1.4a Continuity p. 79-81

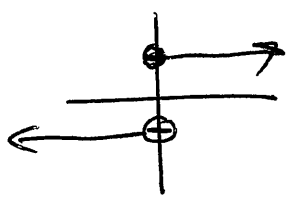
9) $\lim_{x\to 5^+} \frac{x-5}{x^2-25} = \frac{0}{0} \to \lim_{x\to 5^+} \frac{\cancel{x-5}}{(x+5)\cancel{(x-5)}} = \lim_{x\to 5^+} \frac{1}{x+5} = \boxed{\frac{1}{10}}$

11) $\lim_{x\to -3^-} \frac{x}{\sqrt{x^2-9}} = \frac{-3}{\sqrt{0}}$ * limit DNE, but problem is asking about one-sided limit: Test decimals to determine if $+\infty$ or $-\infty$

test -3.1

$\lim_{x\to -3^-} \frac{-3.1}{\sqrt{(-3.1)^2-9}} = \frac{-}{+} = \boxed{-\infty}$

13) $\lim_{x\to 0^-} \frac{|x|}{x} \to |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases} \to \frac{|x|}{x} = \begin{cases} \frac{x}{x} = 1, & x \geq 0 \\ \frac{-x}{x} = -1, & x < 0 \end{cases}$



$\boxed{\lim_{x\to 0^-} \frac{|x|}{x} = -1}$

15) $\lim_{\Delta x\to 0^-} \frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x} = \lim_{\Delta x\to 0^-} \frac{\frac{x-(x+\Delta x)}{x(x+\Delta x)} \cdot \frac{1}{\Delta x}}{\cancel{\Delta x} \cdot \cancel{\frac{1}{\Delta x}}} = \lim_{\Delta x\to 0^-} \frac{\cancel{x}-\cancel{x}-\cancel{\Delta x}}{x(x+\Delta x)} \cdot \frac{1}{\cancel{\Delta x}}$

$\lim_{\Delta x\to 0^-} \frac{-1}{x(x+\Delta x)} = \frac{-1}{x(x+0)} = \boxed{\frac{-1}{x^2}}$

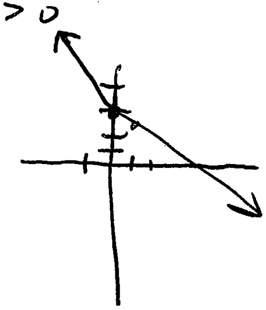
33) $f(x) = \begin{cases} 3-x, & x \leq 0 \\ 3+\frac{1}{2}x, & x > 0 \end{cases}$

$3-x$

x	y
0	3
-1	4
-2	5
-3	6

$3+\frac{1}{2}x$

x	y
0	3
1	3.5
2	4
3	4.5
4	5



Since $f(0)=3$ and $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = 3$, $f(x)$ is continuous on $[-1, 4]$

1.4a

* Removable Discontinuity: variables in denominator that cancel out with numerator (hole in graph)

* Nonremovable Discontinuity: variable (critical value) in denominator (Vertical Asymptote)

43) $f(x) = \frac{x}{x^2 - x} = \frac{\cancel{x}}{\cancel{x}(x-1)}$ hole: $(0, -1)$ (removable disc.)

VA: $x = 1$ (N.D.) nonremovable

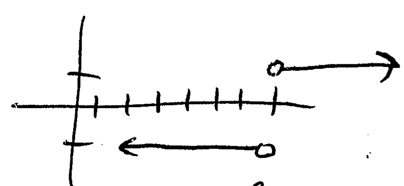
45) $f(x) = \frac{x}{x^2+1}$ continuous for all real x.

47) $f(x) = \frac{x+2}{x^2-3x-10} = \frac{\cancel{x+2}}{(x-5)\cancel{(x+2)}}$ Remov. Disc: $x=-2$ (hole) $(-2, \frac{-1}{7})$

Nonremov Disc: $x = 5$ (VA)

49) $f(x) = \frac{|x+7|}{x+7}$ $|x+7| = \begin{cases} x+7, & x > -7 \\ -x-7, & x < -7 \end{cases}$

$\frac{|x+7|}{x+7} = \begin{cases} \frac{\cancel{x+7}}{\cancel{x+7}} = 1, & x > -7 \\ \frac{-(x+7)}{x+7} = -1, & x < -7 \end{cases}$



1.4b

Nonremovable Discontinuity at $x=7$ since $\lim_{x \to 7^-} f(x) \neq \lim_{x \to 7^+} f(x)$

61) $f(x) = \begin{cases} 3x^2, & x \geq 1 \\ ax-4, & x < 1 \end{cases}$ * Find a and/or b such that function is continuous on number line.

