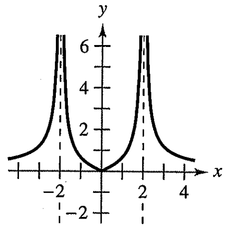
1.5 Exercises

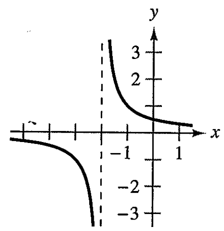
See **CalcChat.com** for tutorial help and worked-out solutions to odd-numbered exercises.

Determining Infinite Limits from a Graph In Exercises 1–4, determine whether $f(x)$ approaches ∞ or $-\infty$ as x approaches -2 from the left and from the right.

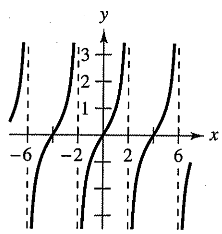
1. $f(x) = 2\left|\dfrac{x}{x^2-4}\right|$



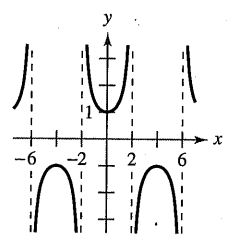
2. $f(x) = \dfrac{1}{x+2}$



3. $f(x) = \tan\dfrac{\pi x}{4}$



4. $f(x) = \sec\dfrac{\pi x}{4}$



Determining Infinite Limits In Exercises 5–8, determine whether $f(x)$ approaches ∞ or $-\infty$ as x approaches 4 from the left and from the right.

5. $f(x) = \dfrac{1}{x-4}$

6. $f(x) = \dfrac{-1}{x-4}$

7. $f(x) = \dfrac{1}{(x-4)^2}$

8. $f(x) = \dfrac{-1}{(x-4)^2}$

Numerical and Graphical Analysis In Exercises 9–12, determine whether $f(x)$ approaches ∞ or $-\infty$ as x approaches -3 from the left and from the right by completing the table. Use a graphing utility to graph the function to confirm your answer.

x	-3.5	-3.1	-3.01	-3.001	-3
$f(x)$					?

x	-2.999	-2.99	-2.9	-2.5
$f(x)$				

9. $f(x) = \dfrac{1}{x^2-9}$

10. $f(x) = \dfrac{x}{x^2-9}$

11. $f(x) = \dfrac{x^2}{x^2-9}$

12. $f(x) = \cot\dfrac{\pi x}{3}$

Finding Vertical Asymptotes In Exercises 13–28, find the vertical asymptotes (if any) of the graph of the function.

13. $f(x) = \dfrac{1}{x^2}$

14. $f(x) = \dfrac{2}{(x-3)^3}$

15. $f(x) = \dfrac{x^2}{x^2-4}$

16. $f(x) = \dfrac{3x}{x^2+9}$

17. $g(t) = \dfrac{t-1}{t^2+1}$

18. $h(s) = \dfrac{3s+4}{s^2-16}$

19. $f(x) = \dfrac{3}{x^2+x-2}$

20. $g(x) = \dfrac{x^3-8}{x-2}$

21. $f(x) = \dfrac{4x^2+4x-24}{x^4-2x^3-9x^2+18x}$

22. $h(x) = \dfrac{x^2-9}{x^3+3x^2-x-3}$

23. $f(x) = \dfrac{x^2-2x-15}{x^3-5x^2+x-5}$

24. $h(t) = \dfrac{t^2-2t}{t^4-16}$

25. $f(x) = \csc \pi x$

26. $f(x) = \tan \pi x$

27. $s(t) = \dfrac{t}{\sin t}$

28. $g(\theta) = \dfrac{\tan\theta}{\theta}$

Vertical Asymptote or Removable Discontinuity In Exercises 29–32, determine whether the graph of the function has a vertical asymptote or a removable discontinuity at $x = -1$. Graph the function using a graphing utility to confirm your answer.

29. $f(x) = \dfrac{x^2-1}{x+1}$

30. $f(x) = \dfrac{x^2-2x-8}{x+1}$

31. $f(x) = \dfrac{x^2+1}{x+1}$

32. $f(x) = \dfrac{\sin(x+1)}{x+1}$

Finding a One-Sided Limit In Exercises 33–48, find the one-sided limit (if it exists).

