Special Trigonometric Limits

1) $\lim_{x\to 0} \frac{\sin(ax)}{(bx)} = \frac{a}{b}$

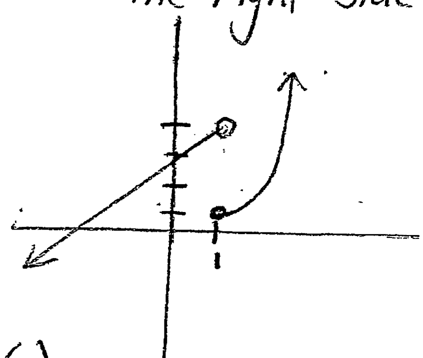
2) $\lim_{x\to 0} \frac{(ax)}{\sin(bx)} = \frac{a}{b}$

3) $\lim_{x\to 0} \frac{1-\cos(ax)}{(ax)} = 0$

Ch.1.4a Notes Continuity and One-sided Limits

A. One-sided Limits – describes the function's behavior from the left or the right side of an x-value.

$\boxed{\text{Ex.1}}$ $f(x) = \begin{cases} x^2, & x \geq 1 \\ x+3, & x < 1 \end{cases}$



a) Left-handed limit: $\lim_{x\to 1^-} f(x) =$ ____

"The limit, (y-value that graph approaches), from the left side of $x=1$ is ____"

b) Right-handed limit: $\lim_{x\to 1^+} f(x) =$ ____

"The limit, (y-value that graph approaches), from the right side of $x=1$ is ____"

* If $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)$, then the limit of $f(x)$ as $x \to c$ exists.

B. Continuity - continuity exists if you can draw the graph without lifting your pencil.

3 Conditions for continuity: *Important*

1) $f(c)$ is defined → point exists

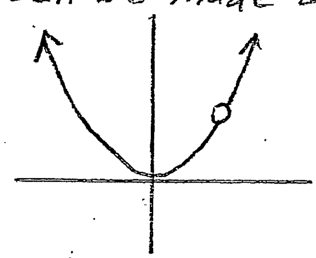
2) $\lim_{x \to c} f(x)$ exists $\left(\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)\right)$ → limit exists

3) $\lim_{x \to c} f(x) = f(c)$ → limit exists where point exist (conditions 1 and 2 agree)

* When checking for discontinuity, step through each of the continuity conditions in order. Stop once you reach a condition that fails.

C. Types of Discontinuity

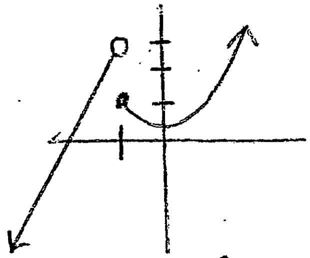
1. Removable Discontinuity (hole in graph) → a graph with removable discontinuity can be made continuous by filling in a single point.



* Removable discontinuity fails continuity condition #3

$\lim_{x \to c} f(x) \neq f(c)$

2. Nonremovable Discontinuity (step, jump discontinuity) - a discontinuit where the graph jumps from one connected piece of graph to another



* Nonremovable discontinuity fails continuity condition #

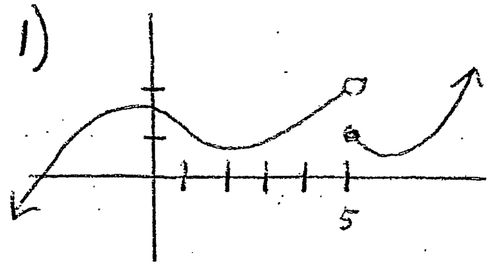
2) $\lim_{x \to c} f(x)$ does not exist: $\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x$

Since $\lim_{x \to -1^-} f(x) = \_\_$ and $\lim_{x \to -1^+} f(x) = \_\_$, $\lim_{x \to -1^-} f(x) \neq \lim_{x \to -1^+} f(x)$

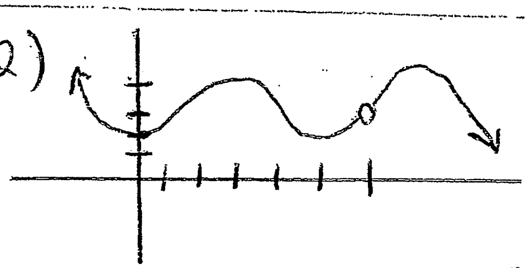
so $\lim_{x \to -1} f(x)$ does not exist

Class
Examples: Using continuity conditions, determine the reason why the following graphs are discontinuous. Then categorize each as removable or nonremovable discontinuity.

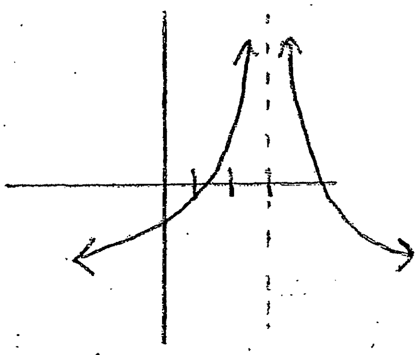
Ex. 1)



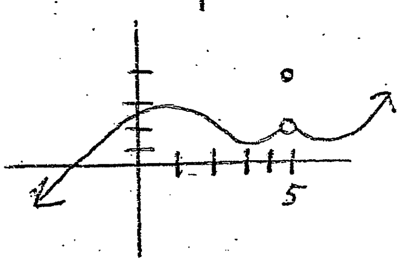
x. 2)



Ex. 3)



x. 4)



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Ex. 5) Find point (x-value) of discontinuity for $f(x) = \frac{x^2-9}{x-3}$
Determine if removable/nonremovable discontinuity