Name: Key Review WS #1 Date: Units 10.1 - 10.6 Period: ____ **Review**

# Mid-Unit 10 Review – Infinite Sequences and Series

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 10.

1. Which of the following series converges?

(A) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{n+3}{3n}\right)$ (B) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{n^2}{2\sqrt{n}}\right)$ (C) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{2\sqrt{n}}{n}\right)$ (D) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{4-n}{n}\right)$

(A) $\lim_{n\to\infty}\frac{n+3}{3n}=\frac{1}{3}\neq 0$ Diverges by nth term test

(B) $\lim_{n\to\infty}\left(\frac{n^2}{2\sqrt{n}}\right)=\infty$

(C) $\boxed{\lim_{n\to\infty}\frac{2\sqrt{n}}{n}\to\frac{2}{\sqrt{n}}=0}$ Converges by AST

(D) $\lim_{n\to\infty}\left(\frac{4-n}{n}\right)=1\neq 0$

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2. What is the value of $\sum_{n=1}^{\infty}\frac{2^{n+1}}{7^n}$? $\to \frac{2^n\cdot 2^1}{7^n}\to\sum_{n=1}^{\infty}2\left(\frac{2}{7}\right)^n$ Geometric Series, $r=2/7$ (converges)

$a_1=2\left(\frac{2}{7}\right)=\frac{4}{7}$

$S=\frac{a_1}{1-r}\to\frac{4/7}{1-2/7}\to\frac{4/7}{5/7}=\boxed{\frac{4}{5}}$

3. Which of the following series can be used with the Limit Comparison Test to determine whether the series $\sum_{n=1}^{\infty}\frac{2^n}{3^n-n^2}$ converges or diverges?

$\frac{2^n}{3^n}\to\left(\frac{2}{3}\right)^n$

(A) $\sum_{n=1}^{\infty}\left(\frac{3}{2}\right)^n$ (B) $\sum_{n=1}^{\infty}\frac{1}{3^n}$ (C) $\boxed{\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n}$ (D) $\sum_{n=1}^{\infty}\frac{1}{n}$

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4. **Calculator active.** Find the sequence of partial sums $S_1, S_2, S_3, S_4$, and $S_5$ for the infinite series $\sum_{n=1}^{\infty}\frac{3}{2^{n-1}}$.

$S_1=\frac{3}{2^0}=\boxed{3}$

$S_2=3+\frac{3}{2}=\boxed{4.5}$

$S_3=4.5+\frac{3}{2^2}=\boxed{5.25}$

$S_4=5.25+\frac{3}{2^3}=\boxed{5.625}$

$S_5=5.625+\frac{3}{2^4}=\boxed{5.8125}$

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5. Verify that the infinite series $\sum_{n=1}^{\infty} \frac{3^n+1}{3^{n+2}}$ diverges by using the $n$th-Term Test for Divergence. Show the value of the limit.

$$\lim_{n\to\infty} \frac{3^n+1}{3^2(3^n)} \to \boxed{\frac{1}{9} \neq 0}$$

Diverges by $n^{th}$-Term Test

6. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{3^n}{n!}$  II. $\sum_{n=1}^{\infty} \frac{n}{8^n}$  III. $\sum_{n=1}^{\infty} \frac{2}{n\sqrt{n}}$

Ratio Test

$$\lim_{n\to\infty} \frac{3^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n}$$

$$\lim_{n\to\infty} \frac{\cancel{3^n} \cdot 3 \cdot \cancel{n!}}{(n+1)\cdot \cancel{n!} \cdot \cancel{3^n}} \to \lim_{n\to\infty} \frac{3}{(n+1)} = 0 < 1$$

Converges by Ratio Test

Ratio Test

$$\lim_{n\to\infty} \frac{n+1}{8^{n+1}} \cdot \frac{8^n}{n} \to \frac{n+1}{n} \cdot \frac{\cancel{8^n}}{\cancel{8^n} \cdot 8}$$

$$\lim_{n\to\infty} \frac{n+1}{8n} = \frac{1}{8} < 1$$

Converges by Ratio Test

$$\to \sum \frac{2}{n \cdot n^{1/2}}$$

$$2 \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$

p-series $p = 3/2 > 1$

Converges

(A) I only  (B) I and II only  (C) I and III only  $\boxed{\text{(D) I, II, and III}}$

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7. For what values of $x$ is the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n}$ conditionally convergent?

