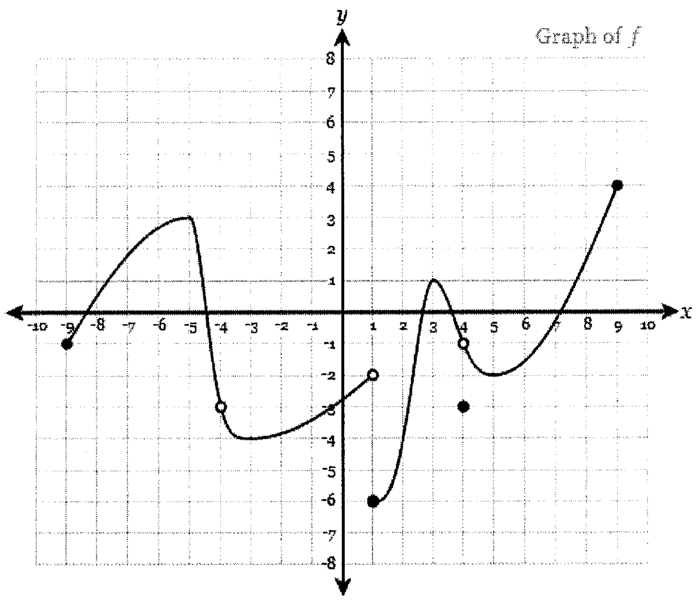
Non-AP Calculus Continuity/Limits 1.4-1.5 Morning Review WS #4

State why each of the graphs of the functions below are not continuous. Your answer must involve conditions for continuity. Then determine type of discontinuity.

1) c = 1 and c = 4



Continuity Conditions

i. f(c) is defined (point exists on the graph)

ii. Find $\lim_{x\to c} f(x)$ $\left[\lim_{x\to c^+} f(x) = \lim_{x\to c^-} f(x)\right]$

iii. f(c) $= \lim_{x\to c} f(x)$

1) c = 1

2) c = 4

3. For $f(x) = \begin{cases} 3 - x^2\ , & x < 2 \\ 2x + 1, & x = 2 \\ x^2 - 5, & x > 2 \end{cases}$, use Continuity Conditions to show that f(x) is discontinuous at x = 2 and state why it is discontinuous there. Then determine if the discontinuity is removable or non-removable and state why.

4. Verify that the Intermediate Value Theorem applies to the indicated interval and find the value of c guaranteed by the theorem:

$f(x) = x^2 + x - 1$ in the interval [0, 5] f(c) = 11

Find the following: (Show all appropriate work for full credit)

5) $\lim_{x \to -2^-} \frac{3-x}{4-x^2} =$

6) $\lim_{x \to -1^+} \frac{x^2-5x-6}{x+1} =$

7) $\lim_{x \to 2^-} \frac{x^2-25}{x^2-2x-3} =$

8) $\lim_{x \to -2^+} \frac{4x^2-14x-8}{x^2-4} =$

9) $\lim_{x \to -3^+} \frac{2-x^2}{x^2-9}$

10) $\lim_{x \to -3} \frac{5-x}{x+3}$

Key

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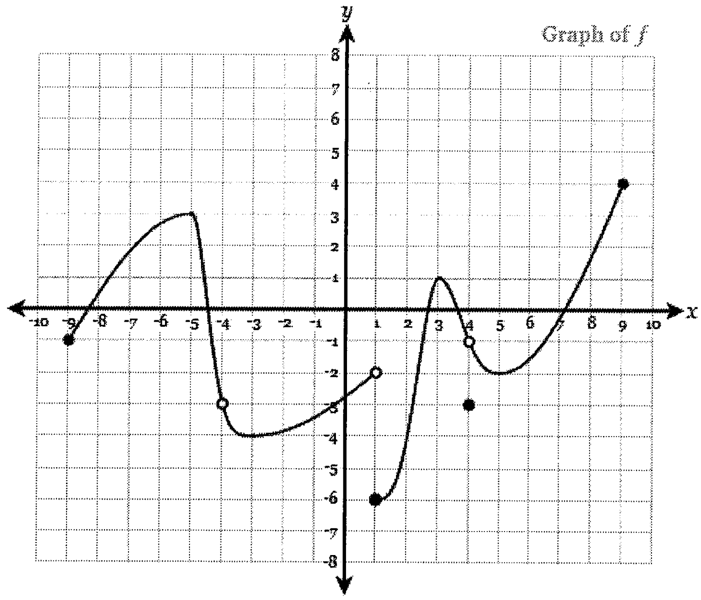
1) c = 1 and c = 4



