10.03 Properties of Logarithms Date: ____________

Opener: Simplify the exponential expression.

1. $x^3 * x^7 * x^1$ $\boxed{x^{11}}$

2. $\frac{m^5 n^2}{m^2 n^9}$ $= \boxed{\frac{m^3}{n^7}}$

3. $(g^4)^{11}$ $\boxed{g^{44}}$

Special properties of exponents and logarithms, where *b* is positive and not 1:

$\log_b 1 = 0$	Why? $b^0 = 1$
$\log_b b = 1$	Why? $b^1 = b$
$\log_b b^x = x$	Why? $b^x = b^x$
$b^{\log_b x} = x$	Why? $\log_b x = \log_b x$ ✓

Properties of Logarithms:

Argument is a Product			
$\log_b u$	$\log_b v$	$\log_b uv$	General Rule: $\log_b u + \log_b v = \log_b uv$
$\log_2 4 =$	$\log_2 8 =$	$\log_2 32 =$	

Argument is a Quotient			
$\log_b u$	$\log_b v$	$\log_b \frac{u}{v}$	General Rule: $\log_b u - \log_b v = \log_b \left(\frac{u}{v}\right)$
$\log_5 3125 =$	$\log_5 25 =$	$\log_5 125 =$	

Argument is a Power		
$\log_b u^n$	$n \log_b u$	General Rule: $\log_b u^n = n \cdot \log_b u$
$\log_2 4^5 =$	$5 \log_2 4 =$	

Examples: Use properties of logarithms to expand each expression. The expanded logarithm expressions should have arguments with no exponent, product, or quotient.

1. $\log_5 2x =$ $\log_5 2 + \log_5 x$

2. $\log_2 8a^2b^5 =$ $\log_2 8 + \log_2 a^2 + \log_2 b^5 \rightarrow \boxed{\log_2 8 + 2\log_2 a + 5\log_2 b}$

3. $\log_7 \frac{g}{h}$ $\log_7 g - \log_7 h$

4. $\log_4 \frac{16w^3}{x^6} \rightarrow$ $\log_4 16 + \log_4 w^3 - \log_4 x^6 \rightarrow \boxed{\log_4 16 + 3\log_4 w - 6\log_4 x}$

5. $\log\sqrt{r} \rightarrow \log_{10} r^{1/2} \rightarrow \frac{1}{2}\log_{10} r$

6. $\ln\frac{a+1}{\sqrt[3]{b-2c}}$ $\log_e \frac{(a+1)}{(b-2c)^{1/3}} \rightarrow \log_e(a+1) - \log_e(b-2c)^{1/3}$

$\boxed{\log_e(a+1) - \frac{1}{3}\log_e(b-2c)}$

Examples: Use properties of logarithms to condense each expression. The condensed logarithm expression should be written as a single logarithm with ~~no coefficient.~~

7. $3\log 4 - 2\log k$ $\log_{10} 4^3 - \log_{10} k^2 = \boxed{\log_{10}\left(\frac{4^3}{k^2}\right)}$

8. $-5\ \log_2(x+1) + 3\log_2(6x)$ $\log_2(6x)^3 - \log_2(x+1)^5 = \log_2\left(\frac{(6x)^3}{(x+1)^5}\right)$

9. $\frac{1}{3}\log_4 10 + \frac{1}{3}\log_4 h - 6\log_4 g$ $\log_4 10^{1/3} + \log_4 h^{1/3} - \log_4 g^6 = \log_4\left(\frac{10^{1/3}h^{1/3}}{g^6}\right)$

10. $\ln(3m+5) - 4\ln m - \ln(m-1)$ $\log_e(3m+5) - \log_e m^4 - \log_e(m-1)$

$\boxed{\log_e\left(\frac{3m+5}{m^4(m-1)}\right) \rightarrow \ln\left(\frac{3m+5}{m^5-m^4}\right)}$

11. $\log 20 + 2\log\frac{1}{2} - \log x + 3\log y$

$\log_{10}20 + \log_{10}\left(\frac{1}{2}\right)^2 - \log_{10}x + \log_{10}y^3 \rightarrow \log_{10}\left(\frac{20\cdot\frac{1}{4}\cdot y^3}{x}\right) \rightarrow \boxed{\log_{10}\left(\frac{5y^3}{x}\right)}$

10.03 Practice

Use properties of logarithms to expand each expression. The expanded logarithm expressions should have arguments with no exponent, product, or quotient.

1. $\ln\frac{4}{5}$

2. $\log_6 3x$

3. $\log\frac{7b}{\sqrt{c}}$

4. $\log_2\frac{m^4}{8n}$

$\log_2 m^4 - \log_2 8 - \log_2 n \rightarrow \boxed{4\log_2 m - \log_2 8 - \log_2 n}$

5. $\ln\sqrt[3]{10g^2}$

$\log_e(10^1 g^2)^{1/3} \rightarrow \log_e 10^{1/3} g^{2/3} \rightarrow \log_e 10^{1/3} + \log_e g^{2/3}$

$\boxed{\frac{1}{3}\log_e 10 + \frac{2}{3}\log_e g}$

6. $\log_3\frac{u-1}{v^5 w^3}$

$\log_3(u-1) - \log_3 v^5 - \log_3 w^3$

$\boxed{\log_3(u-1) - 5\log_3 v - 3\log_3 w}$

7. $\log\frac{a^2 b}{\sqrt[5]{3a-1}}$

$\log_{10}\frac{a^2 b}{(3a-1)^{1/5}} \rightarrow \log_{10} a^2 + \log_{10} b - \log_{10}(3a-1)^{1/5}$

$\boxed{2\log_{10} a + \log_{10} b - \frac{1}{5}\log_{10}(3a-1)}$

Use properties of logarithms to condense each expression. The condensed logarithm expression should be written as a single logarithm with no coefficient.