*Ratio Test

$$\lim_{n\to\infty} \left| \frac{(x+2)^{n+1}}{n+1} \cdot \frac{n}{(x+2)^n} \right| < 1$$

$$\lim_{n\to\infty} \left| \frac{\cancel{(x+2)^n} \cdot (x+2)}{\cancel{(x+2)^n}} \cdot \frac{n}{(n+1)} \right| < 1$$

$$|x+2| < 1$$

$$-1 < x+2 < 1$$

$$-3 < x < -1$$

$\boxed{\text{If } x=-3}$

$$\sum_{n=1}^{\infty} \frac{(-3+2)^n}{n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

Alternating harmonic series converges conditionally

$\boxed{\text{If } x=-1}$

$$\sum_{n=1}^{\infty} \frac{(1)^n}{n}$$

harmonic series diverges

(A) $x > -1$  $\boxed{\text{(B) } x = -3}$  (C) $x = -1$  (D) $x = 3$

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8. Use the Integral Test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^2}$.

Let $f(x) = \dfrac{e^{1/x}}{x^2}$ $\to f(x)$ is continuous, positive, decreasing

$$\lim_{b\to\infty} \int_1^b \frac{e^{x^{-1}}}{x^2} dx \to \lim_{b\to\infty} \int_1^b \frac{e^u}{\cancel{x^2}} \cdot -\cancel{x^2} du$$

$u = x^{-1}$

$\frac{du}{dx} = -1x^{-2}$

$\frac{du}{dx} = \frac{-1}{x^2}$

$-dx = x^2 du$

$dx = -x^2 du$

$$-\int e^u du \to -e^u \to -e^{1/x} \Big]_1^b$$

$$\lim_{b\to\infty} -e^{1/b} - (-e^{1/1})$$

$$-e^0 + e = \boxed{-1+e}$$

Series Converges by Integral Test

9. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n^{-1}}{\sqrt{n}}$ II. $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$ III. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

$\sum \frac{1}{n^{1/2} \cdot n} \rightarrow \frac{1}{n^{3/2}}$

$p = 3/2 > 1$

converges by p-series test

$r = \frac{2}{3} < 1$

converges by Geometric Series Test

Integral Test

let $f(x) = \frac{1}{x \ln x}$

$f(x)$ is positive, continuous, decreasing

$\lim_{b \to \infty} \int_2^b \frac{1}{x \ln x} dx$

$u = \ln x$ | $dx = x\,du$

$\frac{du}{dx} = \frac{1}{x}$

$\int \frac{1}{\cancel{x} \cdot u} \cancel{x}\,du$

$\int \frac{1}{u} du \rightarrow \ln|u| \rightarrow \ln|\ln x| \Big|_2^b$

$\lim_{b \to \infty} \ln|\ln b| - \ln|\ln 2|$

$\infty - \ln|\ln 2|$

Diverges by Integral Test

(A) I only (B) II only (C) III only $\boxed{\text{(D) I and II only}}$ (E) I, II, and III

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10. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{3}{1+2^n}$ is true?

nth term test inconclusive

$\lim_{n \to \infty} \frac{3}{1+2^n} = 0$

(A) Diverges by the $n$th Term test.

(B) Diverges by comparison to $\sum_{n=1}^{\infty} \frac{1}{2^n}$.

$\boxed{\text{(C)}}$ Converges by comparison to $\sum_{n=1}^{\infty} \frac{1}{2^n}$.