Continuity Conditions

i. f(c) is defined (point exists on the graph)

ii. Find $\lim_{x \to c} f(x)$ $[\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x)]$

iii. $f(c) = \lim_{x \to c} f(x)$

1) c = 1

i) $f(1) = -6$

ii) $\lim_{x \to 1^-} f(x) = -2$ $\lim_{x \to 1^+} f(x) = -6$

$\lim_{x \to 1} f(x)$ does not exis

$\boxed{\text{Nonremovable Discontinuity at } x = 1}$

2) c = 4

i) $f(4) = -3$

ii) $\lim_{x \to 4} f(x) = -1$

iii) $f(4) \neq \lim_{x \to 4} f(x)$

$\boxed{\text{Removable Discontinuity at } x = 4}$

3. For $f(x) = \begin{cases} 3 - x^2, & x < 2 \\ 2x + 1, & x = 2 \\ x^2 - 5, & x > 2 \end{cases}$, use Continuity Conditions to show that f(x) is discontinuous at x = 2 and state why it is discontinuous there. Then determine if the discontinuity is removable or non-removable and state why.

i) $f(2) = 5$

ii) $\lim_{x \to 2^-} 3 - x^2 = 3 - 4 = -1$ $\lim_{x \to 2^+} x^2 - 5 = 4 - 5 = -1$ $\lim_{x \to 2} f(x) = -1$

iii) $f(2) \neq \lim_{x \to 2} f(x)$

$\boxed{\text{Removable Discontinuity at } x = 2}$

4. Verify that the Intermediate Value Theorem applies to the indicated interval and find the value of c guaranteed by the theorem:

$f(x) = x^2 + x - 1$ in the interval [0, 5] f(c) = 11

f(x) continuous [0, 5]

$f(0) = -1$

$f(5) = 29$

Since $f(0) < 11 < f(5)$

IVT applies

and $f(c) = 11$ in [0, 5]

$x^2 + x - 1 = 11$

$x^2 + x - 12 = 0$

$(x + 4)(x - 3) = 0$

$\cancel{x = -4}$, $x = 3$

$\boxed{c = 3}$

Find the following: (Show all appropriate work for full credit)

5) $\lim_{x\to -2^-} \frac{3-x}{4-x^2} = \frac{3-(-2)}{4-(-2)^2} = \frac{5}{0}$ V.A. $+\infty$, $-\infty$

test $x=-2.1$

-2.1

-3 -2 -1

$\frac{3-(-2.1)}{4-(-2.1)^2} = \frac{+}{-} = \boxed{-\infty}$

6) $\lim_{x\to -1^+} \frac{x^2-5x-6}{x+1} = \frac{1-5(-1)-6}{-1+1} = \frac{0}{0}$ hole

$\lim_{x\to -1^+} \frac{(x-6)\cancel{(x+1)}}{\cancel{(x+1)}} = -1-6 = \boxed{-7}$

7) $\lim_{x\to 2^-} \frac{x^2-25}{x^2-2x-3} = \frac{2^2-25}{4-4-3} = \frac{-21}{-3}$

$= \boxed{7}$

8) $\lim_{x\to -2^+} \frac{4x^2-14x-8}{x^2-4} = \frac{36}{0}$ $+\infty$, $-\infty$ VA, limit dne

-1.9

-3 -2 -1

$\frac{4(-1.9)^2-14(-1.9)-8}{(-1.9)^2-4} = \frac{+}{-} = \boxed{-\infty}$

9) $\lim_{x\to -3^+} \frac{2-x^2}{x^2-9}$ $\frac{2-(-3)^2}{(-3)^2-9} = \frac{-7}{0}$ $+\infty$, $-\infty$

-2.9

-4 -3 -2

$\frac{2-(-2.9)^2}{(-2.9)^2-9} = \frac{-}{-} = \boxed{+\infty}$

10) $\lim_{x\to -3} \frac{5-x}{x+3} = \frac{5-(-3)}{-3+3} = \frac{8}{0}$

limit does not exist