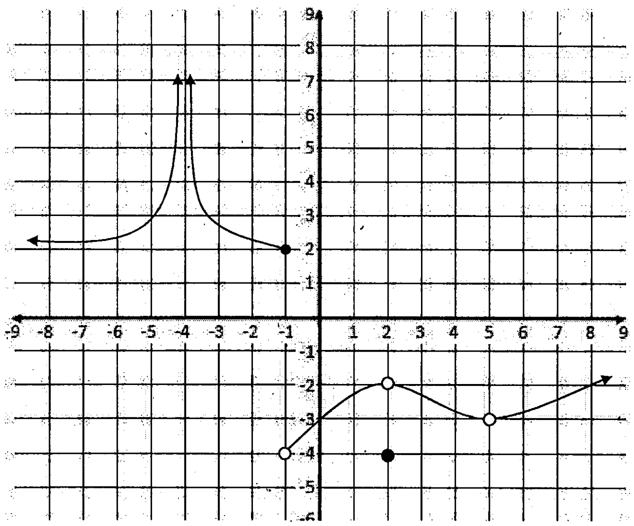
A.P. Calculus AB **Quiz 1-4, 1-5, 3-5** Key A

No calculators!

Name ______ Date ______ Per. ______ 30pts



1. The graph of $f(x)$ (shown above) is discontinuous at x = -4, -1, 2, and 5. The three conditions for discontinuity are given below to the right of the table. For each value of x in the table, write the letter (A, B, or C) that matches why $f(x)$ is discontinuous at that x-value. Then state if the discontinuity is removable or nonremovable. (1 point each blank)

x	Why discontinuous? (write the letter)	Removable or nonremovable?
-4	B	Nonremovable
-1	A	Nonremovable
2	C	Removable
5	B	Removable

8

A. $\lim_{x \to c} f(x)$ DNE

B. $f(c)$ is undefined

C. $\lim_{x \to c} f(x) \neq f(c)$

2. (5 points) If $g(x) = \begin{cases} \dfrac{x^2 - 9}{x+3} & , x \neq -3 \\ 1 - x^2 & , x = -3 \end{cases}$, state the x-value at which $g(x)$ is discontinuous and why. (State Continuity conditions!) Then determine if the discontinuity is removable or non-removable and state why.

5

i) $g(-3) = -8$

ii) $\lim_{x \to -3} \frac{x^2-9}{x+3}$ $\frac{0}{0}$ $\rightarrow \lim_{x \to -3} \frac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}} = -6$

iii) $g(-3) \neq \lim_{x \to -3} g(x)$ Removable Discontinuity at $x = -3$

13

3. (4 pts) Show whether Intermediate Value Theorem applies. Then find all numbers $c \in [a, b]$ for which $f(c) = k$. (Step through conditions!)

$f(x) = x^2 - 4x + 4$; $[-2, 4]$, $k = 9$ $c =$ -1

$f(x)$ continuous $[-2,4]$

$f(-2) = 16$

$f(4) = 4$

By IVT, $f(c) = 9$ in $[-2,4]$

$9 = x^2 - 4x + 4$

$0 = x^2 - 4x - 5$

$0 = (x-5)(x+1)$

$x = 5, -1$

$\boxed{c = -1}$

4

4. Find the following. (1 point each blank)

$$h(x) = \begin{cases} \frac{11+x^2}{3x^4}, & x < -1 \\ 15, & x = -1 \\ \frac{9-3x}{5x+1}, & x > -1 \end{cases}$$

$\lim_{x \to -1^-} \frac{11+x^2}{3x^4} = \frac{12}{3} = 4$

$\lim_{x \to -1^+} \frac{9-3x}{5x+1} = \frac{12}{-4}$

$\lim_{x \to -1^-} h(x) =$ 4

$\lim_{x \to -1^+} h(x) =$ -3

$h(-1) =$ 15

$\lim_{x \to -1} h(x) =$ DNE

$\lim_{x \to \infty} h(x) =$ $-\frac{3}{5}$

$\lim_{x \to \infty} \frac{9-3x}{5x+1}$

$\lim_{x \to -\infty} h(x) =$ 0

$\lim_{x \to -\infty} \frac{11+x^2}{3x^4} = 0$

6

5. (1 point each blank) Let $f(x) = \frac{3x^2-100}{x^2-4}$ and find the following:

