10.05 Solving Logarithmic Equations   Date: ______________

The opposite of taking the *log of both sides* is to take *exponentiate both sides*. This can be used to cancel a logarithm from one or more sides of an equation. To do this, make each side of the equation the exponent of the value of the base of the logarithm(s):

If $log_b\, x = y$, then $b^{log_b x} = b^y$.

This may have the effect of converting the logarithm into its exponential form.

$\cancel{\log_2(-24)}$

Also, the argument of a logarithm ***must be positive***. **Check for extraneous solutions before moving on from each problem. Meaning if the value you get for *x* makes the argument either 0 or a negative, you must exclude that value from the solution set.

Examples: Solve each equation.

1. -3 ln x = -24

$\frac{-3}{-3} \quad \frac{-24}{-3}$

$\ln x = 8$

$\log_e x = 8$

$e^8 = x$

$\boxed{x = e^8}$

2. 4 – 3 log (5x) = 16

$-4 \qquad -4$

$-3\log(5x) = 12$

$\frac{}{-3} \qquad \frac{}{-3}$

$\log_{10}(5x) = -4$

$10^{-4} = 5x$

$5x = \frac{1}{10^4}$

$x = \frac{1}{10^4} \cdot \frac{1}{5} = \boxed{\frac{1}{50000}}$

3. log₃ (x – 1) = -2

$3^{-2} = x - 1$

$\frac{1}{9} = x - 1$

$\boxed{x = \frac{10}{9}}$

$\cancel{\log_3}(x-1) = -2$

$\cancel{3}\qquad 3$

$x - 1 = 3^{-2}$

$\boxed{x = \frac{10}{9}}$

4. log₂ (x² – 4) = log₂ 21

$\cancel{2} \qquad \cancel{2}$

$x^2 - 4 = 21$

$x^2 = 25$

$\boxed{x = \pm 5}$

You may have to use properties to change the equation to have <u>at most</u> one logarithm on each side of the equation.

5. 3 log₇ x = log₇ 64

$\cancel{\log_7} x^3 = \cancel{\log_7} 64$

$x^3 = 64$

$\boxed{x = 4}$

6. log₂ 5 = log₂ 10 – log₂ (x – 4)

$\cancel{\log_2} 5 = \cancel{\log_2}\left(\frac{10}{x-4}\right)$

$\frac{5}{1} = \frac{10}{x-4}$

$5(x-4) = 10$

$5x - 20 = 10$

$5x = 30$

$\boxed{x = 6}$

7. log₄ (x – 3) + log₄ (x + 1) = log₄ (6x – 18)

$\cancel{\log_4}(x-3)(x+1) = \log_4(6x-18)$

$x^2 - 3x + 1x - 3 = 6x - 18$

$x^2 - 2x - 3 - 6x + 18 = 0$

$x^2 - 8x + 15 = 0$

$(x-5)(x-3) = 0$

$x = 5, \quad \cancel{x = 3}$

$\boxed{x = 5}$

8. ln (3x – 4) = 1 + ln (2x + 3)

$\ln(3x-4) - \ln(2x+3) = 1$

$\ln\left(\frac{3x-4}{2x+3}\right) = 1$

$\log_e\left(\frac{3x-4}{2x+3}\right) = 1$

$e^1 = \frac{3x-4}{2x+3}$

$\frac{e}{1} = \frac{3x-4}{2x+3}$

$2xe + 3e = 3x - 4$

$2xe - 3x = -3e - 4$

$x(2e - 3) = -3e - 4$

$x = \frac{-3e-4}{2e-3} \to \cancel{-4.988}$

No solution

Practice:

Solve. Don't forget to check for extraneous solutions!

