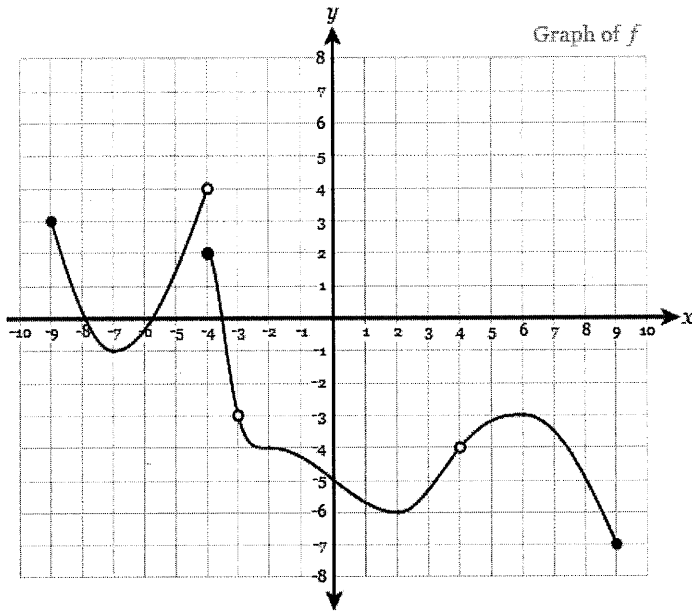


Non-AP Calculus Continuity/Limits 1.4-1.5 Quiz Review WS #3

State why each of the graphs of the functions below are not continuous. Your answer must involve conditions for continuity. Then determine type of discontinuity.

1) $c = -4$ and $c = 4$



Continuity Conditions

- i. Find $f(c)$
- ii. Find $\lim_{x \rightarrow c} f(x)$ $\left[\lim_{x \rightarrow c^+} f(x) = \lim_{x \rightarrow c^-} f(x) \right]$
- iii. $f(c) = \lim_{x \rightarrow c} f(x)$

1) $c = -4$

2) $c = 4$

3. For $f(x) = \begin{cases} 3 - x^2, & x < -1 \\ x^2 - 2x, & x \geq -1 \end{cases}$, use Continuity Conditions to show that $f(x)$ is discontinuous at $x = -1$ and state why it is discontinuous there. Then determine if the discontinuity is removable or non-removable and state why.

4. Verify that the Intermediate Value Theorem applies to the indicated interval and find the value of c guaranteed by the theorem:

$f(x) = x^2 - 7x - 4$ in the interval $[-1, 4]$ $f(c) = -14$

Find the following: (Show all appropriate work for full credit)

$$5) \lim_{x \rightarrow 2^-} \frac{2-x}{4-x^2} =$$

$$6) \lim_{x \rightarrow 3^+} \frac{x^2-5x-6}{x-3} =$$

$$7) \lim_{x \rightarrow 2^-} \frac{x^2-4}{x^2-2x-3} =$$

$$8) \lim_{x \rightarrow 2^+} \frac{4x^2-14x-8}{x^2-16} =$$

$$9) \lim_{x \rightarrow -2^+} \frac{2-x^2}{x^2-4}$$

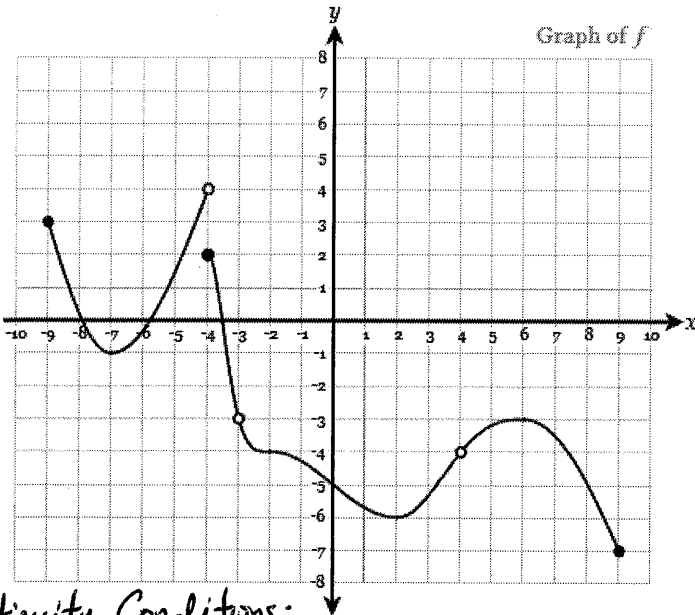
$$10) \lim_{x \rightarrow -4} \frac{5-x}{x+4}$$

Key

Non-AP Calculus Continuity/Limits 1.4-1.5 Quiz Review WS #3

State why each of the graphs of the functions below are not continuous. Your answer must involve conditions for continuity. Then determine type of discontinuity.

