**Calculus Ch. 1.4b Notes Intermediate Value Theorem ( IVT )**

**Review continuity conditions:**

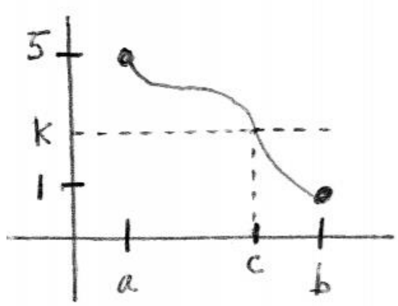
**i**) $f(c)$ is defined **ii**) $\lim_{x \to c} f(x)$ exists ( $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$ ) **iii**) $\lim_{x \to c} f(x) = f(c)$

**Warm-up problem:** Prove that the following is discontinuous at x = 2. Is it removable? If so, redefine f(2) to make the function continuous. (step through continuity conditions)

$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2}, & x \neq 2 \\ x + 5 \ , & x = 2 \end{cases}$$

**Continuity on a closed interval:** If a function is continuous on an open interval (a, b) and $\lim_{x \to a^+} f(x) = f(a)$ and $\lim_{x \to b^-} f(x) = f(b)$, then the function is continuous on the closed interval [a, b].

**Intermediate Value Theorem:** If f is continuous on a closed interval [a, b] and k is any number between f(a) and f(b), then there is at least one number c in [a, b] such that f(c) = k.




*In other words, if a function is continuous, then the graph has to touch all the y-values between the 2 endpoints (at least once)

**Example 1**: Use the IVT to show that there is a zero in the interval [0, 1] for the function $f(x) = x^3 + 2x - 1$.

**Example 2**: Verify the IVT applies to $f(x) = \frac{x^2+x}{x-1}$ on the interval $\left[\frac{5}{2}, 4\right]$ for f(c) = 6 and find c.

**Additional Continuity Practice Problems**:

**Making a Function Continuous** **In Exercises 61–66, find the constant *a*, or the constants *a* and *b*, such that the function is continuous on the entire real number line.**

**Continuity conditions:**

1. $f(c)$ is defined
2. $\lim_{x \to c} f(x)$ exists $\left(\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)\right)$
3. $\lim_{x \to c} f(x) = f(c)$

**61.** $f(x) = \begin{cases} 3x^2, & x \geq 1 \\ ax - 4, & x < 1 \end{cases}$

**66.** $g(x) = \begin{cases} \dfrac{x^2 - a^2}{x - a}, & x \neq a \\ 8, & x = a \end{cases}$