Inverse function: steps
1) swap x and y 2) Solve for y

$y = b^x$ $x = b^y$

$y = \log_b x$

10.02 Intro to Log Function Date: ________________

A logarithmic function is the *inverse* of an exponential function.

Definition: Let b and y be positive numbers with $b \neq 1$. Then, the ***logarithm of y with base b*** is denoted by $\log_b y$ and is defined as follows:

$$\log_b y = x \text{ if and only if } b^x = y$$

$\log_b y = x$

Examples: Convert from exponential form into logarithmic form.

1. $2^3 = 8$

$\log_2 8 = 3$

2. $5^{-3} = \frac{1}{125}$

$\log_5 \frac{1}{125} = -3$

3. $81^{1/4} = 3$

$\log_{81} 3 = \frac{1}{4}$

Examples: Convert from logarithmic form into exponential form.

4. $\log_5 25 = 2$

$5^2 = 25$

5. $\log_7 \frac{1}{343} = -3$

$7^{-3} = \frac{1}{343}$

6. $\log_{32} 2 = \frac{1}{5}$

$32^{1/5} = 2$

$e \approx 2.72$

- ***Common log*** is log base 10
- Denoted as $\log N$, it is understood to mean $\log_{10} N$
- The LOG button on the calculator evaluates $\log_{10} N$

- ***Natural log*** is log base e
- Denoted as $\ln N$, it is understood to mean $\log_e N$
- The LN button on the calculator evaluates $\log_e N$

Examples: Rewrite in exponential form.

7. $\log 100 = 2$

$\log_{10} 100 = 2$

$\boxed{10^2 = 100}$

8. $\ln 7 = x$

$\log_e 7 = x$

$\boxed{e^x = 7}$

Examples: Rewrite in logarithmic form.

9. $10^{-2} = \frac{1}{100}$

$\log_{10} \frac{1}{100} = -2$

$\boxed{\log \frac{1}{100} = -2}$

10. $e^2 = 7.389$

$\log_e 7.389 = 2$

$\boxed{\ln 7.389 = 2}$

Examples: Evaluate the following logarithms.

11. $\log 1 = x$

$\log_{10} 1 = x$

$10^x = 1$ $\boxed{x = 0}$

12. $\log_{64} 8 = x$

$64^x = 8$

$\boxed{x = 1/2}$

13. $\log_5 \frac{1}{625} = x$

$5^x = \frac{1}{625}$ $\boxed{x = -4}$

10.02 Homework: Evaluate each logarithmic expression.

1. $\log_5 125 = x$

$5^x = 125$

$\boxed{x=3}$

2. $\log_8 1 = x$

$8^x = 1$

$\boxed{x=0}$

3. $\log_6 \frac{1}{36} = x$

$6^x = \frac{1}{36}$ $\boxed{x=-2}$

4. $\log_4 2 = x$

$4^x = 2$

$x = 1/2$

5. $\log_7 -49 = x$

$7^x = -49$

$\boxed{\text{no solution}}$

6. $\log_{10} 10{,}000 = x$

$10^x = 10000$

$\boxed{x=4}$

7. $\ln e^2 = x$

$\log_e e^2 = x$

$e^x = e^2$

$\boxed{x=2}$

8. $\log_{256} 4 = x$

$256^x = 4$

$\boxed{x=1/4}$

9. $\log_{1/5} 25 = x$

$\left(\frac{1}{5}\right)^x = 25$

$\boxed{x=-2}$

10. $\log \sqrt{10} = x$

$\log_{10} \sqrt{10} = x$

$10^x = \sqrt{10}$

$10^x = 10^{1/2}$ $\boxed{x=1/2}$

11. $\log_{1/32} 2 = x$

$\left(\frac{1}{32}\right)^x = 2$

$\boxed{x=-1/5}$

12. $\log_{\sqrt{3}} 27 = x$

$(\sqrt{3})^x = 27$

$\cancel{3}^{\frac{1}{2}x} = \cancel{3}^3$

$\frac{1}{2}x = 3$

$\boxed{x=6}$

13. $\log_2 2^9 = x$

$2^x = 2^9$

$\boxed{x=9}$

14. $3 \cdot \ln e^4 = x$

$3\log_e e^4 = x$

$\log_e e^4 = \frac{x}{3}$

$e^{x/3} = e^4$

$\frac{x}{3} = 4$ $\boxed{x=12}$

15. $\ln -5 = x$

$\log_e -5 = x$

$e^x = -5$

$\boxed{\text{No solution}}$

10.03 Properties of Logarithms Date: ______________

Opener: Simplify the exponential expression.

