10.04 Solving Exponential Equations

Date: _____

Recall the **One-to-One Property of Exponential Functions:**
$b^x = b^y$ if and only if $x = y$.

For this property to work, notice that the *bases must be the same*.
Examples: Solve each equation.

1. $32^{x+3} = 4^{2x+10}$

$2^{5(x+3)} = 2^{2(2x+10)}$

$5x+15 = 4x+20$

$1x = 5$ $\quad \boxed{x=5}$

2. $\left(\frac{1}{3}\right)^{2x} = 81^{x-3}$

$3^{-2x} = 3^{4(x-3)}$

$-2x = 4x - 12$

$-6x = -12$

$-6x = -12$
$-6 \quad -6$
$\boxed{x=2}$

There is a similar property of logarithms:
One-to-One Property of Logarithmic Functions:
$\log_b x = \log_b y$ if and only if $x = y$.

Examples: Solve each equation.

3. $\log_4 x = \log_4 3 + \log_4 (x-2)$

$\log_4 x = \log_4 3(x-2)$

$x = 3(x-2)$

$x = 3x - 6$

$-2x = -6$

$\boxed{x=3}$

This property also works backwards: if x = y, then $\log_b x = \log_b y$.
This method is often called "*taking the log of both sides*" and is helpful to solve exponential equations.
Examples: Solve each equation.

4. $4^x = 1.5$

$\log 4^x = \log 1.5$

$\frac{x(\log 4)}{\log 4} = \frac{\log 1.5}{\log 4}$

$\boxed{x = 0.292}$

5. $3.2e^{2x} + 2.5 = 16.9$
$-2.5 \quad -2.5$

$\frac{3.2e^{2x}}{3.2} = \frac{14.4}{3.2}$

$e^{2x} = 4.5$

$\ln e^{2x} = \ln 4.5$

$\frac{2x \ln e}{2} = \frac{\ln 4.5}{2}$

$\boxed{x = 0.752}$

6. $6^{2x+4} = 5^{-x+1}$

$\log 6^{2x+4} = \log 5^{-x+1}$

$(2x+4)\log 6 = (-x+1)\log 5$

$2x\log 6 + 4\log 6 = -x\log 5 + \log 5$

$2x\log 6 + x\log 5 = \log 5 - 4\log 6$

$x(2\log 6 + \log 5) = \log 5 - 4\log 6$

7. $2^{3x+11} = 9^{2x+1}$

$x = \frac{\log 5 - 4\log 6}{2\log 6 + \log 5}$

$\boxed{x = -1.0702}$

let $x = e^x$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

8. $e^{2x} + 2e^x - 8 = 0$

$(e^x+4)(e^x-2) = 0$

$e^x + 4 = 0$ | $e^x - 2 = 0$

$e^x = -4$ | $e^x = 2$

$\ln e^x = \ln(-4)$ | $\ln e^x = \ln 2$

No solution | $x \ln e = \ln 2$

$x = \ln 2$

9. $4e^{2x} + 8e^x = 5$

$4e^{2x} + 8e^x - 5 = 0$

* factor $4x^2 + 8x - 5 = 0$

$(x + \frac{5}{2})(x - \frac{1}{2}) = 0$

$(2x+5)(2x-1) = 0$

$2e^x + 5 = 0$ | $2e^x - 1 = 0$

$e^x = \frac{-5}{2}$ | $2e^x - 1 = 0$

No solution | $e^x = \frac{1}{2}$

$\ln e^x = \ln \frac{1}{2}$

$x \ln e = \ln \frac{1}{2}$

$x = \ln\left(\frac{1}{2}\right)$

-20

10 | -2

4 | 4

8

Practice:
Solve.

1. $4^{x+7} = 8^{x+3}$

2. $\left(\frac{9}{16}\right)^{3x-2} = \left(\frac{3}{4}\right)^{5x+4}$

3. $1.8^x = 9.6$

4. $8^x - 1 = 3.4$

5. $e^{2x} + 5 = 16$

6. $2.5e^{x+4} = 14$

7. $0.75e^{3.4x} - 0.3 = 80.1$

8. $7^{2x+1} = 3^{x+3}$

9. $9^{x+2} = 2^{5x-4}$

10. $e^{2x} - 15e^x + 56 = 0$

11. $6e^{2x} - 5e^x = 6$

12. $300 = \frac{400}{1+3e^{-2x}}$

Pg. 13 10.04 Solving Exponential equations classwork **Odds**

1) $4^{x+7} = 8^{x+3}$

$2^{2(x+7)} = 2^{3(x+3)}$

$2(x+7) = 3(x+3)$

$2x+14 = 3x+9$

$-1x = -5$

$x = 5$

3) $1.8^x = 9.6$

$\log 1.8^x = \log 9.6$

$x(\log 1.8) = \log 9.6$

$x = \frac{\log 9.6}{\log 1.8}$

$x = 3.848$

5) $e^{2x} + 5 = 16$

$e^{2x} = 11$

$\ln e^{2x} = \ln 11$

$2x \ln e = \ln 11$

$2x = \ln 11$

$x = \frac{\ln 11}{2}$

$x = 1.199$

7) $0.75e^{3.4x} - 0.3 = 80.1$

$\frac{0.75e^{3.4x}}{0.75} = \frac{80.4}{0.75}$

$e^{3.4x} = 107.2$

$\ln e^{3.4x} = \ln 107.2$

$3.4x \ln e = \ln 107.2$

$\frac{3.4x}{3.4} = \frac{\ln 107.2}{3.4}$

$x = 1.375$

9) $9^{x+2} = 2^{5x-4}$

$\log 9^{x+2} = \log 2^{5x-4}$

$(x+2)\log 9 = (5x-4)\log 2$

$x\log 9 + 2\log 9 = 5x\log 2 - 4\log 2$

$x\log 9 - 5x\log 2 = -4\log 2 - 2\log 9$

$x(\log 9 - 5\log 2) = -4\log 2 - 2\log 9$

$x = \frac{-4\log 2 - 2\log 9}{\log 9 - 5\log 2}$

$x = 5.650$

11) $6e^{2x} - 5e^{x} = 6$ * let $x = e^{x}$

$6e^{2x} - 5e^{x} - 6 = 0$

* $6x^2 - 5x - 6 = 0$ ← factor this

a·c: -36

$-9 \quad 4$

$6 \quad 6$

-5

$\frac{-9}{6} \to \frac{-3}{2}$

$\frac{4}{6} \to \frac{2}{3}$

$(x - \frac{3}{2})(x + \frac{2}{3}) = 0$

$(2x-3)(3x+2) = 0$

↓

$(2e^{x} - 3)(3e^{x} + 2) = 0$

$2e^{x} - 3 = 0$

$2e^{x} = 3$

$e^{x} = \frac{3}{2}$

$\ln e^{x} = \ln \frac{3}{2}$

$x \cancel{\ln e} = \ln\left(\frac{3}{2}\right)$

$x = \ln\left(\frac{3}{2}\right)$

$3e^{x} + 2 = 0$

$3e^{x} = -2$

$e^{x} = -\frac{2}{3}$

$\ln e^{x} = \cancel{\ln\left(-\frac{2}{3}\right)}$

no solution