1) $c = -4$ and $c = 4$



1) $c = -4$

i) $f(-4) = 2$

ii) $\lim_{x \rightarrow -4^-} f(x) = 4$ $\lim_{x \rightarrow -4^+} f(x) = 2$

$\lim_{x \rightarrow -4} f(x)$ does not exist

Nonremovable Discontinuity
at $x = -4$

2) $c = 4$

i) $f(4)$ undefined

ii) $\lim_{x \rightarrow 4} f(x) = -4$

iii) $f(4) \neq \lim_{x \rightarrow 4} f(x)$

Removable Discontinuity
at $x = 4$.

Continuity Conditions:

i) $f(c)$ exists

ii) $\lim_{x \rightarrow c} f(x)$ exists $\left[\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) \right]$

iii) $f(c) = \lim_{x \rightarrow c} f(x)$

3. For $f(x) = \begin{cases} 3 - x^2, & x < -1 \\ x^2 - 2x, & x \geq -1 \end{cases}$, use Continuity Conditions to show that $f(x)$ is discontinuous at $x = -1$ and state why it is discontinuous there. Then determine if the discontinuity is removable or non-removable and state why.

i) $f(-1) = 1^2 - 2(-1) = 3$

ii) $\lim_{x \rightarrow -1^-} 3 - x^2 = 2$ $\lim_{x \rightarrow -1^+} x^2 - 2x = 3$

$\lim_{x \rightarrow -1} f(x)$ does not exist

Nonremovable discontinuity
at $x = -1$

4. Verify that the Intermediate Value Theorem applies to the indicated interval and find the value of c guaranteed by the theorem:

$f(x) = x^2 - 7x - 4$ in the interval $[-1, 4]$ By IVT, $f(c) = -14$

$f(x)$ continuous $[-1, 4]$

$f(-1) = 4$

$f(4) = -16$

Since $f(-1) < -14 < f(4)$, $f(c) = -14$ in $[-1, 4]$

$-14 = x^2 - 7x - 4$

$0 = x^2 - 7x + 10$

$0 = (x-5)(x-2)$

$x = 2, x = 5$

$c = 2$

Find the following: (Show all appropriate work)

$$5) \lim_{x \rightarrow 2^-} \frac{2-x}{4-x^2} = \frac{0}{0}$$

$$\lim_{x \rightarrow 2^-} \frac{(2-x)}{(2-x)(2+x)} = \lim_{x \rightarrow 2^-} \frac{1}{2+x} = \boxed{\frac{1}{4}}$$

$$6) \lim_{x \rightarrow 3^+} \frac{x^2-5x-6}{x-3} = \frac{9-15-6}{3-3} = \frac{-12}{0}$$

test $x=3.1$

$$\frac{(3.1)^2-5(3.1)-6}{3.1-3} = \frac{-}{+} = \boxed{-\infty}$$

VA
limit dne
 $\rightarrow +\infty$
 $\rightarrow -\infty$

$$7) \lim_{x \rightarrow 2^-} \frac{x^2-4}{x^2-2x-3} = \frac{2^2-4}{4-4-3} = \frac{0}{-3}$$

$$= \boxed{0}$$

$$8) \lim_{x \rightarrow 2^+} \frac{4x^2-14x-8}{x^2-16} = \frac{-20}{0}$$

test $x=2.1$

$$\frac{4(2.1)^2-14(2.1)-8}{(2.1)^2-16} = \frac{-}{+} = \boxed{-\infty}$$

VA: limit dne

$\rightarrow +\infty$
 $\rightarrow -\infty$

$$9) \lim_{x \rightarrow 2^+} \frac{2-x^2}{x^2-4} = \frac{-2}{0}$$

test -1.9

$$\frac{2-(-1.9)^2}{(-1.9)^2-4} = \frac{-}{-} = \boxed{+\infty}$$

$$10) \lim_{x \rightarrow -4} \frac{5-x}{x+4} = \frac{9}{0} \rightarrow \text{Limit dne}$$

does not exist