1. $x^3 * x^7 * x$ $\boxed{x^{11}}$

2. $\frac{m^5 n^2}{m^2 n^9}$ $\boxed{\frac{m^3}{n^7}}$

3. $(g^4)^{11}$ $\boxed{g^{44}}$

Special properties of exponents and logarithms, where *b* is positive and not 1:

$\log_b 1 =$ 0	Why? $b^0 = 1$
$\log_b b =$ 1	Why? $b^1 = b$
$\log_b b^x =$ x	Why? $b^x = b^x$ ✓
$b^{\log_b x} =$ x	Why? $\log_b x = \log_b x$

Properties of Logarithms:

Argument is a Product			
$\log_b u$	$\log_b v$	$\log_b uv$	General Rule:
$\log_2 4 =$	$\log_2 8 =$	$\log_2 32 =$	$\log_b u + \log_b v = \log_b u \cdot v$

Argument is a Quotient			
$\log_b u$	$\log_b v$	$\log_b \frac{u}{v}$	General Rule:
$\log_5 3125 =$	$\log_5 25 =$	$\log_5 125 =$	$\log_b u - \log_b v = \log_b \left(\frac{u}{v}\right)$

Argument is a Power		
$\log_b u^n$	$n \log_b u$	General Rule:
$\log_2 4^5 =$	$5 \log_2 4 =$	$\log_b u^n = n \log_b u$

1) $\log u + \log v = \log uv$

Examples: Use properties of logarithms to expand each expression. The expanded logarithm expressions should have arguments with no exponent, product, or quotient.

2) $\log u - \log v = \log\left(\frac{u}{v}\right)$

3) $\log u^n = n \log u$

1. $\log_5 2x =$ $\log_5 2 + \log_5 x$

2. $\log_2 8a^2b^5 =$ $\log_2 8 + \log_2 a^2 + \log_2 b^5$

$\boxed{\log_2 8 + 2\log_2 a + 5\log_2 b}$

3. $\log_7 \frac{g}{h}$

$\log_7 g - \log_7 h$

4. $\log_4 \frac{16w^3}{x^6}$ $\log_4 16 + \log_4 w^3 - \log_4 x^6 \rightarrow \boxed{\log_4 16 + 3\log_4 w - 6\log_4 x}$

5. $\log \sqrt{r}$ $\log_{10} r^{1/2}$ $\boxed{\frac{1}{2}\log_{10} r}$

6. $\ln \frac{a+1}{\sqrt[3]{b-2c}}$ $\log_e\left(\frac{a+1}{(b-2c)^{1/3}}\right) = \log_e(a+1) - \log_e(b-2c)^{1/3}$

$\boxed{\log_e(a+1) - \frac{1}{3}\log_e(b-2c)}$

Examples: Use properties of logarithms to condense each expression. The condensed logarithm expression should be written as a single logarithm with no coefficient.

7. $3\log 4 - 2\log k$ $\log_{10} 4^3 - \log_{10} k^2$ $\log_{10}\left(\frac{4^3}{k^2}\right) \rightarrow \boxed{\log_{10}\left(\frac{64}{k^2}\right)}$

8. $-5\ \log_2(x+1) + 3\log_2(6x)$ $\log_2(6x)^3 - \log_2(x+1)^5 = \boxed{\log_2 \frac{(6x)^3}{(x+1)^5}}$

9. $\frac{1}{3}\log_4 10 + \frac{1}{3}\log_4 h - 6\log_4 g$ $\log_4 10^{1/3} + \log_4 h^{1/3} - \log_4 g^6 \rightarrow \boxed{\log_4\left(\frac{10^{1/3}h^{1/3}}{g^6}\right)}$

10. $\ln(3m+5) - 4\ln m - \ln(m-1)$ $\log_e(3m+5) - \log_e m^4 - \log_e(m-1)$ $= \log_e\left(\frac{3m+5}{m^4(m-1)}\right)$ or $\ln\left(\frac{3m+5}{m^5-m^4}\right)$

11. $\log 20 + 2\log\frac{1}{2} - \log x + 3\log y$

$\log 20 + \log\left(\frac{1}{2}\right)^2 - \log x + \log y^3 \rightarrow \log\left(\frac{20 \cdot \frac{1}{4} y^3}{x}\right) = \boxed{\log\left(\frac{5y^3}{x}\right)}$