1. $-8\log x = -64$

$-8 \quad -8$

$\log_{10} x = 8$

$10^8 = x$

$\boxed{x = 10^8}$

2. $2 + 3\log 3d = 5$

$-2 \quad -2$

$\frac{3\log(3d)}{3} = \frac{3}{3}$

$\log_{10}(3d) = 1$

$10^1 = 3d$

$\boxed{d = \frac{10}{3}}$

3. $14 + 20\ln 7x = 54$

$-14 \quad -14$

$\frac{20\ln(7x)}{20} = \frac{40}{20}$

$\ln(7x) = 2$

$\log_e(7x) = 2$

$e^2 = 7x$

$\boxed{x = \frac{e^2}{7}}$

4. $7{,}000\ln x = -21{,}000$

5. $log_8(x^2 + 11) = log_8 92$

6. $log_{11}3x = log_{11}(x + 5) - log_{11}2$

7. $\ln x + \ln(x + 7) = \ln 18$

8. $\ln(3x + 1) + \ln(2x - 3) = ln10$

9. $\ln(x - 3) + \ln(2x + 3) = \ln(-4x^2)$

10. $log(5x^2 + 4) = 2log3x^2 - log(2x^2 - 1)$

11. $\log(3x + 2) = 1 + log2x$

12. $log_9 9x - 2 = -log_9 x$

4) $7000 \ln x = -21000$

$\frac{7000 \ln x}{7000} = \frac{-21000}{7000}$

$\ln x = -3$

$\log_e x = -3$

$e^{-3} = x$

$\boxed{x = \frac{1}{e^3}}$

5) $\log_8(x^2+11) = \log_8 92$

$x^2 + 11 = 92$

$x^2 = 81$

$x = \pm 9$

$\boxed{x = 9,\ x = -9}$

6) $\log_{11} 3x = \log_{11}(x+5) - \log_{11}(2)$

$\log_{11} 3x = \log_{11}\left(\frac{x+5}{2}\right)$

$\frac{3x}{1} = \frac{x+5}{2}$

$6x = x + 5$

$5x = 5$

$\boxed{x = 1}$

7) $\ln(x) + \ln(x+7) = \ln 18$

$\ln(x)(x+7) = \ln 18$

$\cancel{\log_e}(x^2+7x) = \cancel{\log_e} 18$

$x^2 + 7x = 18$

$x^2 + 7x - 18 = 0$

$(x-2)(x+9) = 0$

$x = 2,\ x = \cancel{-9}$ extraneous solution

$\boxed{x = 2}$

8) $\ln(3x+1) + \ln(2x-3) = \ln(10)$

$\ln(3x+1)(2x-3) = \ln(10)$

$\ln(6x^2+2x-9x-3) = \ln 10$

$\ln(6x^2-7x-3) = \ln 10$

$\cancel{\log_e}(6x^2-7x-3) = \cancel{\log_e} 10$

$6x^2 - 7x - 3 = 10$

$6x^2 - 7x - 13 = 0$

$\left(x - \frac{13}{6}\right)\left(x + \frac{6}{6}\right)$

$(6x-13)(x+1) = 0$

$x = 13/6,\ x = \cancel{-1}$ (extraneous)

$\boxed{x = 13/6}$

9) $\ln(x-3)+\ln(2x+3)=\ln(-4x^2)$

$\ln(x-3)(2x+3)=\ln(-4x^2)$

$\ln(2x^2-6x+3x-9)=\ln(-4x^2)$

$\log_e(2x^2-3x-9)=\log_e(-4x^2)$

$2x^2-3x-9=-4x^2$

$6x^2-3x-9=0$

$3(2x^2-1x-3)=0$

$3\left(x-\frac{3}{2}\right)\left(x+\frac{2}{2}\right)=0$

$3(2x-3)(x+1)=0$

$2x-3=0$ | $x+1=0$

$x=3/2$ | $x=-1$

Both are solutions. extraneous

$\boxed{\text{No solution}}$

10) $\log(5x^2+4)=2\log 3x^2-\log(2x^2-1)$

$\log(5x^2+4)=\log(3x^2)^2-\log(2x^2-1)$

$\log(5x^2+4)=\log(9x^4)-\log(2x^2-1)$

$\log(5x^2+4)=\log\left(\frac{9x^4}{2x^2-1}\right)$

$\frac{5x^2+4}{1}=\frac{9x^4}{2x^2-1}$

$(5x^2+4)(2x^2-1)=9x^4(1)$

$10x^4+8x^2-5x^2-4=9x^4$

$1x^4+3x^2-4=0$

$(x^2+4)(x^2-1)=0$

$x^2+4=0$ | $x^2-1=0$

$\sqrt{x^2}=\sqrt{-4}$ | $\sqrt{x^2}=\pm\sqrt{1}$

No solution | $\boxed{x=1,-1}$

11) $\log(3x+2) = 1 + \log(2x)$

$\log(3x+2) - \log(2x) = 1$

$\log\left(\frac{3x+2}{2x}\right) = 1$

$\log_{10}\left(\frac{3x+2}{2x}\right) = 1$

$10^1 = \frac{3x+2}{2x}$

$\frac{10}{1} = \frac{3x+2}{2x}$

$20x = 3x + 2$

$17x = 2$

$\boxed{x = \frac{2}{17}}$

12) $\log_9(9x) - 2 = -\log_9(x)$

$\log_9(9x) + \log_9(x) = 2$

$\log_9(9x)(x) = 2$

$\log_9(9x^2) = 2$

$9^2 = 9x^2$

$81 = 9x^2$

$9 = x^2$

$\pm\sqrt{9} = \sqrt{x^2}$

$\pm 3 = x$

$\boxed{x = 3}$, $x \cancel{=} -3$

Extraneous solution