

10.07 Solving Exponential and Log Equations Solve the exponential/log equations

1.

$$5^{2x-1} = 5^4$$

$$2x-1=4$$

$$2x=5$$

$$\boxed{x = 5/2}$$

2.

$$\frac{3 \log_2(x+2)}{3} = \frac{6}{3}$$

$$\log_2(x+2) = 2$$

$$2^2 = x+2$$

$$4-2=x$$

$$\boxed{x=2} \checkmark$$

3.

$$\ln x + \ln 4 = 2$$

$$\ln(4x) = 2$$

$$\log_e(4x) = 2$$

$$e^2 = 4x$$

$$\frac{e^2}{4} = x$$

$$\boxed{x = \frac{e^2}{4}} \checkmark$$

4.

$$2 \log x - \log 4 = 2$$

$$\log x^2 - \log 4 = 2$$

$$\log\left(\frac{x^2}{4}\right) = 2$$

$$\log_{10}\left(\frac{x^2}{4}\right) = 2$$

$$10^2 = \frac{x^2}{4}$$

$$10^2 \cdot 4 = x^2$$

$$400 = x^2$$

$$x = \pm \sqrt{400}$$

$$\boxed{x=20} \quad x=-20$$

Extraneous
solution

5.

$$e^{2x} = 25$$

$$\ln e^{2x} = \ln 25$$

$$2x \ln e = \ln 25$$

$$\boxed{x = \frac{\ln 25}{2}}$$

6.

$$\log(3x-5) = \log(2x-1)$$

$$3x-5 = 2x-1$$

$$1x = 4$$

$$\boxed{x=4} \checkmark$$

7.

$$\frac{1}{25} = 5^{3x+2}$$

$$5^{-2} = 5^{3x+2}$$

$$3x+2 = -2$$

$$3x = -4$$

$$\boxed{x = -\frac{4}{3}}$$

8.

$$3^{2x-7} = 27^x$$

$$3^{2x-7} = 3^{3x}$$

$$3x = 2x-7$$

$$\boxed{x = -7}$$

9.

$$4^{x-1} = 4^3$$

$$x-1 = 3$$

$$\boxed{x=4}$$

10.

$$5^{2x+3} = 125^x$$

$$5^{2x+3} = 5^{3x}$$

$$2x+3 = 3x$$

$$3 = x$$

$$\boxed{x=3}$$

11.

$$\log(6x) = \log(4x+5)$$

$$6x = 4x+5$$

$$2x = 5$$

$$\boxed{x=5/2} \checkmark$$

12.

$$2 \ln e^x = 9$$

$$\frac{2}{2} \ln e^x = \frac{9}{2}$$

$$\ln e^x = \frac{9}{2}$$

$$x \ln e = \frac{9}{2}$$

$$\boxed{x=9/2}$$

13.

$$2 \log(3x) - \log 9 = 1$$

$$\log(3x)^2 - \log 9 = 1$$

$$\log\left(\frac{9x^2}{9}\right) = 1$$

$$\log(x^2) = 1$$

$$\log_{10}(x^2) = 1$$

$$10^1 = x^2$$

$$\boxed{x=\sqrt{10}}$$

$-\sqrt{10}$
extraneous
solution

14.

$$2 \ln x + \ln x^2 = 4$$

$$\ln x^2 + \ln x^2 = 4 \quad \ln x = 1$$

$$\ln x^4 = 4$$

$$\frac{4 \ln x}{4} = \frac{4}{4}$$

$$\log_e x = 1$$

$$e^1 = x$$

$$\boxed{x=e} \checkmark$$

15.

$$\frac{1}{16} = 4^{3x-1}$$

$$4^{-2} = 4^{3x-1}$$

$$3x-1 = -2$$

$$3x = -1$$

$$\boxed{x=-1/3}$$

16.

$$\frac{3 \ln e^{2x}}{3} = \frac{30}{3}$$

$$\ln e^{2x} = 10$$

$$2x \ln e = 10$$

$$2x = 10$$

$$x = \frac{10}{2}$$

$$\boxed{x=5} \checkmark$$

17.

$$\ln e^{x+5} = 17$$

$$(x+5) \ln e = 17$$

$$x+5 = 17$$

$$\boxed{x=12} \checkmark$$

18.

$$\log x - \log 4 = 3$$

$$\log\left(\frac{x}{4}\right) = 3$$

$$\log_{10}\left(\frac{x}{4}\right) = 3$$

$$10^3 = \frac{x}{4}$$

$$1000 \cdot 4 = x$$

$$\boxed{x=4000} \checkmark$$

19.

