Section 1.5 Infinite Limits

1. $\lim_{x\to -2^+} 2\left|\dfrac{x}{x^2-4}\right| = \infty$

$\lim_{x\to -2^-} 2\left|\dfrac{x}{x^2-4}\right| = \infty$

2. $\lim_{x\to -2^+} \dfrac{1}{x+2} = \infty$

$\lim_{x\to -2^-} \dfrac{1}{x+2} = -\infty$

3. $\lim_{x\to -2^+} \tan\dfrac{\pi x}{4} = -\infty$

$\lim_{x\to -2^-} \tan\dfrac{\pi x}{4} = \infty$

4. $\lim_{x\to -2^+} \sec\dfrac{\pi x}{4} = \infty$

$\lim_{x\to -2^-} \sec\dfrac{\pi x}{4} = -\infty$

5. $f(x) = \dfrac{1}{x-4}$

As x approaches 4 from the left, $x - 4$ is a small negative number. So,

$\lim_{x\to 4^-} f(x) = -\infty$

As x approaches 4 from the right, $x - 4$ is a small positive number. So,

$\lim_{x\to 4^+} f(x) = \infty$

6. $f(x) = \dfrac{-1}{x-4}$

As x approaches 4 from the left, $x - 4$ is a small negative number. So,

$\lim_{x\to 4^-} f(x) = \infty.$

As x approaches 4 from the right, $x - 4$ is a small positive number. So,

$\lim_{x\to 4^+} f(x) = -\infty.$

7. $f(x) = \dfrac{1}{(x-4)^2}$

As x approaches 4 from the left or right, $(x-4)^2$ is a small positive number. So,

$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^-} f(x) = \infty.$

8. $f(x) = \dfrac{-1}{(x-4)^2}$

As x approaches 4 from the left or right, $(x-4)^2$ is a small positive number. So,

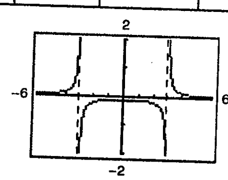
$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x) = -\infty.$

9. $f(x) = \dfrac{1}{x^2-9}$

x	–3.5	–3.1	–3.01	–3.001	–2.999	–2.99	–2.9	–2.5
$f(x)$	0.308	1.639	16.64	166.6	–166.7	–16.69	–1.695	–0.364

$\lim_{x\to -3^-} f(x) = \infty$

$\lim_{x\to -3^+} f(x) = -\infty$

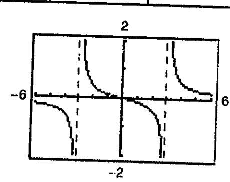


10. $f(x) = \dfrac{x}{x^2-9}$

x	–3.5	–3.1	–3.01	–3.001	–2.999	–2.99	–2.9	–2.5
$f(x)$	–1.077	–5.082	–50.08	–500.1	499.9	49.92	4.915	0.9091

$\lim_{x\to -3^-} f(x) = -\infty$

$\lim_{x\to -3^+} f(x) = \infty$

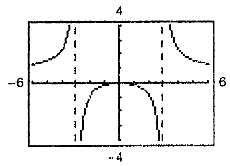


11. $f(x) = \dfrac{x^2}{x^2 - 9}$

x	–3.5	–3.1	–3.01	–3.001	–2.999	–2.99	–2.9	–2.5
$f(x)$	3.769	15.75	150.8	1501	–1499	–149.3	–14.25	–2.273

$\lim_{x\to -3^-} f(x) = \infty$

$\lim_{x\to -3^+} f(x) = -\infty$

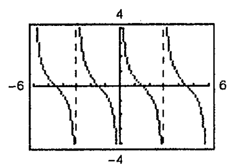


12. $f(x) = \cot\dfrac{\pi x}{3}$

x	–3.5	–3.1	–3.01	–3.001	–2.999	–2.99	–2.9	–2.5
$f(x)$	–1.7321	–9.514	–95.49	–954.9	954.9	95.49	9.514	1.7321

$\lim_{x\to -3^-} f(x) = -\infty$

$\lim_{x\to -3^+} f(x) = \infty$



13. $f(x) = \dfrac{1}{x^2}$

$\lim_{x\to 0^+} \dfrac{1}{x^2} = \infty = \lim_{x\to 0^-} \dfrac{1}{x^2}$

Therefore, $x = 0$ is a vertical asymptote.