(D) Diverges by comparison to $\sum_{n=1}^{\infty} \frac{1}{n}$.

compare with $\frac{1}{2^n}$ (converges by Geometric Series) $\rightarrow \left(\frac{1}{2}\right)^n$

$r = \frac{1}{2} < 1$

$0 \leq \frac{3}{1+2^n} < \frac{1}{2^n}$

By Direct Comparison Test, series also converge.

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11. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{n^2+3}$ is true?

✓ $\lim_{n \to \infty} \frac{n}{n^2+3} = 0$

✓ series is decreasing

(A) The series diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(B) The series diverges by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(C) The series converges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

$\boxed{\text{(D)}}$ The series converges by the Alternating Series Test.

12. Which of the following is required in order to apply the Integral Test to the series $\sum_{n=1}^{\infty} a_n$?

(A) $\lim_{n\to\infty} a_n = 0$ and $\sum_{n=1}^{\infty} a_n$ is a positive series.

(B) $\lim_{n\to\infty} a_n \neq 0$ and $\sum_{n=1}^{\infty} a_n$ is a convergent series.

(C) $a_n = f(n)$ and $f(x)$ is positive, continuous, and increasing on $[1, \infty)$.

$\boxed{\text{(D)}}$ $a_n = f(n)$ and $f(x)$ is positive, continuous, and decreasing on $[1, \infty)$.

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13. If $a_n > 0$ for all $n$ and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{2}{3}$, which of the following series converges? *Ratio Test

(A) $\sum^{\infty} 3^n a_n$

$\lim_{n\to\infty} \left| \frac{3^{n+1} \cdot a_{n+1}}{3^n \cdot a_n} \right|$

$\lim_{n\to\infty} \left| \frac{\cancel{3^n} \cdot 3 \cdot a_{n+1}}{\cancel{3^n}\ a_n} \right|$

$\to 3 \cdot \left(\frac{2}{3}\right) = 2 > 1$ (Diverges)

(B) $\sum^{\infty} \frac{2^n}{a_n}$

$\lim_{n\to\infty} \left| \frac{2^{n+1}}{a_{n+1}} \cdot \frac{a_n}{2^n} \right|$

$\lim_{n\to\infty} \left| \frac{\cancel{2^n} \cdot 2 \cdot a_n}{\cancel{2^n}\ a_{n+1}} \right|$

$2 \cdot \left(\frac{3}{2}\right) = 3 > 1$ (Diverges)

(C) $\sum^{\infty} a_n \left(\frac{7}{2}\right)^n$

$\lim_{n\to\infty} \frac{a_{n+1} \cdot \left(\frac{7}{2}\right)^{n+1}}{a_n \cdot \left(\frac{7}{2}\right)^n}$

$= \frac{2}{3} \cdot \left(\frac{7}{2}\right) = 7/3 > 1$

Diverges

$\boxed{\text{(D) } \sum^{\infty} \frac{(a_n)^2}{3^n}}$

$\lim_{n\to\infty} \frac{(a_{n+1})^2}{3^{n+1}} \cdot \frac{3^n}{(a_n)^2}$

$\to \frac{\cancel{3^n}}{\cancel{3^n} \cdot 3} \cdot \left(\frac{a_{n+1}}{a_n}\right)^2$

$\to \frac{1}{3} \cdot \left(\frac{2}{3}\right)^2 \to \frac{4}{27} < 1$

(converges)

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14. The infinite series $\sum_{n=1}^{\infty} \frac{1}{7^{n+1}}$ has $n$th partial sum $S_n = \frac{1}{6}\left(\frac{1}{7} - \frac{1}{7^{n+1}}\right)$ for $n \geq 1$. What is the sum of the series?

