

BC Calculus Unit 10 "Tests for Convergence" Quiz Review WS #1

Calculators Allowed:

1. Which of the following series converges?

(A) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{n+3}{3n}\right)$

(B) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{n^2}{2\sqrt{n}}\right)$

(C) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{2\sqrt{n}}{n}\right)$

(D) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{4-n}{n}\right)$

2. What is the value of $\sum_{n=1}^{\infty} \frac{2^{n+1}}{7^n}$?

3. Which of the following series can be used with the Limit Comparison Test to determine whether the series

$\sum_{n=1}^{\infty} \frac{2^n}{3^n - n^2}$ converges or diverges?

(A) $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$

(B) $\sum_{n=1}^{\infty} \frac{1}{3^n}$

(C) $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$

(D) $\sum_{n=1}^{\infty} \frac{1}{n}$

4. **Calculator active.** Find the sequence of partial sums $S_1, S_2, S_3, S_4,$ and S_5 for the infinite series $\sum_{n=1}^{\infty} \frac{3}{2^{n-1}}$.

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5. Verify that the infinite series $\sum_{n=1}^{\infty} \frac{3^n + 1}{3^{n+2}}$ diverges by using the n th-Term Test for Divergence. Show the value of the limit.

6. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{3^n}{n!}$

II. $\sum_{n=1}^{\infty} \frac{n}{8^n}$

III. $\sum_{n=1}^{\infty} \frac{2}{n\sqrt{n}}$

(A) I only

(B) I and II only

(C) I and III only

(D) I, II, and III

7. For what values of x is the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n}$ conditionally convergent?

(A) $x > -1$

(B) $x = -3$

(C) $x = -1$

(D) $x = 3$

8. Use the Integral Test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

9. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n^{-1}}{\sqrt{n}}$

II. $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$

III. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

- (A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III
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10. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{3}{1+2^n}$ is true?

(A) Diverges by the n th Term test.

(B) Diverges by comparison to $\sum_{n=1}^{\infty} \frac{1}{2^n}$.

(C) Converges by comparison to $\sum_{n=1}^{\infty} \frac{1}{2^n}$.

(D) Diverges by comparison to $\sum_{n=1}^{\infty} \frac{1}{n}$.

11. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n}{n^2+3}$ is true?

(A) The series diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(B) The series diverges by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(C) The series converges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

(D) The series converges by the Alternating Series Test.

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12. Which of the following is required in order to apply the Integral Test to the series $\sum_{n=1}^{\infty} a_n$?

(A) $\lim_{n \rightarrow \infty} a_n = 0$ and $\sum_{n=1}^{\infty} a_n$ is a positive series.

(B) $\lim_{n \rightarrow \infty} a_n \neq 0$ and $\sum_{n=1}^{\infty} a_n$ is a convergent series.

(C) $a_n = f(n)$ and $f(x)$ is positive, continuous, and increasing on $[1, \infty)$.

(D) $a_n = f(n)$ and $f(x)$ is positive, continuous, and decreasing on $[1, \infty)$.

13. If $a_n > 0$ for all n and $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \frac{2}{3}$, which of the following series converges?

(A) $\sum_{n=1}^{\infty} 3^n a_n$

(B) $\sum_{n=1}^{\infty} \frac{2^n}{a_n}$

(C) $\sum_{n=1}^{\infty} a_n \left(\frac{7}{2}\right)^n$

(D) $\sum_{n=1}^{\infty} \frac{(a_n)^2}{3^n}$

14. The infinite series $\sum_{n=1}^{\infty} \frac{1}{7^{n+1}}$ has n th partial sum $S_n = \frac{1}{6} \left(\frac{1}{7} - \frac{1}{7^{n+1}} \right)$ for $n \geq 1$. What is the sum of the series?

15. For what value of r does the infinite series $\sum_{n=0}^{\infty} 10r^n$ equal 22?

16. Determine whether the series $\sum_{n=1}^{\infty} \frac{\sin \left[\frac{(2n-1)\pi}{2} \right]}{n}$ converges absolutely, converges conditionally, or diverges.