*set piecewise functions equal at $x=1$

$3x^2 = ax - 4$

$3(1)^2 = a(1) - 4$

$3 = a - 4$

$\boxed{7 = a}$

1.4b

65) $f(x) = \begin{cases} 2 & x \le -1 \\ ax+b & -1 < x < 3 \\ -2 & x \ge 3 \end{cases}$

set piecewise functions equal at $x=-1$

$2 = ax+b \qquad 2 = a(-1)+b$

$2 = -a+b$

$a = b-2$

set piecewise functions equal at $x=3$

$ax+b = -2$

$3a+b = -2$

$3(b-2)+b = -2$

$3b-6+b = -2$

$4b = 4$

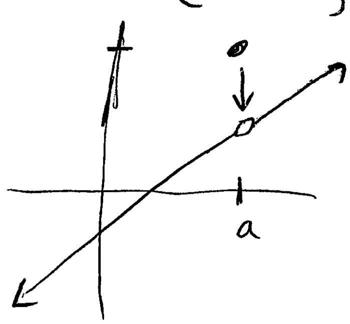
$\underline{\underline{b=1}}$

$a = b-2$

$a = 1-2 = \underline{\underline{-1}}$

$\boxed{a=-1,\ b=1}$

66) $g(x) = \begin{cases} \dfrac{x^2-a^2}{x-a} & x \ne a \\ 8, & x = a \end{cases}$



$\lim_{x\to a} \dfrac{x^2-a^2}{x-a} = 8$

$\lim_{x\to a} \dfrac{\cancel{(x-a)}(x+a)}{\cancel{(x-a)}} = 8$

$a+a = 8$

$2a = 8$

$a = 4$

69) Composite Function: $h(x) = f(g(x))$

$f(x) = \frac{1}{x-6} \quad g(x) = x^2+5$

$f[g(x)] = \dfrac{1}{(x^2+5)-6} = \dfrac{1}{x^2-1} = \dfrac{1}{(x+1)(x-1)}$

$\boxed{\text{Nonremovable Discontinuity at } x = \pm 1}$

1.46

Explain why $f(x)$ has a zero in the given interval

87) $f(x) = \frac{1}{12}x^4 - x^3 + 4$ $[1, 2]$

$f(x)$ is continuous on interval $[1, 2]$

$f(1) = \frac{37}{12}$, $f(2) = -8/3$

By IVT, since $f(2) < 0 < f(1)$ there exists a number c in $[1, 2]$ such that $f(c) = 0$.

88) $f(x) = x^3 + 5x - 3$ $[0, 1]$

$f(x)$ is continuous on interval $[0, 1]$

$f(0) = -3$ $f(1) = 3$

By IVT, since $f(0) < 0 < f(1)$, there exists a number c in $[0, 1]$ such that $f(c) = 0$

Verify that IVT applies:

95) $f(x) = x^2 + x - 1$ $[0, 5]$ $f(c) = 11$

$f(x)$ is continuous on interval $[0, 5]$

$f(0) = -1$ $f(5) = 29$ Since $f(0) < 11 < f(5)$

by IVT there exists a c-value where $f(c) = 11$

$x^2 + x - 1 = 11$

$x^2 + x - 12 = 0$

$(x + 4)(x - 3) = 0$

$x = -4, 3$ $\boxed{c = 3 \text{ since interval is } [0, 5]}$

$\cancel{c = -4}$

1.4b

97) $f(x) = x^3 - x^2 + x - 2 \quad [0, 3] \qquad f(c) = 4$

$f(x)$ is continuous on $[0,3]$ $\quad f(0) = -2, \quad f(3) = 19$

since $f(0) < 4 < f(3)$, by IVT there exists a c in $[0,3]$ where $f(c) = 4$

$x^3 - x^2 + x - 2 = 4$

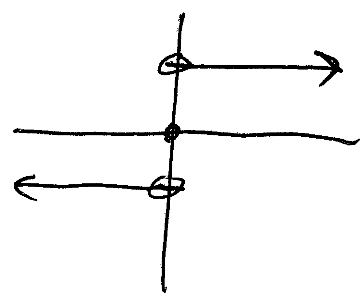
$x^3 - x^2 + x - 6 = 0$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & 1 & -6 \\ & \downarrow & 2 & 2 & 6 \\ \hline & 1 & 1 & 3 & \boxed{0} \end{array}$$

$(x-2)(x^2+x+3) \quad \boxed{c = 2}$

104) True

116) $\text{sgn}(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$



a) $\lim\limits_{x \to 0^-} \text{sgn}(x) = -1$

b) $\lim\limits_{x \to 0^+} \text{sgn}(x) = 1$

c) $\lim\limits_{x \to 0} \text{sgn}(x)$ DNE since $\lim\limits_{x \to 0^-} \text{sgn}(x) \neq \lim\limits_{x \to 0^+} \text{sgn}(x)$