Calculus Ch. 1.5 Notes: Limits Approaching Infinity (Vertical Asymptotes)

Infinite Limits: a limit where the function increases or decreases without bound (towards infinity) as x approaches c

*If the limit as x approaches c from either right or left is $\pm\infty$, then x = c is a vertical asymptote

* Rational Functions: $y = \frac{f(x)}{g(x)}$ If g(x) has no factors that cancel, then there is a vertical asymptote.

Example 1: Find all the vertical asymptotes of $f(x) = \frac{x^2-3x+2}{x^2-4}$

Finding One-Sided Limits approaching Vertical Asymptotes:

Steps:

1) Evaluate Limit using the argument (plug in the value)
2) If Limit is undefined ($\frac{nonzero}{zero}$) then there is a vertical asymptote
3) To further evaluate the one-sided limit (determining the direction of arrows as $+\infty$ *or* $-\infty$)
    a. Test decimals 0.1 to the left of the argument x-value
    b. Test decimal 0.1 to the right of the argument x-value
4) Determine if the resulting fraction is a positive or negative value
    a. A positive decimal value indicates the one-sided limit is $+\infty$
    b. A negative decimal value indicates the one-sided limit is $-\infty$

Example 2: Determine $\lim_{x\to 2} f(x)$ for $f(x) = \frac{x+1}{x-2}$

Algebraic Steps (for x approaching Real Number): **1)** Plug in x-value first (IGNORE one-sided limit) **2)** If result is a real number value, the value is the limit. **3)** If the result is $\frac{0}{0}$ (indeterminate form) then reduce by **i)** factoring **ii)** conjugate method **iii)** simplify complex fraction **4)** Re-evaluate the reduced Expression 4) If result is undefined, and it's a one-sided limit, then test using decimals.

Find the following:

3) $\lim_{x\to -3^-} \frac{9-x^2}{x-4} =$

4) $\lim_{x\to 0^-} \frac{5x-x^2}{x^2-x} =$

5) $\lim_{x\to -2^-} \frac{x^2+1}{x+2} =$

6) $\lim_{x\to 5} \frac{3x^2-1}{25-x^2} =$

7) $\lim_{x\to -3^+} \frac{2x^2+3x-9}{x+3}$

8) $\lim_{x\to -4^+} \frac{2x^2-1}{x^2-16}$

9) $\lim_{x\to 1^+} \frac{x^2-2}{x^2+2x+1} =$

10) $\lim_{x\to 3^+} \frac{4\,x^2-14x+6}{x-3} =$