14. $f(x) = \dfrac{2}{(x-3)^3}$

$\lim_{x\to 3^-} \dfrac{2}{(x-3)^3} = -\infty$

$\lim_{x\to 3^+} \dfrac{2}{(x-3)^3} = \infty$

Therefore, $x = 3$ is a vertical asymptote.

15. $f(x) = \dfrac{x^2}{x^2 - 4} = \dfrac{x^2}{(x+2)(x-2)}$

$\lim_{x\to -2^-} \dfrac{x^2}{x^2-4} = \infty$ and $\lim_{x\to -2^+} \dfrac{x^2}{x^2-4} = -\infty$

Therefore, $x = -2$ is a vertical asymptote.

$\lim_{x\to 2^-} \dfrac{x^2}{x^2-4} = -\infty$ and $\lim_{x\to 2^+} \dfrac{x^2}{x^2-4} = \infty$

Therefore, $x = 2$ is a vertical asymptote.

16. $f(x) = \dfrac{3x}{x^2 + 9}$

No vertical asymptotes because the denominator is never zero.

17. $g(t) = \dfrac{t-1}{t^2+1}$

No vertical asymptotes because the denominator is never zero.

18. $h(s) = \dfrac{3s+4}{s^2-16} = \dfrac{3s+4}{(s-4)(s+4)}$

$\lim_{s\to 4^-} \dfrac{3s+4}{s^2-16} = -\infty$ and $\lim_{s\to 4^+} \dfrac{3s+4}{s^2-16} = \infty$

Therefore, $s = 4$ is a vertical asymptote.

$\lim_{s\to -4^-} \dfrac{3s+4}{s^2-16} = -\infty$ and $\lim_{s\to -4^+} \dfrac{3s+4}{s^2-16} = \infty$

Therefore, $s = -4$ is a vertical asymptote.

19. $f(x) = \dfrac{3}{x^2+x-2} = \dfrac{3}{(x+2)(x-1)}$

$\lim_{x\to -2^-} \dfrac{3}{x^2+x-2} = \infty$ and $\lim_{x\to -2^+} \dfrac{3}{x^2+x-2} = -\infty$

Therefore, $x = -2$ is a vertical asymptote.

$\lim_{x\to 1^-} \dfrac{3}{x^2+x-2} = -\infty$ and $\lim_{x\to 1^+} \dfrac{3}{x^2+x-2} = \infty$

Therefore, $x = 1$ is a vertical asymptote.

20. $g(x) = \dfrac{x^3-8}{x-2} = \dfrac{(x-2)(x^2+2x+4)}{x-2}$

$= x^2 + 2x + 4,\ x \neq 2$

$\lim_{x\to 2} g(x) = 4 + 4 + 4 = 12$

There are no vertical asymptotes. The graph has a hole at $x = 2$.

21. $f(x) = \dfrac{4(x^2 + x - 6)}{x(x^3 - 2x^2 - 9x + 18)}$

$= \dfrac{4(x + 3)(x - 2)}{x(x - 2)(x^2 - 9)}$

$= \dfrac{4}{x(x - 3)}, x \neq -3, 2$

$\lim_{x\to 0^-} f(x) = \infty$ and $\lim_{x\to 0^+} f(x) = -\infty$

Therefore, $x = 0$ is a vertical asymptote.

$\lim_{x\to 3^-} f(x) = -\infty$ and $\lim_{x\to 3^+} f(x) = \infty$

Therefore, $x = 3$ is a vertical asymptote.

$\lim_{x\to 2} f(x) = \dfrac{4}{2(2 - 3)}$

$= -2$ and $\lim_{x\to -3} f(x)$

$= \dfrac{4}{-3(-3 - 3)} = \dfrac{2}{9}$

Therefore, the graph has holes at $x = 2$ and $x = -3$.

