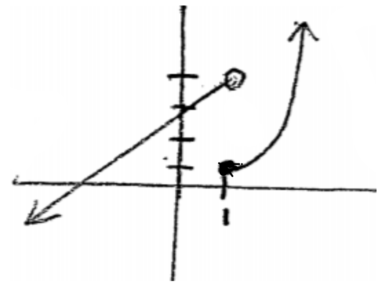
Calculus **Ch. 1.4a Notes** **Continuity and One-Sided Limits**

Definition: **One-Sided Limits** – describes the function's behavior from the left or the right side of an x-value

Example 1:

$$f(x)=\begin{cases} x^2 & , \quad x \geq 1 \\ x+3 & , \quad x < 1 \end{cases}$$



a) $\lim\limits_{x \to 1} f(x)$ =

b) left handed limit: $\lim\limits_{x \to 1^-} f(x)$ =

In other words: "The Limit (y-value that the graph approaches) **from the left side** of x = 1 is ____

c) right handed limit: $\lim\limits_{x \to 1^+} f(x)$ =

In other words: "The Limit (y-value that the graph approaches) **from the right side** of x = 1 is ____

- Recall that the limit of $f(x)$ as $x \to c$ exists only if $\lim\limits_{x \to c^-} f(x) = \lim\limits_{x \to c^+} f(x)$.

Continuity

can I walk along the graph without any interruptions? Can I draw the graph without ever lifting my pen/pencil? If so, the path or graph is continuous at that point.

Continuity Conditions: (*IMPORTANT: KNOW THIS*)

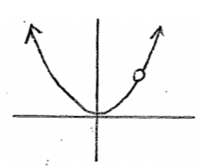
For a function, *f*, to be continuous at *c*, the following 3 conditions must be met.

1. $f(c)$ is defined *point exists
2. $\lim\limits_{x \to c} f(x)$ exists ($\lim\limits_{x \to c^-} f(x) = \lim\limits_{x \to c^+} f(x)$) *the limit exists
3. $\lim\limits_{x \to c} f(x) = f(c)$ * the limit exists at same location as point

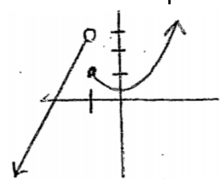
- When checking for discontinuity, step through each of the conditions above in order.

Types of Continuity:

1) **Removable Discontinuity** (hole in graph) – a graph with removable discontinuity can be made continuous by filling in a single point.



2) **Nonremovable Discontinuity** (step, jump discontinuity) – this is a discontinuity where the graph jumps from one connected piece of graph to another.



*Non-removable discontinuity fails the 2nd continuity condition:

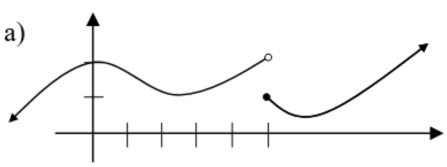
$\lim_{x \to -1^-} f(x) =$ $\lim_{x \to -1^+} f(x) =$ then $\lim_{x \to -1} f(x) =$

Continuity Conditions revisited

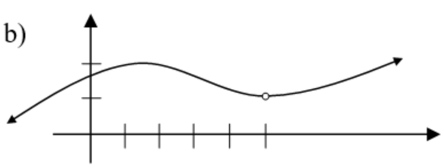
$i.\ f(c) is\ defined$	*If first condition fails, function is not continuous at the point, but continue to test next condition(s) to categorize removable/nonremovable
$ii.\ \lim_{x \to c} f(x)$ exists $(\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x))$	*If 2nd condition fails, then the limit does not exist, and this function must have **non-removable discontinuity** at that point *Test 3rd condition **only** if 2nd condition passes.
$iii.\ \lim_{x \to c} f(x) = f(c)$	*If 2nd condition passes, but 3rd condition fails, then this function must have **removable discontinuity** at that point *If all 3 condition passes, then the function is continuous at that point.

Class Example 2: Using continuity conditions, determine the reason why the following graphs are discontinuous. Then categorize as removable or nonremovable discontinuity

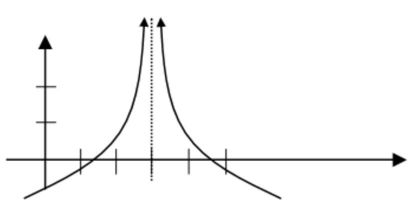
a)



b)



c)



d) Find the point (x-value) of discontinuity for the function $f(x) = \frac{x^2 - 9}{x - 3}$. Is it removable? If so, what would we need to set f(x) equal to at that value for the function to be continuous? (Step through continuity conditions to support your answer)