33. $\displaystyle\lim_{x\to-1^+}\frac{1}{x+1}$

34. $\displaystyle\lim_{x\to1^-}\frac{-1}{(x-1)^2}$

35. $\displaystyle\lim_{x\to2^+}\frac{x}{x-2}$

36. $\displaystyle\lim_{x\to2^-}\frac{x^2}{x^2+4}$

37. $\displaystyle\lim_{x\to-3^-}\frac{x+3}{x^2+x-6}$

38. $\displaystyle\lim_{x\to(-1/2)^+}\frac{6x^2+x-1}{4x^2-4x-3}$

39. $\displaystyle\lim_{x\to0^-}\left(1+\frac{1}{x}\right)$

40. $\displaystyle\lim_{x\to0^+}\left(6-\frac{1}{x^3}\right)$

41. $\displaystyle\lim_{x\to-4^-}\left(x^2+\frac{2}{x+4}\right)$

42. $\displaystyle\lim_{x\to3^+}\left(\frac{x}{3}+\cot\frac{\pi x}{2}\right)$

43. $\displaystyle\lim_{x\to0^+}\frac{2}{\sin x}$

44. $\displaystyle\lim_{x\to(\pi/2)^+}\frac{-2}{\cos x}$

45. $\displaystyle\lim_{x\to\pi^+}\frac{\sqrt{x}}{\csc x}$

46. $\displaystyle\lim_{x\to0^-}\frac{x+2}{\cot x}$

47. $\lim_{x\to(1/2)^-} x \sec \pi x$

48. $\lim_{x\to(1/2)^+} x^2 \tan \pi x$

One-Sided Limit **In Exercises 49–52, use a graphing utility to graph the function and determine the one-sided limit.**

49. $f(x) = \dfrac{x^2 + x + 1}{x^3 - 1}$

$\lim_{x\to 1^+} f(x)$

50. $f(x) = \dfrac{x^3 - 1}{x^2 + x + 1}$

$\lim_{x\to 1^-} f(x)$

51. $f(x) = \dfrac{1}{x^2 - 25}$

$\lim_{x\to 5^-} f(x)$

52. $f(x) = \sec \dfrac{\pi x}{8}$

$\lim_{x\to 4^+} f(x)$

WRITING ABOUT CONCEPTS

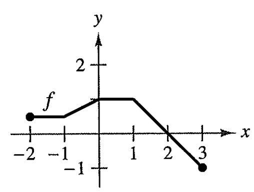
53. Infinite Limit In your own words, describe the meaning of an infinite limit. Is ∞ a real number?

54. Asymptote In your own words, describe what is meant by an asymptote of a graph.

55. Writing a Rational Function Write a rational function with vertical asymptotes at $x = 6$ and $x = -2$, and with a zero at $x = 3$.

56. Rational Function Does the graph of every rational function have a vertical asymptote? Explain.

57. Sketching a Graph Use the graph of the function f (see figure) to sketch the graph of $g(x) = 1/f(x)$ on the interval $[-2, 3]$. To print an enlarged copy of the graph, go to *MathGraphs.com.*



58. Relativity According to the theory of relativity, the mass m of a particle depends on its velocity v. That is,

$$m = \frac{m_0}{\sqrt{1 - (v^2/c^2)}}$$

where m_0 is the mass when the particle is at rest and c is the speed of light. Find the limit of the mass as v approaches c from the left.

59. Numerical and Graphical Analysis Use a graphing utility to complete the table for each function and graph each function to estimate the limit. What is the value of the limit when the power of x in the denominator is greater than 3?

x	1	0.5	0.2	0.1	0.01	0.001	0.0001
$f(x)$							

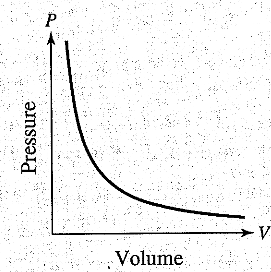
(a) $\lim_{x\to 0^+} \dfrac{x - \sin x}{x}$

(b) $\lim_{x\to 0^+} \dfrac{x - \sin x}{x^2}$

(c) $\lim_{x\to 0^+} \dfrac{x - \sin x}{x^3}$

(d) $\lim_{x\to 0^+} \dfrac{x - \sin x}{x^4}$

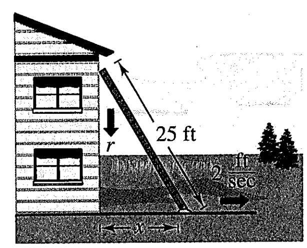
60. HOW DO YOU SEE IT? For a quantity of gas at a constant temperature, the pressure P is inversely proportional to the volume V. What is the limit of P as V approaches 0 from the right? Explain what this means in the context of the problem.



61. Rate of Change A 25-foot ladder is leaning against a house (see figure). If the base of the ladder is pulled away from the house at a rate of 2 feet per second, then the top will move down the wall at a rate of

$$r = \frac{2x}{\sqrt{625 - x^2}} \text{ ft/sec}$$

where x is the distance between the base of the ladder and the house, and r is the rate in feet per second.



(a) Find the rate r when x is 7 feet.

(b) Find the rate r when x is 15 feet.

(c) Find the limit of r as x approaches 25 from the left.

62. Average Speed

On a trip of d miles to another city, a truck driver's average speed was x miles per hour. On the return trip, the average speed was y miles per hour. The average speed for the round trip was 50 miles per hour.