a) $\lim_{x \to 2^-} f(x) =$ 1.9

$\lim_{x \to 2^-} \frac{3x^2-100}{x^2-4} = \frac{-}{-} = \boxed{+\infty}$

b) $\lim_{x \to 2^+} f(x) =$ $\boxed{-\infty}$

$\lim_{x \to 2^+} \frac{3x^2-100}{x^2-4} = \frac{-}{+} = -$

c) $\lim_{x \to 2} f(x) =$ DNE

d) $\lim_{x \to -\infty} f(x) =$ $= \boxed{3}$

$\lim_{x \to -\infty} \frac{3x^2-100}{x^2-4} = \frac{3}{1}$

e) $\lim_{x \to \infty} f(x) =$ $= \boxed{3}$

$\lim_{x \to \infty} \frac{3x^2-100}{x^2-4} = \frac{3}{1}$

5

6. (2 points) Find all horizontal asymptotes of $f(x) = \frac{5x-7}{\sqrt{9x^2+8x-25}}$ (Show appropriate work!)

$\lim_{x \to \infty} \frac{5x}{\sqrt{9x^2}}$

$\boxed{y = \frac{5}{3},\ y = -\frac{5}{3}}$

2

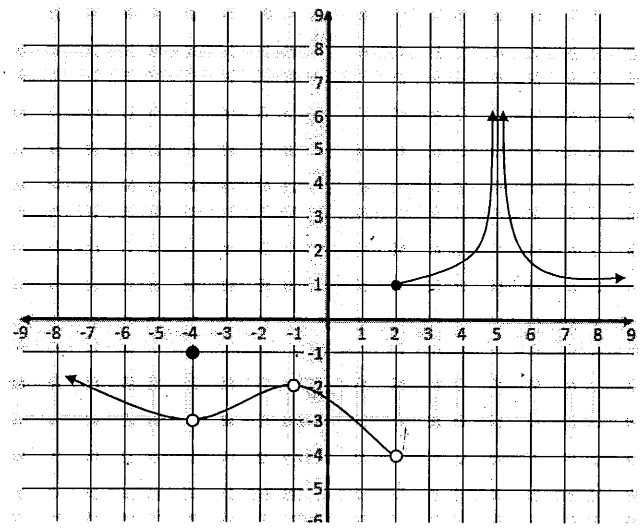
17

30pts

A.P. Calculus AB **Quiz 1-4, 1-5, 3-5**

No calculators!

Name Key B Date Per.



1. The graph of $f(x)$ (shown above) is discontinuous at $x = -4, -1, 2$, and 5. The three conditions for discontinuity are given below to the right of the table. For each value of x in the table, write the letter (A, B, or C) that matches why $f(x)$ is discontinuous at that x-value. Then state if the discontinuity is removable or nonremovable. (1 point each blank)

x	Why discontinuous? (write the letter)	Removable or nonremovable?
-4	C	Removable
-1	B	Removable
2	A	Non Removable
5	B	Nonremovable

A. $\lim_{x\to c} f(x)$ DNE

B. $f(c)$ is undefined

C. $\lim_{x\to c} f(x) \neq f(c)$

2. (5 points) If $g(x) = \begin{cases} \dfrac{x^2-16}{x+4}, & x \neq -4 \\ 2-x^2, & x=-4 \end{cases}$, state the x-value at which $g(x)$ is discontinuous and <u>why</u>. (Use Continuity conditions!) Then determine if the discontinuity is removable or non-removable and <u>state why</u>.

i) $g(-4) = 2-(-4)^2 = -14$

ii) $\lim\limits_{x\to -4} \dfrac{x^2-16}{x+4} = \dfrac{0}{0}$ $\lim\limits_{x\to -4} \dfrac{\cancel{(x+4)}(x-4)}{\cancel{(x+4)}} = -8$

iii) $g(-4) \neq \lim\limits_{x\to -4} g(x)$ Removable Discontinuity at $x=-4$

13

3. (4 pts) Show whether Intermediate Value Theorem applies. Then find all numbers $c \in [a, b]$ for which $f(c) = k$. (Step through conditions!)