22. $h(x) = \dfrac{x^2 - 9}{x^3 + 3x^2 - x - 3}$

$= \dfrac{(x - 3)(x + 3)}{(x - 1)(x + 1)(x + 3)}$

$= \dfrac{x - 3}{(x + 1)(x - 1)}, x \neq -3$

$\lim_{x\to -1^-} h(x) = -\infty$ and $\lim_{x\to -1^+} h(x) = \infty$

Therefore, $x = -1$ is a vertical asymptote.

$\lim_{x\to 1^-} h(x) = \infty$ and $\lim_{x\to 1^+} h(x) = -\infty$

Therefore, $x = 1$ is a vertical asymptote.

$\lim_{x\to -3} h(x) = \dfrac{-3 - 3}{(-3 + 1)(-3 - 1)} = -\dfrac{3}{4}$

Therefore, the graph has a hole at $x = -3$.

23. $f(x) = \dfrac{x^2 - 2x - 15}{x^3 - 5x^2 + x - 5}$

$= \dfrac{(x - 5)(x + 3)}{(x - 5)(x^2 + 1)}$

$= \dfrac{x + 3}{x^2 + 1}, x \neq 5$

$\lim_{x\to 5} f(x) = \dfrac{5 + 3}{5^2 + 1} = \dfrac{15}{26}$

There are no vertical asymptotes. The graph has a hole at $x = 5$.

24. $h(t) = \dfrac{t^2 - 2t}{t^4 - 16} = \dfrac{t(t - 2)}{(t - 2)(t + 2)(t^2 + 4)}$

$= \dfrac{t}{(t + 2)(t^2 + 4)}, t \neq 2$

$\lim_{t\to -2^-} h(t) = \infty$ and $\lim_{t\to -2^+} h(t) = -\infty$

Therefore, $t = -2$ is a vertical asymptote.

$\lim_{t\to 2} h(t) = \dfrac{2}{(2 + 2)(2^2 + 4)} = \dfrac{1}{16}$

Therefore, the graph has a hole at $t = 2$.

25. $f(x) = \csc \pi x = \dfrac{1}{\sin \pi x}$

Let n be any integer.

$\lim_{x\to n} f(x) = -\infty$ or ∞

Therefore, the graph has vertical asymptotes at $x = n$.

26. $f(x) = \tan \pi x = \dfrac{\sin \pi x}{\cos \pi x}$

$\cos \pi x = 0$ for $x = \dfrac{2n + 1}{2}$, where n is an integer.

$\lim_{x\to \frac{2n+1}{2}} f(x) = \infty$ or $-\infty$

Therefore, the graph has vertical asymptotes at $x = \dfrac{2n + 1}{2}$.

27. $s(t) = \dfrac{t}{\sin t}$

$\sin t = 0$ for $t = n\pi$, where n is an integer.

$\lim_{t\to n\pi} s(t) = \infty$ or $-\infty$ (for $n \neq 0$)

Therefore, the graph has vertical asymptotes at $t = n\pi$, for $n \neq 0$.

$\lim_{t\to 0} s(t) = 1$

Therefore, the graph has a hole at $t = 0$.

28. $g(\theta) = \dfrac{\tan \theta}{\theta} = \dfrac{\sin \theta}{\theta \cos \theta}$

$\cos \theta = 0$ for $\theta = \dfrac{\pi}{2} + n\pi$, where n is an integer.

$\lim_{\theta\to \frac{\pi}{2}+n\pi} g(\theta) = \infty$ or $-\infty$

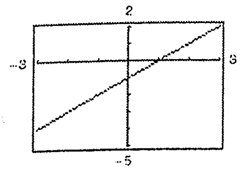
Therefore, the graph has vertical asymptotes at $\theta = \dfrac{\pi}{2} + n\pi$.

$\lim_{\theta\to 0} g(\theta) = 1$

Therefore, the graph has a hole at $\theta = 0$.