(a) Verify that

$$y = \frac{25x}{x - 25}.$$

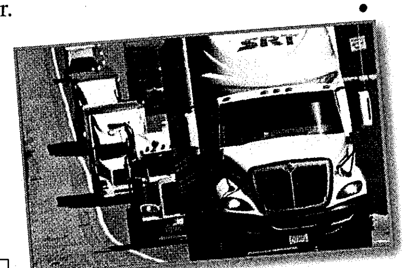
What is the domain?

(b) Complete the table.

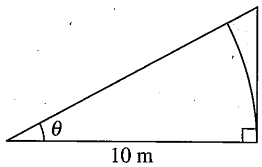
x	30	40	50	60
y				

Are the values of y different than you expected? Explain.

(c) Find the limit of y as x approaches 25 from the right and interpret its meaning.



63. **Numerical and Graphical Analysis** Consider the shaded region outside the sector of a circle of radius 10 meters and inside a right triangle (see figure).



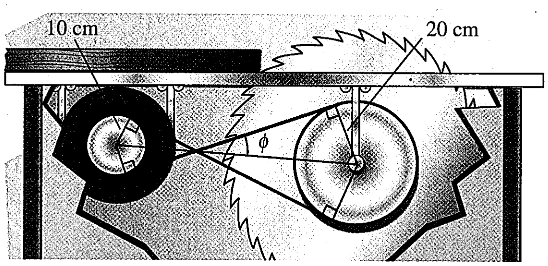
(a) Write the area $A = f(\theta)$ of the region as a function of θ. Determine the domain of the function.

(b) Use a graphing utility to complete the table and graph the function over the appropriate domain.

θ	0.3	0.6	0.9	1.2	1.5
$f(\theta)$					

(c) Find the limit of A as θ approaches $\pi/2$ from the left.

64. **Numerical and Graphical Reasoning** A crossed belt connects a 20-centimeter pulley (10-cm radius) on an electric motor with a 40-centimeter pulley (20-cm radius) on a saw arbor (see figure). The electric motor runs at 1700 revolutions per minute.



(a) Determine the number of revolutions per minute of the saw.

(b) How does crossing the belt affect the saw in relation to the motor?

(c) Let L be the total length of the belt. Write L as a function of ϕ, where ϕ is measured in radians. What is the domain of the function? (*Hint:* Add the lengths of the straight sections of the belt and the length of the belt around each pulley.)

(d) Use a graphing utility to complete the table.

ϕ	0.3	0.6	0.9	1.2	1.5
L					

(e) Use a graphing utility to graph the function over the appropriate domain.

(f) Find $\lim_{\phi \to (\pi/2)^-} L$. Use a geometric argument as the basis of a second method of finding this limit.

(g) Find $\lim_{\phi \to 0^+} L$.

True or False? **In Exercises 65–68, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

65. The graph of a rational function has at least one vertical asymptote.

66. The graphs of polynomial functions have no vertical asymptotes.

67. The graphs of trigonometric functions have no vertical asymptotes.

68. If f has a vertical asymptote at $x = 0$, then f is undefined at $x = 0$.

69. **Finding Functions** Find functions f and g such that $\lim_{x \to c} f(x) = \infty$ and $\lim_{x \to c} g(x) = \infty$, but $\lim_{x \to c} [f(x) - g(x)] \neq 0$.

70. **Proof** Prove the difference, product, and quotient properties in Theorem 1.15.

71. **Proof** Prove that if $\lim_{x \to c} f(x) = \infty$, then $\lim_{x \to c} \frac{1}{f(x)} = 0$.

72. **Proof** Prove that if

$$\lim_{x \to c} \frac{1}{f(x)} = 0$$

then $\lim_{x \to c} f(x)$ does not exist.

Infinite Limits **In Exercises 73 and 74, use the ε-δ definition of infinite limits to prove the statement.**

73. $\lim_{x \to 3^+} \frac{1}{x - 3} = \infty$

74. $\lim_{x \to 5^-} \frac{1}{x - 5} = -\infty$

SECTION PROJECT

Graphs and Limits of Trigonometric Functions

Recall from Theorem 1.9 that the limit of $f(x) = (\sin x)/x$ as x approaches 0 is 1.

(a) Use a graphing utility to graph the function f on the interval $-\pi \leq x \leq \pi$. Explain how the graph helps confirm this theorem.

(b) Explain how you could use a table of values to confirm the value of this limit numerically.

(c) Graph $g(x) = \sin x$ by hand. Sketch a tangent line at the point $(0, 0)$ and visually estimate the slope of this tangent line.

(d) Let $(x, \sin x)$ be a point on the graph of g near $(0, 0)$, and write a formula for the slope of the secant line joining $(x, \sin x)$ and $(0, 0)$. Evaluate this formula at $x = 0.1$ and $x = 0.01$. Then find the exact slope of the tangent line to g at the point $(0, 0)$.

(e) Sketch the graph of the cosine function $h(x) = \cos x$. What is the slope of the tangent line at the point $(0, 1)$? Use limits to find this slope analytically.

(f) Find the slope of the tangent line to $k(x) = \tan x$ at $(0, 0)$.