$f(x) = 3x^2 + x - 2$; $[-1, 2]$, $k = 8$ $c =$ $5/3$

$f(x)$ continuous $[-1, 2]$

$f(-1) = 0$

$f(2) = 12$

By IVT, $f(c) = 8$ in $[-1, 2]$

$f(c) = 8$

$8 = 3x^2 + x - 2$

$0 = 3x^2 + x - 10$

$0 = (3x - 5)(x + 2)$

$x = 5/3, -2$

4. Find the following. (1 point each blank) $h(x) = \begin{cases} \frac{7+x^2}{4x^4}, & x < -1 \\ 10, & x = -1 \\ \frac{9-5x}{8x+1}, & x > -1 \end{cases}$

$\lim_{x \to -1^-} \frac{7+x^2}{4x^4} = \frac{7+1}{4} = \frac{8}{4} = 2$ $\lim_{x \to -1^+} \frac{9-5x}{8x+1} = \frac{9+5}{-8+1}$

$\lim_{x \to -1^-} h(x) =$ 2 $\lim_{x \to -1^+} h(x) =$ -2 $h(-1) =$ 10

$\lim_{x \to \infty} \frac{9-5x}{8x+1}$

$\lim_{x \to -1} h(x) =$ DNE $\lim_{x \to \infty} h(x) =$ $-\frac{5}{8}$ $\lim_{x \to -\infty} h(x) =$ 0

$\lim_{x \to -\infty} \frac{7+x^2}{4x^4} = 0$

5. (1 points each blank) Let $f(x) = \frac{5x^2-100}{(x-2)^2}$ and find the following:

a) $\lim_{x \to 2^-} f(x) =$ 1.9 b) $\lim_{x \to 2^+} f(x) = \boxed{-\infty}$ c) $\lim_{x \to 2} f(x) =$ DNE $(-\infty)$

$\lim_{x \to 2^-} \frac{5x^2-100}{(x-2)^2} = \frac{5(x^2-25)}{(x-2)^2}$ $\frac{-}{+}$ $\boxed{-\infty}$ $\frac{5(2.1)^2-100}{(2.1-2)^2} = \frac{-}{+} = \boxed{-\infty}$

d) $\lim_{x \to -\infty} f(x) =$ 5 e) $\lim_{x \to \infty} f(x) =$ 5

$\lim_{x \to -\infty} \frac{5x^2-100}{x^2-4x+4} =$

6. (2 points) Find all horizontal asymptotes of $f(x) = \frac{7x-2}{\sqrt{16x^2+8x-25}}$ (Show appropriate work!)

$\lim_{x \to +\infty} \frac{7x-2}{\sqrt{16x^2+8x-25}} = \frac{7}{4}$

$\boxed{y = 7/4, y = -7/4}$

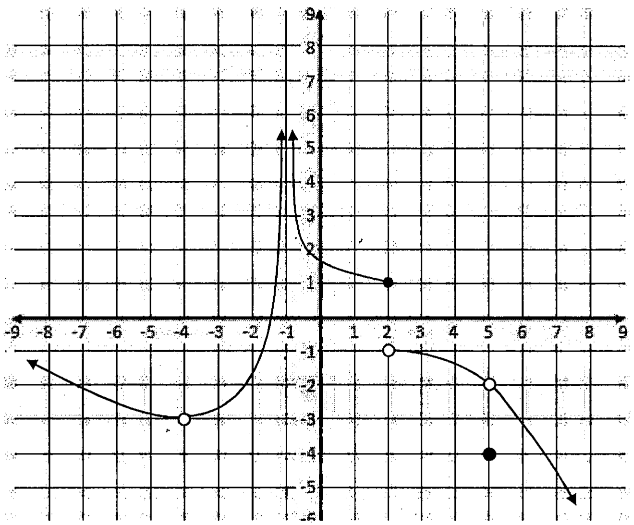
$\lim_{x \to -\infty} f(x) = -\frac{7}{4}$

17

30 pts

A.P. Calculus AB **Quiz 1-4, 1-5, 3-5** Name Key C Date Period

No calculators!



