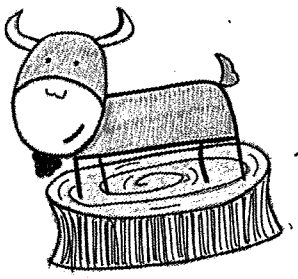
10.06 p. 16-17

Name: _____ Date: _____ Period: _____



DOODLE-ING MATH

Why do Lumberjacks Make Good Music?

Directions: For each problem, solve the exponential or logarithmic function. Doodle or color on the lumberjack below according to your answer choice.

1. $2^{3x-19} = 4$

$2^{3x-19} = 2^2$

$3x - 19 = 2$
$+19 \quad +19$

$3x = 21$

$\boxed{x = 7}$

If your answer is 7 color his pants grey.
If your answer is -7 color his pants blue.

2. $\log_2 x + \log_2(x+7) = 3$

$\log_2 x(x+7) = 3$

$\log_2 x^2 + 7x = 3$

$2^3 = x^2 + 7x$

$x^2 + 7x - 8 = 0$

$(x+8)(x-1) = 0$

$x = \cancel{-8}$ AND $\boxed{1}$

CHECK:
① $\cancel{\log_2 -8} + \log \ldots$
② $\log_2 1 + \log_2(1+7) = 3$
$\log_2 1 + \log_2 8 = 3$
$\log_2 8 = 3$
$3 = 3$ ✓

If your answer is 1 and -8 draw the following hat:

If your answer is 1 draw the following hat:

3. $2e^{2x} - 7e^x + 6 = 0$

12
3 4

$u = e^x$

$2u^2 - 7u + 6 = 0$

$(u - \frac{4}{2})(u - \frac{3}{2}) = 0$

$(u-2)(2u-3) = 0$

$u = 2 \quad 2u = 3$
$u = \frac{3}{2}$

$e^x = 2$
$\ln e^x = \ln 2$
$x = \ln 2 \approx \boxed{.69}$

$e^x = \frac{3}{2}$
$\ln e^x = \ln \frac{3}{2}$
$x = \ln \frac{3}{2} \approx \boxed{.41}$

If your answer is .69 and .41 color his hat red.
If your answer is 4.48 and 7.39 color his hat blue.

4. $\log(x+1) - \log(3x-2) = \log\left(\frac{2}{x}\right)$

$\cancel{\log} \frac{x+1}{3x-2} = \cancel{\log} \frac{2}{x}$

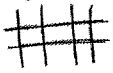
$\frac{x+1}{3x-2} = \frac{2}{x} \Rightarrow x^2 + x = 6x - 4$
$-6x - 4 \quad -6x + 4$

$5x + 4 = 0$

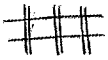
$\boxed{x = 4 \text{ AND } 1}$

CHECK:
$\log 5 - \log 10 = \log \frac{1}{2}$
$\log \frac{5}{10} = \log \frac{1}{2}$ ✓
$\log 2 - \log 1 = \log 2$
$\log 2 = \log 2$ ✓

If your answer is 1 draw the following pattern on his hat:



If your answer is 1 and 4 draw the following pattern on his hat:



5. $7^x = 156$

$\ln 7^x = \ln 156$

$\frac{x \ln 7}{\ln 7} = \frac{\ln 156}{\ln 7}$

$x = \frac{\ln 156}{\ln 7} \approx 2.595$

If your answer is 22.3 draw a sledge hammer in his hands.

If your answer is 2.595 draw a guitar in his hands.

6. $3^{x^2-7} = 27^{2x}$

$3^{x^2-7} = 3^{3(2x)}$

$x^2 - 7 = 6x$
$-6x$

$x^2 - 6x - 7 = 0$

$(x-7)(x+1)$

$\boxed{x = 7 \text{ AND } -1}$

If your answer is 7 and -1, draw the following shoes on the lumberjack:

If your answer is -7 and 1 draw the following shoes on the lumberjack:

7. $\frac{6\ln(2x)}{6} = \frac{12}{6}$

$\ln(2x) = 2$

$e^{\ln(2x)} = e^2$

$\frac{2x}{2} = \frac{e^2}{2} \quad x = \frac{e^2}{2} \approx 3.69$

If your answer is 3.69 color his shoes yellow.
If your answer is 0 color his shoes brown.