* Sum of Series is $\lim_{n\to\infty} S_n$

$\lim_{n\to\infty} \frac{1}{6}\left[\frac{1}{7} - \frac{1}{7^{n+1}}\right] = \frac{1}{6}\left(\frac{1}{7} - 0\right) = \boxed{\frac{1}{42}}$

15. For what value of $r$ does the infinite series $\sum_{n=0}^{\infty} 10r^n$ equal 22?

Geometric Series

$S = \frac{a_1}{1-r}$

$a_1 = 10r^0 = 10$

$S = \frac{a_1}{1-r}$

$22 = \frac{10}{1-r}$

$10 = 22(1-r)$

$10 = 22 - 22r$

$22r = 12$

$r = \frac{12}{22}$

$\boxed{r = \frac{6}{11}}$

16. Determine whether the series $\sum_{n=1}^{\infty} \frac{\sin\left[\frac{(2n-1)\pi}{2}\right]}{n}$ converges absolutely, converges conditionally, or diverges.

$n=1 \rightarrow \sin\left(\frac{\pi}{2}\right) = 1$

$n=2 \rightarrow \sin\left(\frac{3\pi}{2}\right) = -1$

$n=3 \rightarrow \sin\left(\frac{5\pi}{2}\right) = 1$

$n=4 \rightarrow \sin\left(\frac{7\pi}{2}\right) = -1$

Same as $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ → Alternating Harmonic series

→ $\lim_{n\to\infty} \frac{1}{n} = 0$ ✓

$\frac{1}{n}$ is decreasing ✓

$\boxed{\text{Series converges conditionally}}$

17. Determine the convergence of the infinite $p$-series $\sum_{n=1}^{\infty} n^{-\pi}$.

$\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)^{\pi} \rightarrow \frac{1}{n^{\pi}}$ $\boxed{p = \pi > 1}$

By p-series test, Series converges.

18. The $n$th-Term Test can be used to determine divergence for which of the following series?

I. $\sum_{n=1}^{\infty} \frac{2}{n+1}$

$\lim_{n\to\infty} \frac{2}{n+1} = 0$

(Inconclusive)

II. $\sum_{n=1}^{\infty} (-1)^{n+1}\left(\frac{n}{4n+1}\right)$

$\lim_{n\to\infty} \frac{n}{4n+1} = \frac{1}{4} \neq 0$

Diverges

III. $\sum_{n=1}^{\infty} \frac{n(n-2)^2}{3n^3+1}$ → $\frac{n(n^2-4n+4)}{3n^3+1}$

$\lim_{n\to\infty} \frac{n^3-4n^2+4n}{3n^3+1} = \frac{1}{3} \neq 0$

Diverges

(A) III only

$\boxed{\text{(B) II and III only}}$

(C) I and III only

(D) I, II, and III

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**BC Calculus Unit 10 "Tests for Convergence" Quiz Review WS #2** (10.1 - 10.6)

Calculators Allowed:

1. The infinite series $\sum_{n=1}^{\infty} a_n$ has $n$th partial sum $S_n = \frac{4^n - 1}{4^{n+1}}$ for $n \geq 1$. What is the sum of the series?

*Sum of Series is $\lim_{n\to\infty} S_n$

$$\lim_{n\to\infty} \frac{4^n - 1}{4^{n+1}} \rightarrow \lim_{n\to\infty} \frac{4^n - 1}{4(4^n)} = \boxed{\frac{1}{4}}$$

2. Which of the following series diverge?

Limit Comparison Test

LCT

I. $\sum_{n=1}^{\infty} \frac{1}{n^2(n+3)} \rightarrow \frac{1}{n^3+3n^2}$

Compare with $\frac{1}{n^3}$ (convergent p-series)

$$\lim_{n\to\infty} \frac{1}{n^3+3n^2} \cdot \frac{n^3}{1} \rightarrow \boxed{1}$$ converges by LCT