8. $\log_5 8 - \log_5 12$

9. $3\ln x + 5\ln y$

10. $10\log k - 2\log 3$

11. $\frac{1}{2}\log_5 36 + \log_5 r - 3\log_5 p$

$\log_5 36^{1/2} + \log_5 r - \log_5 p^3$

$\log_5\left(\frac{36^{1/2} r}{p^3}\right) \rightarrow \boxed{\log_5\left(\frac{6r}{p^3}\right)}$

12. $2\log_8 9 - 3\log_8 c - 4\log_8 d$

$\log_8 9^2 - \log_8 c^3 - \log_8 d^4 \rightarrow \boxed{\log_8\left(\frac{81}{c^3 d^4}\right)}$

13. $3\log n - \frac{1}{2}\log(6-n) + \log 7$

$\log_{10} n^3 - \log_{10}(6-n)^{1/2} + \log_{10} 7$

$\boxed{\log_{10}\left(\frac{7n^3}{(6-n)^{1/2}}\right)}$

14. $\frac{2}{5}\ln 32 - \left(3\ln j - \frac{1}{2}\ln 9\right)$

$\log_e 32^{2/5} - \ln j^3 + \ln 9^{1/2}$

$(32^{1/5})^2 \rightarrow (2)^2 = 4$

$\log_e\left(\frac{32^{2/5} \cdot 9^{1/2}}{j^3}\right) \rightarrow \log_e\left(\frac{4 \cdot 3}{j^3}\right) \rightarrow \boxed{\log_e\left(\frac{12}{j^3}\right)}$

10.03 More Practice with Log Properties Date: _______________

Choose "A" or "B" as the correct answer. Then, explain the mistake in the wrong answer.

		Answer A	Answer B
1.	Expand: $\log\left(\frac{j}{kp}\right)$ $\log j - \log k - \log p$	$\log j - \log k + \log p$	$\log j - \log k - \log p$ ✓
2.	Condense: $\frac{\log a}{4}$ $\frac{1}{4}\log a \to \log a^{1/4}$	$\log\left(\frac{a}{4}\right)$	$\log a^{1/4}$ ✓
3.	Expand: $\log cd^3$ $\log_{10} c + \log_{10} d^3$ $\log c + 3\log d$	$\log c + 3\log d$ ✓	$3\log c + 3\log d$
4.	Condense: $\frac{1}{2}\log m - 4\log r + \log u$ $\log m^{1/2} - \log r^4 + \log u$ $\log\left(\frac{m^{1/2}u}{r^4}\right)$	$\log\frac{\sqrt{m}}{r^4 u}$	$\log\frac{u\sqrt{m}}{r^4}$ ✓
5.	Expand: $\ln\sqrt[5]{z^2}$ $\log_e z^{2/5} \to \frac{2}{5}\log_e z = \frac{2}{5}\ln z$	✓ $\frac{2\ln z}{5}$	$\frac{5\ln z}{2}$
6.	Condense: $\log_2(x+3) + \log_2(x-2)$ $\log_2(x+3)(x-2)$ $\log_2(x^2+3x-2x-6)$	$\log_2(x+1)$	$\log_2(x^2+x-6)$ ✓
7.	Which is equivalent to: $5^x = 100$ $5^x = 100 \to \log_5 100 = x$	$x = \log_5 100$ ✓	$x = \frac{100}{5}$
8.	Which is equivalent to: $e^2 = x$ $\log_e x = 2$ $\boxed{\ln x = 2}$	$\log x = 2$	$\ln x = 2$ ✓
9.	Which is equivalent to: $\log_3 3^{2x}$ $2x$	9^x	$2x$ ✓

$\cancel{\log\left(\frac{a}{4}\right)}$

10.04 Solving Exponential Equations

Date: ______________

Recall the **One-to-One Property of Exponential Functions:**
$b^x = b^y$ if and only if $x = y$.

For this property to work, notice that the *bases must be the same*.
Examples: Solve each equation.

1. $32^{x+3} = 4^{2x+10}$

$\cancel{2}^{5(x+3)} = \cancel{2}^{2(2x+10)}$

$5x+15 = 4x+20$

$1x = 5$ $\boxed{x=5}$

2. $\left(\frac{1}{3}\right)^{2x} = 81^{x-3}$

$\cancel{3}^{-2x} = \cancel{3}^{4(x-3)}$

$-2x = 4x-12$

$-6x = -12$

$\frac{-6x}{-6} = \frac{-12}{-6}$

$\boxed{x=2}$

There is a similar property of logarithms:
One-to-One Property of Logarithmic Functions:
$log_b\, x = log_b\, y$ if and only if $x = y$.

Examples: Solve each equation.
3. $\log_4 x = \log_4 3 + \log_4 (x-2)$

$\cancel{\log_4} x = \cancel{\log_4} 3(x-2)$

$x = 3(x-2)$

$x = 3x-6$

$-2x = -6$

$\boxed{x=3}$

This property also works backwards: if x = y, then $\log_b x = \log_b y$.
This method is often called "*taking the log of both sides*" and is helpful to solve exponential equations.
Examples: Solve each equation.

4. $4^x = 1.5$

5. $3.2e^{2x} + 2.5 = 16.9$

6. $6^{2x+4} = 5^{-x+1}$

7. $2^{3x+11} = 9^{2x+1}$