8. $\log(x-3) = \log(7x-23) - \log(x+1)$

$\log(x-3) = \log \frac{7x-23}{x+1}$

$x - 3 = \frac{7x-23}{x+1}$

$(x+1)(x-3) = 7x - 23$

$x^2 - 2x - 3 = 7x - 23$

$x^2 - 9x + 20 = 0$

$(x-5)(x-4) = 0$

$\boxed{x = 4 \text{ AND } 5}$

CHECK:
$\log 1 = \log 5 - \log 5$
$\log 1 = \log \frac{5}{5}$
$\log 1 = \log 1$ ✓
$\log 2 = \log 12 - \log 6$
$\log 2 = \log \frac{12}{6}$
$\log 2 = \log 2$ ✓

If your answer is -10 and 2 color his jacket red.
If your answer is 4 and 5 color his jacket blue.

9. $2\log_4 x - \log_4(x-1) = 1$

$\log_4 x^2 - \log_4(x-1) = 1$

$\log_4 \frac{x^2}{x-1} = 1$

$\left(\frac{x^2}{x-1}\right) = 4^1$

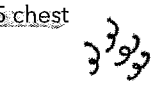
$\frac{x^2}{x-1} = 4$

$x^2 = 4x - 4$
$x^2 - 4x + 4 = 0$
$(x-2)(x-2) = 0$

$\boxed{x = 2}$

CHECK:
$\underbrace{\log_4 4}_{=1} - \underbrace{\log_4 1}_{=0} = 1$
$1 + 0 = 1 \quad 1 = 1$ ✓

If your answer is 1 draw 3 chest hairs on the lumberjack.
If your answer is 2 draw 5 chest hairs on the lumberjack.



10. $\log_2(x+2)-\log_2(x-5)=3$

$\frac{x+2}{x-5}=2^3$

$\frac{x+2}{x-5}=8$

$x+2=8x-40$
$-8x-2 \quad -8x-2$

$-7x=-42$

$\boxed{x=6}$

CHECK:
$\underbrace{\log_2 8}_{\log_2 2^3} - \underbrace{\log_2 1}_{=0} = 3$

$3-0=3$

$3=3$ ✓

If your answer is 6, draw a pile of 6 logs next to the lumberjack.
If your answer is -4, draw a pile of 4 logs next to the lumberjack.

11. $27^{2x+4}=9^{x-2}$

$3^{3(2x+4)}=3^{2(x-2)}$

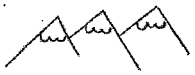
$3^{6x+12}=3^{2x-4}$

$6x+12=2x-4$
$-2x-12 \quad -2x-12$

$4x=-16$

$\boxed{x=-4}$

If your answer is -6, draw 3 snowcapped mountains:



If your answer is -4, draw 3 mountains:

12. $\log_4(x-6)=-2$

$x-6=4^{-2}$

$x-6=\frac{1}{16}+6$
$+6 \quad +6$

$x=\frac{1}{16}+\frac{96}{16}=\boxed{\frac{97}{16}}$

CHECK:
$\log_4\left(\frac{97}{16}-6\right)=-2$

$\log_4\left(\frac{97}{16}-\frac{96}{16}\right)=-2$

$\log_4\left(\frac{1}{16}\right)=-2$

$4^{-2}=\frac{1}{16}$ ✓

If your answer is $\frac{291}{48}$ draw 2 trees near your log pile.
If your answer is $\frac{97}{16}$ draw 1 tree near your log pile.

13. $\ln\sqrt{2x-4}=0$

$e \qquad e$

$(\sqrt{2x-4})^2=(1)^2$

$2x-4=1$
$+4 \quad +4$

$\frac{2x}{2}=\frac{5}{2}$

$\boxed{x=\frac{5}{2}}$

If your answer is 2, draw the following mustache on the lumberjack:

If your answer is 2.5 draw the following mustache on the lumberjack:

14. $e^{2x}-5e^x+6=0$

$(e^x-3)(e^x-2)=0$

$e^x=3 \qquad e^x=2$

$x=\ln 3 \qquad x=\ln 2$
$\approx .69 \qquad \approx 1.099$

If your answer is .69 and 1.099 color his beard brown and his mustache grey.
If your answer is 1.792 color his beard grey and his mustache brown.

