f(x) = x^3 - x^2 + x - 2, \quad [0, 3], \quad f(c) = 4$

$f(x)$ continuous $[0,3]$

$f(0) = -2$

$f(3) = 19$

By IVT, since $f(0) < 4 < f(3)$
$f(c) = 4$ in interval $[0,3]$

$x^3 - x^2 + x - 2 = 4$

$x^3 - x^2 + x - 6 = 0$

$(x-2)(x^2+x+3) = 0$

$x - 2 = 0$ | $x^2 + x + 3 = 0$

$x = 2$ | no solution

$\boxed{c = 2}$

Find the following:

1) $\lim_{x\to -3^-} \frac{9-x^2}{x-4} = \frac{9-(-3)^2}{-3-4} = \frac{0}{-7}$

$\boxed{0}$

2) $\lim_{x\to 0^-} \frac{5x-x^2}{x^2-x} = \frac{0}{0}$

$\lim_{x\to 0^-} \frac{\cancel{x}(5-x)}{\cancel{x}(x-1)} \to \lim_{x\to 0^-} \frac{5-x}{x-1} = \frac{5-0}{0-1} \quad \frac{5}{-1}$

$= \boxed{-5}$

3) $\lim_{x\to -2^-} \frac{x^2+1}{x+2} = \frac{(-2)^2+1}{-2+2} = \frac{5}{0}$

VA., Limit DNE $\nearrow +\infty$, $\searrow -\infty$

test $x=-2.1$

$\frac{(-2.1)^2+1}{-2.1+2} = \frac{+}{-} = \boxed{-\infty}$

4) $\lim_{x\to 5} \frac{3x^2-1}{25-x^2} = \frac{3(5)^2-1}{25-5^2} = \frac{74}{0}$ $\nearrow +\infty$, $\searrow -\infty$

$\boxed{\text{limit does not exist}}$

5) $\lim_{x\to -3^+} \frac{2x^2+3x-9}{x+3}$ $\quad 2(-3)^2-9-9 = \frac{0}{0}$

$\lim_{x\to -3^+} \frac{(2x-3)\cancel{(x+3)}}{\cancel{(x+3)}} = 2(-3)-3 = \boxed{-9}$

6) $\lim_{x\to -4^+} \frac{2x^2-1}{x^2-16} = \frac{2(-4)^2-1}{(-4)^2-16} = \frac{31}{0}$ $\nearrow +\infty$, $\searrow -\infty$

$\frac{2(-3.9)^2-1}{(-3.9)^2-16} = \frac{+}{-} = \boxed{-\infty}$

7) $\lim_{x\to 1^+} \frac{x^2-2}{x^2+2x+1} = \frac{1-2}{1^2+2+1} = \boxed{\frac{-1}{4}}$

8) $\lim_{x\to 3^+} \frac{4x^2-14x+6}{x-3} = \frac{4(9)-14(3)+6}{3-3} \quad \frac{0}{0}$

$\lim_{x\to 3^+} \frac{2(2x^2-7x+3)}{(x-3)}$

$\lim_{x\to 3^+} \frac{2(2x-1)\cancel{(x-3)}}{\cancel{(x-3)}} = 2(2(3)-1) = 2(5)$

$= \boxed{10}$

Finding a One-Sided Limit **In Exercises 33–48, find the one-sided limit (if it exists).**

33. $\lim_{x \to -1^+} \frac{1}{x+1}$

V.A. $\frac{1}{0}$ $+\infty$ $-\infty$

-0.9

$\frac{1}{-0.9+1} = \frac{+}{+} = \boxed{+\infty}$

34. $\lim_{x \to 1^-} \frac{-1}{(x-1)^2}$ $\rightarrow \frac{-1}{0^2}$ V.A. $+\infty$ $-\infty$

$x=0.9$

$\frac{-1}{(0.9-1)^2} = \frac{-}{+} = \boxed{-\infty}$

35. $\lim_{x \to 2^+} \frac{x}{x-2}$ $\rightarrow \frac{2}{0}$ VA $+\infty$ $-\infty$

test $x=2.1$

$\frac{2.1}{2.1-2} = \frac{+}{+} = \boxed{+\infty}$

36. $\lim_{x \to 2^-} \frac{x^2}{x^2+4} = \frac{2^2}{2^2+4} = \frac{4}{8} = \boxed{\frac{1}{2}}$

37. $\lim_{x \to -3^-} \frac{x+3}{x^2+x-6}$ $\frac{-3+3}{3^2+-3-6} = \frac{0}{0}$

$\lim_{x \to -3^-} \frac{\cancel{(x+3)}}{\cancel{(x+3)}(x-2)}$

$\lim_{x \to -3^-} \frac{1}{x-2} = \frac{1}{-3-2} = \boxed{\frac{-1}{5}}$

50. $f(x) = \frac{x^3-1}{x^2+x+1}$

$\lim_{x \to 1^-} f(x)$

$\lim_{x \to 1^-} \frac{x^3-1}{x^2+x+1} = \frac{1-1}{1+1+1} = \frac{0}{3} = \boxed{0}$

51. $f(x) = \frac{1}{x^2-25}$

$\lim_{x \to 5^-} f(x)$

$\lim_{x \to 5^-} \frac{1}{x^2-25} = \frac{1}{0}$ VA $+\infty$ $-\infty$

test $x=4.9$

$\frac{1}{4.9^2-25} = \frac{+}{-} = \boxed{-\infty}$