29. $\lim_{x \to -1} \frac{x^2 - 1}{x + 1} = \lim_{x \to -1}(x - 1) = -2$

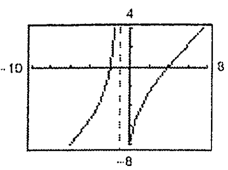
Removable discontinuity at $x = -1$



30. $\lim_{x \to -1^-} \frac{x^2 - 2x - 8}{x + 1} = \infty$

$\lim_{x \to -1^+} \frac{x^2 - 2x - 8}{x + 1} = -\infty$

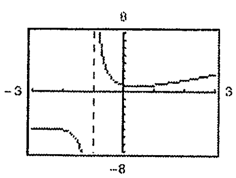
Vertical asymptote at $x = -1$



31. $\lim_{x \to -1^+} \frac{x^2 + 1}{x + 1} = \infty$

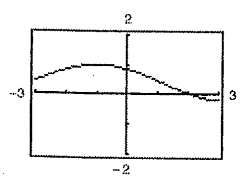
$\lim_{x \to -1^-} \frac{x^2 + 1}{x + 1} = -\infty$

Vertical asymptote at $x = -1$



32. $\lim_{x \to -1} \frac{\sin(x + 1)}{x + 1} = 1$

Removable discontinuity at $x = -1$



33. $\lim_{x \to -1^+} \frac{1}{x + 1} = \infty$

34. $\lim_{x \to 1^-} \frac{-1}{(x - 1)^2} = -\infty$

35. $\lim_{x \to 2^+} \frac{x}{x - 2} = \infty$

36. $\lim_{x \to 2^-} \frac{x^2}{x^2 + 4} = \frac{4}{4 + 4} = \frac{1}{2}$

37. $\lim_{x \to -3^-} \frac{x + 3}{(x^2 + x - 6)} = \lim_{x \to -3^-} \frac{x + 3}{(x + 3)(x - 2)}$

$= \lim_{x \to -3^-} \frac{1}{x - 2} = -\frac{1}{5}$

38. $\lim_{x \to -(1/2)^+} \frac{6x^2 + x - 1}{4x^2 - 4x - 3} = \lim_{x \to -(1/2)^+} \frac{(3x - 1)(2x + 1)}{(2x - 3)(2x + 1)}$

$= \lim_{x \to -(1/2)^+} \frac{3x - 1}{2x - 3} = \frac{5}{8}$

39. $\lim_{x \to 0^-} \left(1 + \frac{1}{x}\right) = -\infty$

40. $\lim_{x \to 0^+} \left(6 - \frac{1}{x^3}\right) = -\infty$

41. $\lim_{x \to -4^-} \left(x^2 + \frac{2}{x + 4}\right) = -\infty$

42. $\lim_{x \to 3^+} \left(\frac{x}{3} + \cot \frac{\pi x}{2}\right) = \infty$

43. $\lim_{x \to 0^+} \frac{2}{\sin x} = \infty$

44. $\lim_{x \to (\pi/2)^+} \frac{-2}{\cos x} = \infty$

45. $\lim_{x \to \pi^+} \frac{\sqrt{x}}{\csc x} = \lim_{x \to \pi^+} \left(\sqrt{x} \sin x\right) = 0$

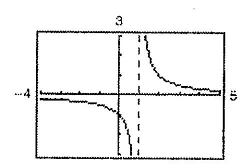
46. $\lim_{x \to 0^-} \frac{x + 2}{\cot x} = \lim_{x \to 0^-} \left[(x + 2) \tan x\right] = 0$

47. $\lim_{x \to (1/2)^-} x \sec \pi x = \lim_{x \to (1/2)^-} \frac{x}{\cos \pi x} = \infty$

48. $\lim_{x \to (1/2)^+} x^2 \tan \pi x = -\infty$

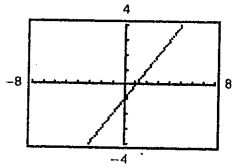
49. $f(x) = \frac{x^2 + x + 1}{x^3 - 1} = \frac{x^2 + x + 1}{(x - 1)(x^2 + x + 1)}$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{1}{x - 1} = \infty$