15. $\log_2(3x-1)=3$

$2 \qquad 2$

$3x-1=8$
$+1 \quad +1$

$3x=9$

$\boxed{x=3}$

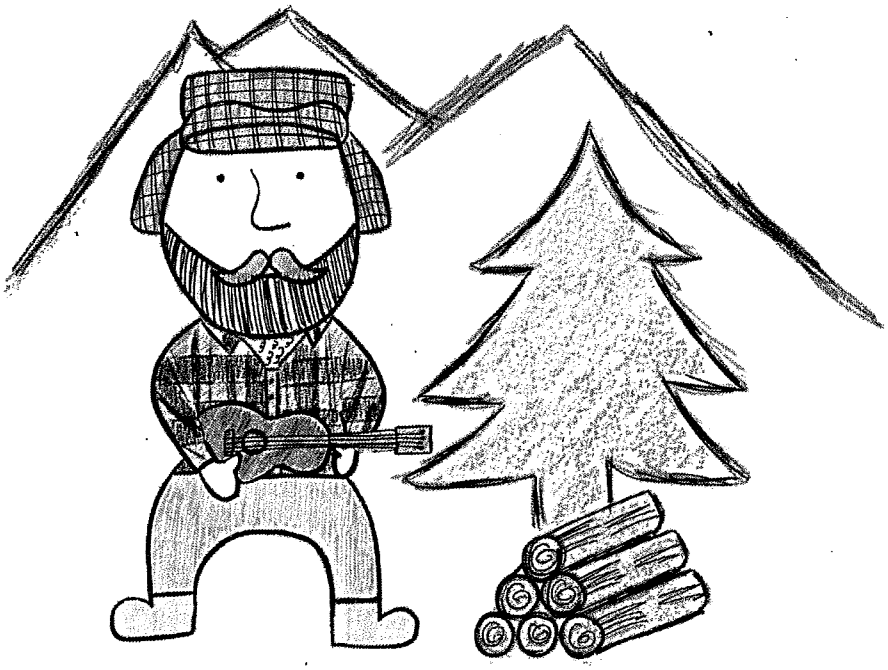
CHECK:
$\underbrace{\log_2 8}_{\log_2 2^3}=3$

$3=3$ ✓

If your answer is 3, the answer to the puzzle is: "because they've got natural logarithm."
If your answer is $\frac{10}{3}$ the answer to the puzzle is: "because they've got great logarithm."

Why do Lumberjacks Make Good Music?

15. BECAUSE THEY'VE GOT NATURAL LOGARITHM!



$\boxed{10.06}$ $\boxed{p.16}$

1) $2^{3x-19} = 4$

$2^{3x-19} = 2^2$

$3x - 19 = 2$

$3x = 21$

$\boxed{x = 7}$

2) $\log_2 x + \log_2(x+7) = 3$

$\log_2 x(x+7) = 3$

$\log_2(x^2+7x) = 3$

$2^3 = x^2 + 7x$

$x^2 + 7x - 8 = 0$

$(x+8)(x-1) = 0$

$\cancel{x = -8}$, $x = 1$

$\boxed{x = 1}$

3) $2e^{2x} - 7e^x + 6 = 0$

* let $x = e^x$

$* \; 2x^2 - 7x + 6 = 0$

a·c

12

-4 -3

2 2

-7

$(x-2)(x-3/2)$

$(x-2)(2x-3) = 0$

original scenario $(e^x - 2)(2e^x - 3) = 0$

$e^x - 2 = 0$

$e^x = 2$

$\log e^x = \log 2$

$x \log e = \log 2$

$x = \frac{\log 2}{\log e} = \boxed{0.693}$

$2e^x - 3 = 0$

$e^x = \frac{3}{2}$

$\log e^x = \log(3/2)$

$x \log e = \log(3/2)$

$x = \frac{\log(3/2)}{\log e} = \boxed{0.405}$

4) $\log(x+1) - \log(3x-2) = \log\left(\frac{2}{x}\right)$

$\cancel{\log}\left(\frac{x+1}{3x-2}\right) = \cancel{\log}\left(\frac{2}{x}\right)$