1. The graph of $f(x)$ (shown above) is discontinuous at x = -4, -1, 2, and 5. The three conditions for discontinuity are given below to the right of the table. For each value of x in the table, write the letter (A, B, or C) that matches why $f(x)$ is discontinuous at that x-value. Then state if the discontinuity is removable or nonremovable. (1 point each blank)

x	Why discontinuous? (write the letter)	Removable or nonremovable?
-4	B	Removable
-1	B	Nonremovable
2	A	Nonremovable
5	C	Removable

A. $\lim_{x \to c} f(x)$ DNE

B. $f(c)$ is undefined

C. $\lim_{x \to c} f(x) \neq f(c)$

2. (5 points) If $g(x) = \begin{cases} \dfrac{x^2 - 25}{x+5}, & x \neq -5 \\ 5 - x^2, & x = -5 \end{cases}$, state the x-value at which $g(x)$ is discontinuous and why. Then determine if the discontinuity is removable or non-removable and state why.

notation

i) $g(-5) = 5 - (-5)^2 = -20$

ii) $\lim_{x \to -5} \frac{x^2 - 25}{x+5} = \frac{0}{0} = \lim_{x \to -5} \frac{(x-5)\cancel{(x+5)}}{\cancel{(x+5)}} = -10$

iii) $g(-5) \neq \lim_{x \to -5} g(x)$ Removable Discontinuity at $x = -5$

13

3. (4 pts) Show that the Intermediate Value Theorem applies. Then find all numbers $c \in [a, b]$ for which $f(c) = k$.

$f(x) = 2x^2 + x - 7$; $[-1, 3]$, $k = 8$

$c =$ $5/2$

$f(x)$ continuous $[-1, 3]$

$f(-1) = -6$

$f(3) = 14$

By IVT, $f(c) = 8$ in $[-1, 3]$

$8 = 2x^2 + x - 7$

$0 = 2x^2 + x - 15$

$0 = (2x-5)(x+3)$

$x = 5/2, -3$

$c = 5/2$

4. Find the following. (1 point each blank)

$$h(x) = \begin{cases} \frac{9+x^2}{2x^4}, & x < -1 \\ 12, & x = -1 \\ \frac{9-6x}{2x+5}, & x > -1 \end{cases}$$

$\lim_{x\to -1^-} \frac{9+x^2}{2x^4} = \frac{10}{2}$

$\lim_{x\to -1^+} \frac{9-6x}{2x+5} = \frac{15}{3}$

$\lim_{x\to -1^-} h(x) =$ 5

$\lim_{x\to -1^+} h(x) =$ 5

$h(-1) =$ 12

$\lim_{x\to\infty} \frac{9-6x}{2x+5}$

$\lim_{x\to -1} h(x) =$ 5

$\lim_{x\to\infty} h(x) =$ -3

$\lim_{x\to -\infty} h(x) =$ 0

$\lim_{x\to -\infty} \frac{9+x^2}{2x^4} = 0$

5. (1 points each blank) Let $f(x) = \frac{100-4x^2}{x^2-9}$ and find the following:

$64/0$

a) $\lim_{x\to 3^-} f(x) =$

$\lim_{x\to 3^-} \frac{100-4(2.9)^2}{2.9^2-9} = \frac{+}{-} = \boxed{-\infty}$

b) $\lim_{x\to 3^+} f(x) = \frac{100-4(3.1)^2}{(3.1)^2-9} = \frac{+}{+}$

$\boxed{+\infty}$

c) $\lim_{x\to 3} f(x) = \boxed{DNE}$

d) $\lim_{x\to -\infty} f(x) = -4$

e) $\lim_{x\to\infty} f(x) = -4$

$\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} \frac{100-4x^2}{x^2-9}$

6. (2 points) Find all horizontal asymptotes of $f(x) = \frac{9x-2}{\sqrt{4x^2+8x-25}}$

$y = \frac{9}{\sqrt{4}}, \frac{-9}{\sqrt{4}}$

$\boxed{y = \frac{9}{2}, y = \frac{-9}{2}}$

17