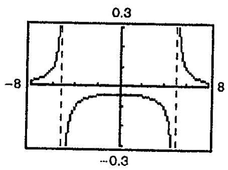
50. $f(x) = \dfrac{x^3 - 1}{x^2 + x + 1} = \dfrac{(x - 1)(x^2 + x + 1)}{x^2 + x + 1}$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x - 1) = 0$



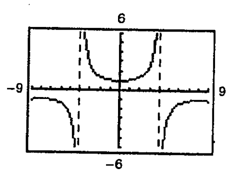
51. $f(x) = \dfrac{1}{x^2 - 25}$

$\lim_{x \to 5^-} f(x) = -\infty$



52. $f(x) = \sec \dfrac{\pi x}{8}$

$\lim_{x \to 4^+} f(x) = -\infty$



53. A limit in which $f(x)$ increases or decreases without bound as x approaches c is called an infinite limit. ∞ is not a number. Rather, the symbol

$\lim_{x \to c} f(x) = \infty$

says how the limit fails to exist.

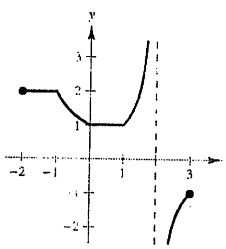
54. The line $x = c$ is a vertical asymptote if the graph of f approaches $\pm\infty$ as x approaches c.

55. One answer is

$$f(x) = \frac{x - 3}{(x - 6)(x + 2)} = \frac{x - 3}{x^2 - 4x - 12}.$$

56. No. For example, $f(x) = \dfrac{1}{x^2 + 1}$ has no vertical asymptote.

57.

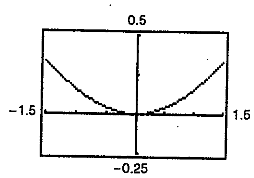


58. $m = \dfrac{m_0}{\sqrt{1 - (v^2/c^2)}}$

$\lim_{v \to c^-} m = \lim_{v \to c^-} \dfrac{m_0}{\sqrt{1 - (v^2/c^2)}} = \infty$

59. (a)

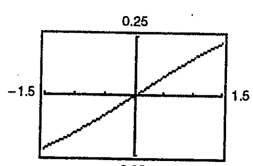
x	1	0.5	0.2	0.1	0.01	0.001	0.0001
$f(x)$	0.1585	0.0411	0.0067	0.0017	≈ 0	≈ 0	≈ 0



$\lim_{x \to 0^+} \dfrac{x - \sin x}{x} = 0$

(b)

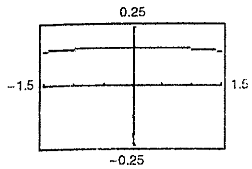
x	1	0.5	0.2	0.1	0.01	0.001	0.0001
$f(x)$	0.1585	0.0823	0.0333	0.0167	0.0017	≈ 0	≈ 0



$\lim_{x \to 0^+} \dfrac{x - \sin x}{x^2} = 0$

(c)

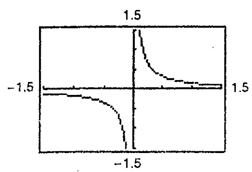
x	1	0.5	0.2	0.1	0.01	0.001	0.0001
$f(x)$	0.1585	0.1646	0.1663	0.1666	0.1667	0.1667	0.1667



$$\lim_{x\to 0^+} \frac{x - \sin x}{x^3} = 0.1667\ (1/6)$$

(d)

x	1	0.5	0.2	0.1	0.01	0.001	0.0001
$f(x)$	0.1585	0.3292	0.8317	1.6658	16.67	166.7	1667.0



$$\lim_{x\to 0^+} \frac{x - \sin x}{x^4} = \infty$$

or $n > 3$, $\displaystyle\lim_{x\to 0^+} \frac{x - \sin x}{x^n} = \infty$.