$\cancel{\frac{x+1}{3x-2} = \frac{2}{x}}$

$x(x+1) = 2(3x-2)$

$x^2 + x = 6x - 4$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$\boxed{x = 4,\ x = 1}$

5) $7^x = 156$

$\log 7^x = \log 156$

$x \log 7 = \log 156$

$x = \frac{\log 156}{\log 7} = \boxed{2.595}$

6) $3^{x^2-7} = 27^{2x}$

$\cancel{3}^{x^2-7} = \cancel{3}^{3(2x)}$

$x^2 - 7 = 6x$

$x^2 - 6x - 7 = 0$

$(x-7)(x+1) = 0$

$\boxed{x = 7,\ x = -1}$

7) $\frac{6\ln(2x)}{6} = \frac{12}{6}$

$\ln(2x) = 2$

$\log_e(2x) = 2$

$e^2 = 2x$

$\frac{e^2}{2} = x$

$\boxed{x = \frac{e^2}{2}} \approx \boxed{3.695}$

8) $\log(x-3) = \log(7x-23) - \log(x+1)$

$\cancel{\log}(x-3) = \cancel{\log}\left(\frac{7x-23}{x+1}\right)$

$\frac{x-3}{1} = \frac{7x-23}{x+1}$

$(x-3)(x+1) = (7x-23)$

$x^2-2x-3 = 7x-23$

$x^2-9x+20=0$

$(x-4)(x-5)=0$

$\boxed{x=4,\ x=5}$

9) $2\log_4 x - \log_4(x-1) = 1$

$\log_4 x^2 - \log_4(x-1) = 1$

$\log_4\left(\frac{x^2}{x-1}\right) = 1$

$\frac{4^1}{1} = \frac{x^2}{x-1}$

$x^2 = 4x-4$

$x^2-4x+4=0$

$(x-2)(x-2)=0$

$\boxed{x=2}$

10) $\log_2(x+2) - \log_2(x-5) = 3$

$\log_2\left(\frac{x+2}{x-5}\right) = 3$

$2^3 = \frac{x+2}{x-5}$

$\frac{8}{1} = \frac{x+2}{x-5}$

$8(x-5) = x+2$

$8x-40 = x+2$

$7x = 42$

$\boxed{x=6}$

11) $27^{2x+4} = 9^{x-2}$

$\cancel{3}^{3(2x+4)} = \cancel{3}^{2(x-2)}$

$3(2x+4) = 2(x-2)$

$6x+12 = 2x-4$

$4x = -16$

$\boxed{x = -4}$

12) $\log_4(x-6) = -2$

$4^{-2} = x-6$

$\frac{1}{4^2} = x-6$

$\frac{1}{16} + 6 = x$

$x = \frac{1}{16} + \frac{96}{16} = \boxed{\frac{97}{16}}$

13) $\ln\sqrt{2x-4} = 0$

$\log_e\sqrt{2x-4} = 0$

$e^0 = \sqrt{2x-4}$

$1 = \sqrt{2x-4}$

$(1)^2 = (\sqrt{2x-4})^2$

$1 = 2x-4$

$5 = 2x$

$\boxed{5/2 = x}$

14) $e^{2x} - 5e^{x} + 6 = 0$ *let $x = e^{x}$

$* \; x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

Bring e^x back $\rightarrow$ $(e^x - 3)(e^x - 2) = 0$

$e^x - 3 = 0$	$e^x - 2 = 0$
$e^x = 3$	$e^x = 2$
$\log e^x = \log 3$	$\ln e^x = \ln 2$
$x \log e = \log 3$	$x \cancel{\ln e} = \ln 2$
$x = \frac{\log 3}{\log e} = \boxed{0.691}$	$x = \ln 2$ $\boxed{x = 1.099}$

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15) $\log_2(3x-1) = 3$

$2^3 = 3x - 1$	$3x = 9$
$8 = 3x - 1$	$\boxed{x = 3}$ ✓
$9 = 3x$	