$$\log(8-3x) = \log(7-5x)$$

$$8-3x = 7-5x$$

$$1 = -2x$$

$$\boxed{-\frac{1}{2} = x} \checkmark$$

20.

$$\frac{5 \ln(3x-2)}{5} = \frac{15}{5}$$

$$\ln(3x-2) = 3$$

$$\log_e(3x-2) = 3$$

$$e^3 = 3x-2$$

$$\frac{e^3+2}{3} = \frac{3x}{3}$$

$$\boxed{x = \frac{e^3+2}{3}} \checkmark$$

21.

$$\log(3x-2) = 3$$

$$\log_{10}(3x-2) = 3$$

$$10^3 = 3x-2$$

$$1000 = 3x-2$$

$$1002 = 3x$$

$$\frac{1002}{3} = x$$

$$\boxed{x=334} \checkmark$$

22.

$$4^{x-1} = 64^x$$

$$4^{x-1} = 4^{3(x)}$$

$$x-1 = 3x$$

$$-1 = 2x$$

$$\boxed{-\frac{1}{2} = x}$$

23.

$$\frac{4 \ln x}{4} = \frac{-2}{4}$$

$$\ln x = -\frac{1}{2}$$

$$\log_e x = -\frac{1}{2}$$

$$e^{-\frac{1}{2}} = x \rightarrow \boxed{x = \frac{1}{e^{1/2}}} \checkmark$$

$$\ln x^4 = -2$$

$$\log_e x^4 = -2$$

$$e^{-2} = x^4$$

$$\frac{1}{e^2} = x^4$$

$$\sqrt[4]{\frac{1}{e^2}} = x$$

$$x = \left(\frac{1}{e^2}\right)^{1/4} = \boxed{\frac{1}{e^{1/2}}}$$

24.

$$e^{x-4} = 2$$

$$\ln e^{(x-4)} = \ln 2$$

$$(x-4) \ln e = \ln 2$$

$$x-4 = \ln 2$$

$$\boxed{x = 4 + \ln 2}$$

25.

$$\ln x + \ln 4x = 16$$

$$\ln x(4x) = 16 \quad e^{16} = 4x^2$$

$$\ln(4x^2) = 16$$

$$\log_e(4x^2) = 16$$

$$\frac{e^{16}}{4} = x^2$$

$$\pm \sqrt{\frac{e^{16}}{4}} \rightarrow \left(\frac{e^{16}}{4}\right)^{1/2} \rightarrow x = \frac{e^8}{2}, \quad -\frac{e^8}{2} \text{ extraneous solution}$$

26.

$$4 \log x = -4$$

$$\log_{10} x = -1$$

$$10^{-1} = x$$

$$x = \frac{1}{10} \quad \checkmark$$

27.

$$\frac{1}{4} = 2^{2x-3}$$

$$\cancel{x}^{-2} = \cancel{2}^{2x-3}$$

$$2x-3 = -2$$

$$2x = 1$$

$$x = \frac{1}{2}$$

28.

$$\log 5x = \log(9 + 8x)$$

$$5x = 9 + 8x$$

$$-3x = 9$$

$$x = -3$$

Extraneous solution
No solution

29.

$$\ln(x-1) - \ln 2 = 3$$

$$\ln\left(\frac{x-1}{2}\right) = 3 \quad \frac{e^3}{1} = \frac{x-1}{2}$$

$$\log_e\left(\frac{x-1}{2}\right) = 3$$

$$x-1 = 2e^3$$

$$x = 2e^3 + 1 \quad \checkmark$$

$$e^3 = \frac{x-1}{2}$$

30.

$$\ln x = -1$$

$$\log_e x = -1$$

$$e^{-1} = x$$

$$x = \frac{1}{e}$$

31.

$$e^{\frac{x}{5}} = 32$$

$$\ln e^{\frac{x}{5}} = \ln 32$$

$$\cancel{\frac{x}{5}} \ln e = \ln 32$$

$$x = 5 \ln 32$$

32.

$$\frac{-2 \log_3 6x}{-2} = \frac{2}{-2}$$

$$\log_3(6x) = -1$$

$$3^{-1} = 6x$$

$$\frac{1}{3} = 6x$$

$$\frac{1}{3} \cdot \frac{1}{6} = x$$

$$x = \frac{1}{18} \quad \checkmark$$