AP Calculus – 1.5 Notes Infinite Limits (Vertical Asymptotes) & Limits at Infinity (Horizontal Asymptotes)

I. Infinite Limits and Vertical Asymptotes

Use the function $f(x) = \frac{x^2+2x-8}{x^2+x-12}$ to answer the following.

1. Identify all vertical asymptotes.	2. Evaluate $\lim\limits_{x\to 3^-} f(x)$	3. Evaluate $\lim\limits_{x\to 3^+} f(x)$

Find the limit.

4. $\lim\limits_{x\to 3^+} \frac{1-x}{x-3}$	5. $\lim\limits_{x\to 1} \frac{x-3}{x^2-2x+1}$

Practice Problems:

Identify the vertical asymptotes of each function.

1. $f(x) = \frac{x-6}{x^2-9x+18}$	2. $f(x) = \frac{2x^2-x-3}{3x^2+4x+1}$	3. $f(x) = \frac{x^2-x-12}{x+7}$
4. $f(x) = \frac{3x^2-11x+10}{x-2}$	5. $f(x) = \frac{x^3+2x^2-24x}{x^2-x}$	6. $f(x) = \frac{7x^2+4x-3}{7x-3}$

II. Limits at Infinity and Horizontal Asymptotes

Horizontal Asymptotes: (End-behavior)

What does the y-value approach as the x-value approaches negative infinity AND positive infinity? Does it approach a specific number, or is it growing without bound?

Checking for Horizontal Asymptotes (H.A.) $\left(\lim_{x\to\infty} f(x) \quad or \quad \lim_{x\to-\infty} f(x)\right)$

If $f(x) = \frac{p(x)}{q(x)}$, then **compare the degrees between numerator and denominator**

i) If Numerator degree < Denominator degree, then the H.A. is y = 0

Example 1: $\lim_{x\to\infty} \frac{3x^2-7}{2x^3+1} =$

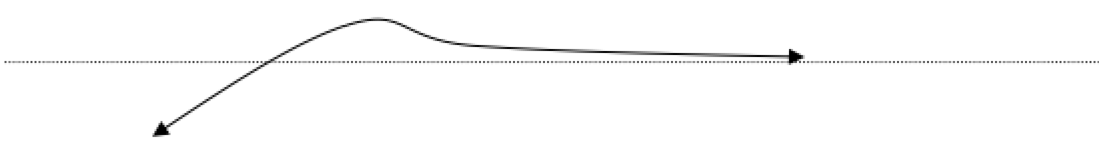
ii) If Denominator degree = Numerator degree, then H.A. is $y = \frac{numerator\ coefficient}{denominator\ coefficient}$

Example 2: $\lim_{x\to\infty} \frac{5x^2+3}{2x^2+4x-9} =$

iii) If Numerator degree > Denominator degree, then H.A. does not exist (limit is therefore $+\infty\ or -\infty$)

Example 3: $\lim_{x\to\infty} \frac{2x^3+1}{7x^2+5x+10} =$

Note: a H.A. is a description of end behavior, not a boundary that the graph can't cross. A function can NEVER cross a vertical asymptote, but it might cross a horizontal asymptote.



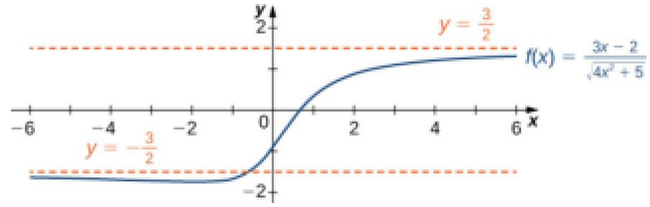
Use Horizontal Asymptote Rules for the following:

4) $\lim_{x\to\infty} \frac{3x^2+1}{2x-5}$	5) $\lim_{x\to-\infty} \frac{3x^2+1}{2x-5}$	6) $\lim_{x\to-\infty} \frac{3x+1}{5-2x}$
7) $\lim_{x\to\infty} \frac{3x+1}{5-2x}$	8) $\lim_{x\to\infty} \frac{3x+1}{2x^2-5}$	9) $\lim_{x\to-\infty} \frac{3x^3+1}{2x^2-5}$

B. Finding Horizontal Asymptotes with **Radicals in denominator**

Ex. 10: Find the Horizontal asymptotes for:

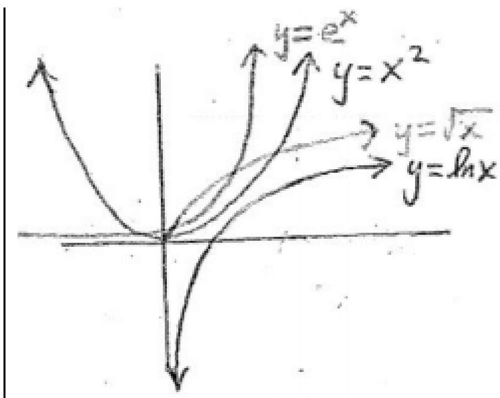
$$y = \frac{3x-2}{\sqrt{4x^2+5}}$$



C. Comparative Growth Rates

*Families of Functions grow at predictable rates in relations to each other as x **approaches** $+\infty$

***Logarithms < Radicals < Polynomial (Algebraic) < Exponential**
(slowest) (fastest)



*Note: Comparative Growth Rates relationship **<u>only apply</u>** when limit approaches infinity. (NOT $-\infty$)

Ex. 11 $\lim\limits_{x\to\infty} \frac{\sqrt{5000x+1000}}{x^2}$

Ex. 13 $\lim\limits_{x\to\infty} \frac{\ln(40000000x)}{2x}$

Ex. 12 $\lim\limits_{x\to\infty} \frac{-e^{2x}}{1000x^4+x^5}$

Ex. 14 $\lim\limits_{x\to\infty} \frac{-\sqrt{3000x-4}}{\ln(5x+1)}$

Unit Review Problems

5. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2+6x+8}{x+2}$ when $x \neq -2$, then $f(-2) =$

6. Let f be the function defined by $f(x) = \begin{cases} \frac{x^2+8x+12}{x+6}, & x \neq -6 \\ b, & x = -6 \end{cases}$. For what value of b is f continuous at $x = -6$?

Evaluate the limit.		
7. $\lim\limits_{x\to\infty} \sin\left(\frac{x+3\pi x^2}{2x^2}\right)$	8. $\lim\limits_{x\to -5^-} \frac{-3}{25-x^2}$	9. $\lim\limits_{x\to\infty} \frac{\sin x}{x}$
10. $\lim\limits_{x\to\infty} \frac{4x^5-2x^2+3}{3x^2+2x^5-x^4}$	11. $\lim\limits_{x\to -1} \frac{x^2+1}{x+1}$	12. $\lim\limits_{x\to\infty} x^5 3^{-x}$

13. Identify all horizontal asymptotes of $f(x) = \frac{\sqrt{16\ \ ^6+x^3+5x}}{5x^3-8x}$.