17. Determine the convergence of the infinite p -series $\sum_{n=1}^{\infty} n^{-p}$.

18. The n th-Term Test can be used to determine divergence for which of the following series?

I. $\sum_{n=1}^{\infty} \frac{2}{n+1}$

II. $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{n}{4n+1} \right)$

III. $\sum_{n=1}^{\infty} \frac{n(n-2)^2}{3n^3+1}$

(A) III only

(B) II and III only

(C) I and III only

(D) I, II, and III

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BC Calculus Unit 10 "Tests for Convergence" Quiz Review WS #2

Calculators Allowed:

1. The infinite series $\sum_{n=1}^{\infty} a_n$ has n th partial sum $S_n = \frac{4^n - 1}{4^{n+1}}$ for $n \geq 1$. What is the sum of the series?

2. Which of the following series diverge?

I. $\sum_{n=1}^{\infty} \frac{1}{n^2(n+3)}$

II. $\sum_{n=1}^{\infty} \frac{n^2 2^{n+1}}{3^n}$

III. $\sum_{n=1}^{\infty} \frac{n!}{n4^n}$

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

3. The n th-Term Test can be used to determine divergence for which of the following series?

I. $\sum_{n=1}^{\infty} \frac{2n+1}{1-n}$

II. $\sum_{n=0}^{\infty} 5\left(\frac{2}{3}\right)^n$

III. $\sum_{n=1}^{\infty} \frac{2n(n-1)^2}{4n^2 - 3n^3}$

(A) I and II only (B) II and III only (C) I and III only (D) I, II, and III

4. If b and t are real numbers such that $0 < |t| < |b|$, what is the sum of $b^2 \sum_{n=0}^{\infty} \left(\frac{t^2}{b^2}\right)^n$?

5. Explain why the Integral Test does not apply to the series $\sum_{n=1}^{\infty} \frac{3}{n^{-2}}$.

6. For what values of p will the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^{3p+1}}$ converge?

7. **Calculator active.** Which of the following series matches the following sequence of partial sums 0.1667, 0.3333, 0.4833, 0.6167, 0.7357, ...?

(A) $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$

(B) $\sum_{n=1}^{\infty} \frac{n}{(n+1)(n+2)}$

(C) $\sum_{n=1}^{\infty} \frac{n+1}{n+2}$

(D) $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+3)}$

8. For what values of x is the series $\sum_{n=1}^{\infty} \frac{(7x-3)^n}{n}$ conditionally convergent?

(A) $x = \frac{2}{7}$

(B) $x = \frac{4}{7}$

(C) $x > \frac{4}{7}$

(D) $x < \frac{2}{7}$

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9. Which of the following series can be used with the Limit Comparison Test to determine whether the series

$$\sum_{n=1}^{\infty} \frac{3n+2}{n^3-2n} \text{ converges or diverges?}$$

(A) $\sum_{n=1}^{\infty} \frac{1}{n}$

(B) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

(C) $\sum_{n=1}^{\infty} \frac{1}{n^3}$

(D) $\sum_{n=1}^{\infty} \frac{1}{n^3-2n}$

10. Verify that the infinite series $\sum_{n=1}^{\infty} \frac{7^{n+1}-2}{7^{n+2}}$ diverges by using the n th-Term Test for Divergence. Show the value of the limit.

11. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{2^n}{9^{n+n}}$ is true?

(A) The series diverges by the n th Term Test.

(B) The series diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(C) The series converges by comparison with $\sum_{n=1}^{\infty} \frac{2^n}{9^n}$.

(D) The series converges by comparison with $\sum_{n=1}^{\infty} \frac{1}{9^n}$.

12. Which of the following series converge by the Alternating Series Test?

I. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

II. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$

III. $\sum_{n=1}^{\infty} (-1)^n \left(\frac{\pi}{e}\right)^n$

A. I only

B. I and II only

C. I and III only

D. I, II, and III

13. Which of the following series is absolutely convergent?

I. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n^4}}$

II. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

III. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

(A) I only

(B) I and II only

(C) I and III only

(D) I, II, and III

14. Use the Integral test to determine if the series $\sum_{n=1}^{\infty} \frac{3n^2}{n^3 + 1}$ converges or diverges.

15. Which of the following statements are true about the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n+1}{n}$?

I. $a_{n+1} \leq a_n$ for all $n \geq 1$.

II. $\lim_{n \rightarrow \infty} a_n \neq 0$

III. The series converges by the Alternating Series Test

A. I only

B. I and II only

C. II and III only

D. I, II, and III

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16. What are all values of $x > 0$ for which the series $\sum_{n=1}^{\infty} \frac{n^2 x^{n+1}}{7^n}$ converges.