Ratio Test

II. $\sum_{n=1}^{\infty} \frac{n^2 2^{n+1}}{3^n}$

$$\lim_{n\to\infty} \left| \frac{(n+1)^2 \cdot 2^{n+2}}{3^{n+1}} \cdot \frac{3^n}{n^2(2^{n+1})} \right|$$

$$\lim_{n\to\infty} \left| \frac{\cancel{2^n} \cdot 2^2 \cdot \cancel{3^n}}{\cancel{2^n} \cdot 2 \cdot \cancel{3^n} \cdot 3} \cdot \frac{(n+1)^2}{n^2} \right| = \frac{2}{3} < 1 \text{ (converges)}$$

Ratio Test:

III. $\sum_{n=1}^{\infty} \frac{n!}{n4^n}$

$$\lim_{n\to\infty} \left| \frac{(n+1)!}{(n+1)4^{n+1}} \cdot \frac{n \cdot 4^n}{n!} \right|$$

$$\lim_{n\to\infty} \left| \frac{n \cdot \cancel{4^n} \cdot \cancel{(n+1)} \cdot \cancel{n!}}{\cancel{4^n} \cdot 4 \cdot \cancel{n!} \cdot \cancel{(n+1)}} \right|$$

$$\lim_{n\to\infty} \left(\frac{n}{4}\right) = \infty > 1$$ Diverges

(A) I only (B) II only $\boxed{\text{(C) III only}}$ (D) I and II only (E) I, II, and III

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3. The $n$th-Term Test can be used to determine divergence for which of the following series?

I. $\sum_{n=1}^{\infty} \frac{2n+1}{1-n}$

$$\lim_{n\to\infty} \frac{2n+1}{1-n} = \frac{2}{-1} = -2 \neq 0$$

(Diverges)

II. $\sum_{n=0}^{\infty} 5\left(\frac{2}{3}\right)^n$

Geometric Series

$r = \frac{2}{3} < 1$

Converges by GST

III. $\sum_{n=1}^{\infty} \frac{2n(n-1)^2}{4n^2 - 3n^3} \rightarrow \frac{2n(n^2-2n+1)}{4n^2-3n^3}$

$$\lim_{n\to\infty} \frac{2n^3 - 4n^2 + 2n}{4n^2 - 3n^3} \rightarrow \frac{-2}{3} \neq 0$$

(Diverges)

(A) I and II only (B) II and III only $\boxed{\text{(C) I and III only}}$ (D) I, II, and III

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4. If $b$ and $t$ are real numbers such that $0 < |t| < |b|$, what is the sum of $b^2 \sum_{n=0}^{\infty} \left(\frac{t^2}{b^2}\right)^n$?

Geometric Series $\sum_{n=0}^{\infty} a(r)^n$

$r = \frac{t^2}{b^2} < 1$

(converging Geometric Series)

$a_1 = b^2\left(\frac{t^2}{b^2}\right)^0 = b^2$

$$S = \frac{a_1}{1-r} \rightarrow \frac{b^2}{1 - \frac{t^2}{b^2}}$$

$$\frac{b^2}{\frac{b^2 - t^2}{b^2}} \rightarrow b^2 \cdot \frac{b^2}{b^2 - t^2}$$

$$\boxed{S = \frac{b^4}{b^2 - t^2}}$$

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5. Explain why the Integral Test does not apply to the series $\sum_{n=1}^{\infty} \frac{3}{n^{-2}}$. $\rightarrow \sum_{n=1}^{\infty} 3n^2 \leftarrow$ $\boxed{3n^2 \text{ is not decreasing for } n \geq 1}$

Series does not fulfill condition of function decreasing.

6. For what values of $p$ will the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^{3p+1}}$ converge?

* Converges if $3p+1 > 1$

$3p > 0$

$\boxed{p > 0}$

7. **Calculator active.** Which of the following series matches the following sequence of partial sums 0.1667, 0.3333, 0.4833, 0.6167, 0.7357, ...?