60. $\displaystyle\lim_{V\to 0^+} P = \infty$

As the volume of the gas decreases, the pressure increases.

61. (a) $r = \dfrac{2(7)}{\sqrt{625 - 49}} = \dfrac{7}{12}$ ft/sec

(b) $r = \dfrac{2(15)}{\sqrt{625 - 225}} = \dfrac{3}{2}$ ft/sec

(c) $\displaystyle\lim_{x\to 25^-} \frac{2x}{\sqrt{625 - x^2}} = \infty$

62. (a) $\text{Average speed} = \dfrac{\text{Total distance}}{\text{Total time}}$

$$50 = \frac{2d}{(d/x) + (d/y)}$$

$$50 = \frac{2xy}{y + x}$$

$$50y + 50x = 2xy$$

$$50x = 2xy - 50y$$

$$50x = 2y(x - 25)$$

$$\frac{25x}{x - 25} = y$$

Domain: $x > 25$

(b)

x	30	40	50	60
y	150	66.667	50	42.857

(c) $\displaystyle\lim_{x\to 25^+} \frac{25x}{\sqrt{x - 25}} = \infty$

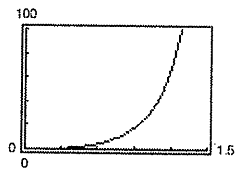
As x gets close to 25 mi/h, y becomes larger and larger.

63. (a) $A = \frac{1}{2}bh - \frac{1}{2}r^2\theta = \frac{1}{2}(10)(10\tan\theta) - \frac{1}{2}(10)^2\theta = 50\tan\theta - 50\theta$

Domain: $\left(0, \frac{\pi}{2}\right)$

(b)

θ	0.3	0.6	0.9	1.2	1.5
$f(\theta)$	0.47	4.21	18.0	68.6	630.1



(c) $\lim_{\theta\to\pi/2^-} A = \infty$

64. (a) Because the circumference of the motor is half that of the saw arbor, the saw makes $1700/2 = 850$ revolutions per minute.

(b) The direction of rotation is reversed.

(c) $2(20\cot\phi) + 2(10\cot\phi)$: straight sections. The angle subtended in each circle is $2\pi - \left(2\left(\frac{\pi}{2} - \phi\right)\right) = \pi + 2\phi$.

So, the length of the belt around the pulleys is $20(\pi + 2\phi) + 10(\pi + 2\phi) = 30(\pi + 2\phi)$.

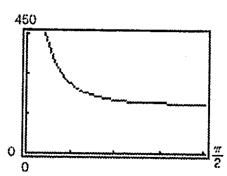
Total length $= 60\cot\phi + 30(\pi + 2\phi)$

Domain: $\left(0, \frac{\pi}{2}\right)$

(d)

ϕ	0.3	0.6	0.9	1.2	1.5
L	306.2	217.9	195.9	189.6	188.5

(e)



(f) $\lim_{\phi\to(\pi/2)^-} L = 60\pi \approx 188.5$

(All the belts are around pulleys.)

(g) $\lim_{\phi\to0^+} L = \infty$

65. False. For instance, let

$$f(x) = \frac{x^2 - 1}{x - 1} \text{ or}$$

$$g(x) = \frac{x}{x^2 + 1}.$$

66. True

67. False. The graphs of $y = \tan x$, $y = \cot x$, $y = \sec x$ and $y = \csc x$ have vertical asymptotes.

68. False. Let

$$f(x) = \begin{cases} \frac{1}{x}, & x \neq 0 \\ 3, & x = 0. \end{cases}$$

The graph of f has a vertical asymptote at $x = 0$, but $f(0) = 3$.

69. Let $f(x) = \frac{1}{x^2}$ and $g(x) = \frac{1}{x^4}$, and $c = 0$.

$\lim_{x\to 0}\frac{1}{x^2} = \infty$ and $\lim_{x\to 0}\frac{1}{x^4} = \infty$, but $\lim_{x\to 0}\left(\frac{1}{x^2} - \frac{1}{x^4}\right) = \lim_{x\to 0}\left(\frac{x^2 - 1}{x^4}\right) = -\infty \neq 0$.