17. Which of the following is a convergent p -series?

(A) $\sum_{n=1}^{\infty} n^4$

(B) $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$

(C) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2}}$

(D) $\sum_{n=1}^{\infty} \left(\frac{1}{n^3}\right)^{\frac{1}{2}}$

18. Consider the series $\sum_{n=1}^{\infty} a_n$. If $\frac{a_{n+1}}{a_n} = \frac{1}{2}$ for all integers $n \geq 1$, and $\sum_{n=1}^{\infty} a_n = 64$, then $a_1 = ?$

Answers to Mid-Unit 10 · WS#2

1. $\frac{1}{4}$	2. C	3. C	4. $\frac{b^4}{b^2-t^2}$	5. $f(n)$ is not a decreasing function for $n \geq 1$.
6. $p > 0$	7. B	8. A	9. B	
10. Diverges by n th-Term Test, $\lim_{n \rightarrow \infty} a_n = \frac{1}{7}$	11. C	12. B	13. B	
14. $\int_1^{\infty} f(x) dx = \infty$, Series Diverges	15. B	16. $x < 7$	17. D	18. 32

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BC Calculus Unit 10 "Tests for Convergence" Review WS #3

Calculators Allowed:

- 1) The infinite series $\sum_{n=1}^{\infty} \frac{3}{4^{n+1}}$ has n th partial sum $S_n = \frac{1}{4} - \frac{1}{4^{n+1}}$. What is the sum of the series?

Use the nth-Term Test for Divergence to determine if the series diverges.		
<p>2. $\sum_{n=0}^{\infty} \frac{\pi^{n+1}}{7^n}$</p>	<p>3. $\sum_{n=1}^{\infty} \frac{2(n-2)^2}{3(n+4)^2}$</p>	<p>4. $\sum_{n=1}^{\infty} \frac{1}{e^n}$</p>

- 5) If the infinite series $\sum_{n=1}^{\infty} a^n$ has n th partial sum $S_n = \frac{4}{3}(4^n - 1)$ for $n \geq 1$. What is the sum of the series?

- 6) Does the series $\sum_{n=1}^{\infty} \left(\frac{1}{2n-1} - \frac{1}{2n+1} \right)$ converge or diverge? If it converges find its sum.

- 7) What is the sum of the infinite geometric series $11 + -\frac{11}{3} + \frac{11}{9} + -\frac{11}{27} + \dots$?

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8) What is the value of $\sum_{n=1}^{\infty} \frac{(-e)^{n+1}}{9^n}$?

9) For what value of a does the infinite series $\sum_{n=0}^{\infty} a \left(-\frac{3}{5}\right)^n$ equal 15?

10) The n th-Term Test can be used to determine divergence for which of the following series?

I. $\sum_{n=1}^{\infty} \frac{(n+1)^3}{3n^3 - 2n + 1}$

II. $\sum_{n=1}^{\infty} \frac{(n+1)^2}{2n^2 - 3n^3 + 1}$

III. $\sum_{n=1}^{\infty} \ln \frac{1}{n}$

- A. III only
- B. I and III only
- C. II and III only
- D. I, II, and III

11) Verify that the infinite series $\sum_{n=1}^{\infty} \frac{6^n + 1}{6^{n+1}}$ diverges by using the n th-Term Test for Divergence. Show the value of the limit.

12) Use the Integral Test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^5}$.

13) Prove the Integral Test applies to the series $\sum_{n=1}^{\infty} \frac{1}{(n+1)^3}$. Determine the convergence or divergence of the series.

14) Use the Integral Test to determine if the series $\sum_{n=1}^{\infty} \frac{4n}{2n^2+1}$ converges or diverges.

15) For what values of p will the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^{1-p}}$ converge?

16) For what values of p will both infinite series $\sum_{n=1}^{\infty} \left(\frac{3}{p}\right)^n$ and $\sum_{n=1}^{\infty} \frac{1}{n^{5-p}}$ converge?

17) Which of the following is a divergent p -series?

A. $\sum_{n=1}^{\infty} n^{-\pi}$

B. $\sum_{n=1}^{\infty} \frac{1}{n}$

C. $\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^n$

D. $\sum_{n=1}^{\infty} \frac{1}{n^3}$

18) Which of the following series converges?