(A) $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$ $\boxed{\text{(B)}}$ $\sum_{n=1}^{\infty} \frac{n}{(n+1)(n+2)}$ (C) $\sum_{n=1}^{\infty} \frac{n+1}{n+2}$ (D) $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+3)}$

$S_1 = \frac{1}{6}$

$S_2 = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

$S_3 = \frac{1}{3} + \frac{3}{20} = .4833$

$S_4 = 0.4833 + \frac{2}{15} \approx 0.6167$

$S_5 = 0.616 + \frac{5}{42} \approx 0.7357$

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8. For what values of $x$ is the series $\sum_{n=1}^{\infty} \frac{(7x-3)^n}{n}$ conditionally convergent?

* Ratio Test

$\lim_{n\to\infty} \left| \frac{(7x-3)^{n+1}}{n+1} \cdot \frac{n}{(7x-3)^n} \right| < 1$

$\lim_{n\to\infty} \left| \frac{\cancel{(7x-3)^n}(7x-3)}{\cancel{(7x-3)^n}} \cdot \frac{n}{n+1} \right| < 1$

$|7x-3| < 1$

$-1 < 7x-3 < 1$

$2 < 7x < 4$

$\frac{2}{7} < x < \frac{4}{7}$

test $x = 2/7$

$\frac{[7(2/7)-3]^n}{n} \rightarrow \frac{(2-3)^n}{n} \rightarrow \frac{(-1)^n}{n}$

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ is alternating harmonic series

$\hookrightarrow$ conditionally convergent

$\boxed{\text{(A) } x = \frac{2}{7}}$ (B) $x = \frac{4}{7}$ (C) $x > \frac{4}{7}$ (D) $x < \frac{2}{7}$

9. Which of the following series can be used with the Limit Comparison Test to determine whether the series $\sum_{n=1}^{\infty} \frac{3n+2}{n^3-2n}$ converges or diverges? $\frac{3n}{n^3} \to \frac{1}{n^2}$ (Comparison partner for series)

(A) $\sum_{n=1}^{\infty} \frac{1}{n}$  $\boxed{\text{(B)}}$ $\sum_{n=1}^{\infty} \frac{1}{n^2}$  (C) $\sum_{n=1}^{\infty} \frac{1}{n^3}$  (D) $\sum_{n=1}^{\infty} \frac{1}{n^3-2n}$

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10. Verify that the infinite series $\sum_{n=1}^{\infty} \frac{7^{n+1}-2}{7^{n+2}}$ diverges by using the $n$th-Term Test for Divergence. Show the value of the limit.

$\lim_{n\to\infty} \frac{7(7^n)-2}{7^2(7^n)} \to \frac{7}{49} \to \frac{1}{7} \neq 0$ Diverges by $n^{th}$ term test.

11. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{2^n}{9^n+n}$ is true? $\to$ comparison partner $\frac{2^n}{9^n}$

$\left(\frac{2}{9}\right)^n$ Converges by GST since $r = \frac{2}{9} < 1$

(A) The series diverges by the $n$th Term Test.

(B) The series diverges by comparison with $\sum \frac{1}{n}$.

$\boxed{\text{(C)}}$ The series converges by comparison with $\sum_{n=1}^{\infty} \frac{2^n}{9^n}$.

(D) The series converges by comparison with $\sum_{n=1}^{\infty} \frac{1}{9^n}$.

$0 \leq \frac{2^n}{9^n+n} < \frac{2^n}{9^n}$

series converges by Direct Comparison Test

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12. Which of the following series converge by the Alternating Series Test? (AST)

I. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

$\lim_{n\to\infty} \frac{1}{n} = 0$ ✓

$\frac{1}{n}$ is decreasing ✓

converges by (AST)

II. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$

$\lim_{n\to\infty} \frac{1}{n^{1/2}} = 0$ ✓

$\frac{1}{n^{1/2}}$ is decreasing ✓

converges by AST

III. $\sum_{n=1}^{\infty} (-1)^n \left(\frac{\pi}{e}\right)^n$

$\lim_{n\to\infty} \left(\frac{\pi}{e}\right)^n = \infty$

Diverges by $n^{th}$ term test

A. I only  $\boxed{\text{B.}}$ I and II only  C. I and III only  D. I, II, and III

13. Which of the following series is absolutely convergent?

I. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n^4}}$

Ratio Test II. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

III. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

$\sum_{n=1}^{\infty} \frac{1}{n^{4/3}}$ converges by p-series test

$p = 4/3 > 1$

(Absolute Convergence)

$\lim_{n\to\infty} \left| \frac{1}{(n+1)!} \cdot \frac{n!}{1} \right|$

$\lim_{n\to\infty} \left| \frac{\cancel{n!}}{(n+1)\cancel{n!}} \right| = 0 < 1$

Converges by Ratio Test (Absolute Convergence)

$\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (harmonic series)

(A) I only $\boxed{\text{(B)}}$ I and II only (C) I and III only (D) I, II, and III

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14. Use the Integral test to determine if the series $\sum_{n=1}^{\infty} \frac{3n^2}{n^3+1}$ converges or diverges.

let $f(x) = \frac{3x^2}{x^3+1}$

$\lim_{b\to\infty} \int_1^b \frac{3x^2}{x^3+1} dx$ $\quad dx = \frac{du}{3x^2}$

$u = x^3+1$

$\frac{du}{dx} = 3x^2$

$\int \frac{\cancel{3x^2}}{u} \cdot \frac{du}{\cancel{3x^2}}$

$\int \frac{1}{u} du = \ln|u| \rightarrow \ln|x^3+1| \Big]_1^b$

$\lim_{b\to\infty} \ln|b^3+1| - \ln|1+1| = \infty$

$\boxed{\text{Series diverge by Integral Test}}$

15. Which of the following statements are true about the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n+1}{n}$ ?

✓ I. $a_{n+1} \le a_n$ for all $n \ge 1$.

✓ II. $\lim_{n\to\infty} a_n \neq 0$

X III. The series converges by the Alternating Series Test

$\lim_{n\to\infty} \frac{n+1}{n} = 1 \neq 0$ Series diverge by $n^{th}$ term test.

A. I only $\boxed{\text{B.}}$ I and II only C. II and III only D. I, II, and III

16. What are all values of $x > 0$ for which the series $\sum_{n=1}^{\infty} \frac{n^2 x^{n+1}}{7^n}$ converges. *Ratio Test

$$\lim_{n\to\infty} \left| \frac{(n+1)^2 x^{n+2}}{7^{n+1}} \cdot \frac{7^n}{n^2 \cdot x^{n+1}} \right| < 1$$

$$\lim_{n\to\infty} \left| \frac{\cancel{x^n} \cdot x^2 \cdot \cancel{7^n}}{\cancel{x^n} \cdot x \cdot \cancel{7^n} \cdot 7} \cdot \frac{(n+1)^2}{n^2} \right| < 1$$

$$\left|\frac{x}{7}\right| < 1$$

$$|x| < 7$$

$$\boxed{0 < x < 7}$$

17. Which of the following is a convergent $p$-series?

Diverges nth term test

converges geometric series test $r = 1/2$

$\frac{1}{n^{2/3}}$, $p = 2/3 < 1$ diverges by p-series test.

$\sum \frac{1}{n^{3/2}}$ $p = 3/2 > 1$ converges by p-series test

(A) $\sum_{n=1}^{\infty} n^4$ (B) $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$ (C) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2}}$ $\boxed{\text{(D)}}$ $\sum_{n=1}^{\infty} \left(\frac{1}{n^3}\right)^{\frac{1}{2}}$

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18. Consider the series $\sum_{n=1}^{\infty} a_n$. If $\frac{a_{n+1}}{a_n} = \frac{1}{2}$ for all integers $n \geq 1$, and $\sum_{n=1}^{\infty} a_n = 64$, then $a_1 = ?$