70. Given $\lim_{x\to c} f(x) = \infty$ and $\lim_{x\to c} g(x) = L$:

(1) Difference:

Let $h(x) = -g(x)$. Then $\lim_{x\to c} h(x) = -L$, and $\lim_{x\to c}[f(x) - g(x)] = \lim_{x\to c}[f(x) + h(x)] = \infty$, by the Sum Property.

(2) Product:

If $L > 0$, then for $\varepsilon = L/2 > 0$ there exists $\delta_1 > 0$ such that $|g(x) - L| < L/2$ whenever $0 < |x - c| < \delta_1$. So, $L/2 < g(x) < 3L/2$. Because $\lim_{x\to c} f(x) = \infty$ then for $M > 0$, there exists $\delta_2 > 0$ such that $f(x) > M(2/L)$ whenever $|x - c| < \delta_2$. Let δ be the smaller of δ_1 and δ_2. Then for $0 < |x - c| < \delta$, you have $f(x)g(x) > M(2/L)(L/2) = M$. Therefore $\lim_{x\to c} f(x)g(x) = \infty$. The proof is similar for $L < 0$.

(3) Quotient: Let $\varepsilon > 0$ be given.

There exists $\delta_1 > 0$ such that $f(x) > 3L/2\varepsilon$ whenever $0 < |x - c| < \delta_1$ and there exists $\delta_2 > 0$ such that $|g(x) - L| < L/2$ whenever $0 < |x - c| < \delta_2$. This inequality gives us $L/2 < g(x) < 3L/2$. Let δ be the smaller of δ_1 and δ_2. Then for $0 < |x - c| < \delta$, you have

$$\left|\frac{g(x)}{f(x)}\right| < \frac{3L/2}{3L/2\varepsilon} = \varepsilon.$$

Therefore, $\lim_{x\to c}\frac{g(x)}{f(x)} = 0$.

71. Given $\lim_{x\to c} f(x) = \infty$, let $g(x) = 1$. Then

$\lim_{x\to c}\frac{g(x)}{f(x)} = 0$ by Theorem 1.15.

72. Given $\lim_{x\to c}\frac{1}{f(x)} = 0$. Suppose $\lim_{x\to c} f(x)$ exists and equals L.

Then, $\lim_{x\to c}\frac{1}{f(x)} = \frac{\lim_{x\to c} 1}{\lim_{x\to c} f(x)} = \frac{1}{L} = 0$.

This is not possible. So, $\lim_{x\to c} f(x)$ does not exist.

73. $f(x) = \frac{1}{x - 3}$ is defined for all $x > 3$.

Let $M > 0$ be given. You need $\delta > 0$ such that

$f(x) = \frac{1}{x - 3} > M$ whenever $3 < x < 3 + \delta$.

Equivalently, $x - 3 < \frac{1}{M}$ whenever

$|x - 3| < \delta, x > 3$.

So take $\delta = \frac{1}{M}$. Then for $x > 3$ and

$|x - 3| < \delta, \frac{1}{x - 3} > \frac{1}{8} = M$ and so $f(x) > M$.

74. $f(x) = \frac{1}{x - 5}$ is defined for all $x < 5$. Let $N < 0$ be given. You need $\delta > 0$ such that $f(x) = \frac{1}{x - 5} < N$ whenever $5 - \delta < x < 5$. Equivalently, $x - 5 > \frac{1}{N}$ whenever $|x - 5| < \delta, x < 5$. Equivalently, $\frac{1}{|x - 5|} < -\frac{1}{N}$ whenever $|x - 5| < \delta, x < 5$. So take $\delta = -\frac{1}{N}$. Note that $\delta > 0$ because $N < 0$. For $|x - 5| < \delta$ and $x < 5, \frac{1}{|x - 5|} > \frac{1}{\delta} = -N$, and $\frac{1}{x - 5} = -\frac{1}{|x - 5|} < N$.