(A) $\sum_{n=1}^{\infty} \frac{3n}{2n^2 + 1}$

(B) $\sum_{n=1}^{\infty} \frac{3n^2}{n + 2n^2}$

(C) $\sum_{n=1}^{\infty} \left(\frac{\pi}{e}\right)^n$

(D) $\sum_{n=1}^{\infty} \frac{3n^2}{2n^3 + 3n}$

(E) $\sum_{n=1}^{\infty} \frac{n-1}{n4^n}$

19) Which of the following series can be used with the Limit Comparison Test to determine whether the series

$$\sum_{n=1}^{\infty} \frac{5^n}{7^n - n^2}$$

diverges or converges?

(A) $\sum_{n=1}^{\infty} \frac{1}{n}$

(B) $\sum_{n=1}^{\infty} \frac{1}{5^n}$

(C) $\sum_{n=1}^{\infty} \frac{1}{7^n}$

(D) $\sum_{n=1}^{\infty} \left(\frac{5}{7}\right)^n$

20) Use the Comparison Test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n-2}{n5^n}$. You must identify the series you are using for comparison.

21) Determine whether the series $\sum_{n=1}^{\infty} \frac{n5^n}{4n^4 - 3}$ converges or diverges. Identify the test for convergence used.

22) Explain why the Alternating Series Test does not apply to the series $\sum_{n=1}^{\infty} \frac{(-1)^n \cos(n\pi)}{n^2}$.

23) Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n n}{\ln(n+1)}$.

24) Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{(-1)^n}{4^n}$

II. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\pi^n}$

III. $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{1+n^2}$

- A. I only B. I and II only C. I and III only D. I, II, and III

25) Which of the following statements is true?

A. $\sum_{n=1}^{\infty} \frac{(-1)^n (1-n)}{n}$ converges by the Alternating Series Test.

B. $\sum_{n=1}^{\infty} \frac{(-1)^n (n+1)}{2n}$ converges by the Alternating Series Test.

C. $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{4\sqrt{n}}$ converges by the Alternating Series Test.

D. $\sum_{n=1}^{\infty} \frac{(-1)^n 2\sqrt{n}}{n}$ converges by the Alternating Series Test.

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26) Use the Ratio Test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n^4}{3^n}$.

27) If the Ratio Test is applied to the series $\sum_{n=0}^{\infty} \frac{6^n}{(n+1)^n}$, which of the following inequalities results, implying that the series converges?

A. $\lim_{n \rightarrow \infty} \frac{6^n}{(n+1)^n} < 1$

B. $\lim_{n \rightarrow \infty} \frac{6(n+1)^n}{(n+2)^{n+1}} < 1$

C. $\lim_{n \rightarrow \infty} \frac{6^{n+1}}{(n+1)^n} < 1$

D. $\lim_{n \rightarrow \infty} \frac{6^{n+1}}{(n+1)^{n+1}} < 1$

28) If $a_n > 0$ for all n and $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = 5$, which of the following series converges?

A. $\sum_{n=1}^{\infty} \frac{a_n}{n^2}$

B. $\sum_{n=1}^{\infty} \frac{a_n}{2^n}$

C. $\sum_{n=1}^{\infty} \frac{a_n}{n^5}$

D. $\sum_{n=1}^{\infty} \frac{a_n}{7^n}$

29) What are all values of $x > 0$ for which the series $\sum_{n=1}^{\infty} \frac{6n^3}{x^n}$ converges?

30) Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{1}{n!}$

II. $\sum_{n=1}^{\infty} \frac{9^n}{n^5}$

III. $\sum_{n=1}^{\infty} \frac{5n}{2n-1}$

A. I only

B. I and II only

C. I and III only

D. I, II, and III

31) For what values of x is the series $\sum_{n=0}^{\infty} (-1)^n (5x+1)^n$ absolutely convergent?

32) For what values of x is the series $\sum_{n=1}^{\infty} \frac{(5x-2)^n}{n}$ conditionally convergent?

A. $x > \frac{3}{5}$

B. $x = \frac{3}{5}$

C. $x = \frac{1}{5}$

D. $x < \frac{1}{5}$

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33) Which of the following statements is true about the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2\sqrt{n}}$.

- A. The series converges conditionally.
- B. The series converges absolutely.
- C. The series converges but neither conditionally nor absolutely.
- D. The series diverges.

34) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+5}$ converges absolutely, converges conditionally, or diverges.

35) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)!}$ converges absolutely, converges conditionally, or diverges.