Key

**Calculus** **Ch. 1.5 Notes: Limits Approaching Infinity (Vertical Asymptotes)**

**Infinite Limits**: a limit where the function increases or decreases without bound (towards infinity) as x approaches c

*If the limit as x approaches c from either right or left is $\pm\infty$ , then x = c is a vertical asymptote

* Rational Functions: $y = \frac{f(x)}{g(x)}$ If g(x) has no factors that cancel, then there is a vertical asymptote.

**Example 1**: Find all the vertical asymptotes of $f(x) = \frac{x^2-3x+2}{x^2-4} = \frac{(x-1)\cancel{(x-2)}}{\underbrace{\cancel{(x-2)}}_{\text{hole}}\underbrace{(x+2)}_{\text{vertical asymptote}}}$

vertical asymptote exists at $\boxed{x=-2}$

**Finding One-Sided Limits approaching Vertical Asymptotes:**

Steps:

1) Evaluate Limit using the argument (plug in the value)
2) If Limit is undefined $(\frac{nonzero}{zero})$ then there is a vertical asymptote
3) To further evaluate the one-sided limit (determining the direction of arrows as $+\infty$ $or$ $-\infty$ )
   a. Test decimals 0.1 to the left of the argument x-value
   b. Test decimal 0.1 to the right of the argument x-value
4) Determine if the resulting fraction is a positive or negative value
   a. A positive decimal value indicates the one-sided limit is $+\infty$
   b. A negative decimal value indicates the one-sided limit is $-\infty$

**Example 2**: Determine $\lim_{x\to 2} f(x)$ for $f(x) = \frac{x+1}{x-2}$

$\lim_{x\to 2} \frac{x+1}{x-2} \to \frac{2+1}{2-2} \to \frac{3}{0}$

This indicates that vertical asymptote exists at $x=2$.

$\lim_{x\to 2} f(x) = d.n.e$ (but we can use one-sided limits to further subcategorize) (either $+\infty$ or $-\infty$)

$\lim_{x\to 2^-} \frac{x+1}{x-2} \to \frac{1.9+1}{1.9-2} \to \frac{+}{-} \to \boxed{-\infty}$

test using $x=1.9$

$\lim_{x\to 2^+} \frac{x+1}{x-2} \to \frac{3}{0} \to \lim_{x\to 2^+} \frac{x+1}{x-2} \to \frac{2.1+1}{2.1-2} \to \frac{+}{+} = \boxed{+\infty}$

test $x=2.1$

$\boxed{\lim_{x\to 2} f(x) = \text{does not exist}}$

No subcategory since arrows are not same direction

**Find the following**:

3) $\lim_{x\to -3^-} \frac{9-x^2}{x-4} = \frac{9-(-3)^2}{-3-4} = \frac{0}{-7}$

$\boxed{0}$

4) $\lim_{x\to 0^-} \frac{5x-x^2}{x^2-x} = \frac{0}{0}$

$\lim_{x\to 0^-} \frac{\cancel{x}(5-x)}{\cancel{x}(x-1)} \to \lim_{x\to 0^-} \frac{5-x}{x-1} = \frac{5-0}{0-1}\ \frac{5}{-1}$

$= \boxed{-5}$

5) $\lim_{x\to -2^-} \frac{x^2+1}{x+2} = \frac{(-2)^2+1}{-2+2} = \frac{5}{0}$

VA., Limit DNE $\nearrow +\infty$ $\searrow -\infty$

test $x=-2.1$

$\frac{(-2.1)^2+1}{-2.1+2} = \frac{+}{-} = \boxed{-\infty}$

6) $\lim_{x\to 5} \frac{3x^2-1}{25-x^2} = \frac{3(5)^2-1}{25-5^2} = \frac{74}{0}$ $\nearrow +\infty$ $\searrow -\infty$

limit does not exist

7) $\lim_{x\to -3^+} \frac{2x^2+3x-9}{x+3}$ $\quad 2(-3)^2-9-9 = \frac{0}{0}$

$\lim_{x\to -3^+} \frac{(2x-3)\cancel{(x+3)}}{\cancel{(x+3)}} = 2(-3)-3 = \boxed{-9}$

8) $\lim_{x\to -4^+} \frac{2x^2-1}{x^2-16} = \frac{2(-4)^2-1}{(-4)^2-16} = \frac{31}{0}$ $\nearrow +\infty$ $\searrow -\infty$

$\frac{2(-3.9)^2-1}{(-3.9)^2-16} = \frac{+}{-} = \boxed{-\infty}$

9) $\lim_{x\to 1^+} \frac{x^2-2}{x^2+2x+1} = \frac{1-2}{1^2+2+1} = \boxed{\frac{-1}{4}}$

10) $\lim_{x\to 3^+} \frac{4x^2-14x+6}{x-3} = \frac{4(9)-14(3)+6}{3-3}\ \frac{0}{0}$

$\lim_{x\to 3^+} \frac{2(2x^2-7x+3)}{(x-3)}$

$\lim_{x\to 3^+} \frac{2(2x-1)\cancel{(x-3)}}{\cancel{(x-3)}} = 2(2(3)-1) = 2(5)$

$= \boxed{10}$