*Geometric Series

$$S = \frac{a_1}{1-r}$$

$$S = 64$$

$$r = 1/2$$

$$64 = \frac{a_1}{1 - 1/2}$$

$$64 = \frac{a_1}{1/2} \rightarrow a_1 = \left(\tfrac{1}{2}\right)(64)$$

$$\boxed{a_1 = 32}$$

## Answers to Mid-Unit 10 Corrective Assignment

| | | | | |
|---|---|---|---|---|
| 1. $\frac{1}{4}$ | 2. C | 3. C | 4. $\frac{b^4}{b^2 - t^2}$ | 5. $f(n)$ is not a decreasing function for $n \geq 1$. |
| 6. $p > 0$ | 7. B | 8. A | 9. B | |
| 10. Diverges by $n$th-Term Test, $\lim_{n\to\infty} a_n = \frac{1}{7}$ | 11. C | 12. B | 13. B | |
| 14. $\int_1^{\infty} f(x)\,dx = \infty$, Series Diverges | 15. B | 16. $x < 7$ | 17. D | 18. 32 |

Name: Key WS #3 Date: ____________ Period: ______ (11)

**BC Calculus Unit 10 "Tests for Convergence" Review WS #3**

Calculators Allowed:

1) The infinite series $\sum_{n=1}^{\infty} \frac{3}{4^{n+1}}$ has $n$th partial sum $S_n = \frac{1}{4} - \frac{1}{4^{n+1}}$. What is the sum of the series?

*Sum of series is $\lim_{n\to\infty} S_n$

$\lim_{n\to\infty} \frac{1}{4} - \frac{1}{4^{n+1}} \to \frac{1}{4} - 0 = \boxed{\frac{1}{4}}$

**Use the $n$th-Term Test for Divergence to determine if the series diverges.** If inconclusive, find appropriate test. (find sum if possible)

2. $\sum_{n=0}^{\infty} \frac{\pi^{n+1}}{7^n} \to \sum_{n=0}^{\infty} \pi\left(\frac{\pi}{7}\right)^n$

$\lim_{n\to\infty} \pi\left(\frac{\pi}{7}\right)^n = 0$ ($n^{th}$ term inconclusive)

$r = \frac{\pi}{7} < 1$

Converges by GST

$S = \frac{a_1}{1-r}$

$a_1 = \pi \quad r = \pi/7$

$S = \frac{\pi}{1-\pi/7} \to \frac{\pi}{\frac{7-\pi}{7}} \to \pi \cdot \frac{7}{7-\pi} \to \boxed{\frac{7\pi}{7-\pi}}$

3. $\sum_{n=1}^{\infty} \frac{2(n-2)^2}{3(n+4)^2}$

$\lim_{n\to\infty} \frac{2(n-2)^2}{3(n+4)^2} = \frac{2}{3} \neq 0$

Diverges by $n^{th}$ term test.

4. $\sum_{n=1}^{\infty} \frac{1}{e^n} \to \lim_{n\to\infty} \frac{1}{e^n} = 0$ (Inconclusive)

$r = \frac{1}{e} < 1$

$a_1 = \frac{1}{e}$

$S = \frac{a_1}{1-r}$

$S = \frac{1/e}{1-1/e}$

$S = \frac{\frac{1}{e}}{\frac{e-1}{e}}$

$S = \frac{1}{e} \cdot \frac{e}{e-1} \to \boxed{\frac{1}{e-1}}$

Converges by GST

5) If the infinite series $\sum_{n=1}^{\infty} a^n$ has $n$th partial sum $S_n = \frac{4}{3}(4^n - 1)$ for $n \geq 1$. What is the sum of the series?

$\lim_{n\to\infty} \frac{4}{3}(4^n - 1) = \infty$ $\boxed{\text{Series Diverges}}$

6) Does the series $\sum_{n=1}^{\infty} \left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)$ converge or diverge? If it converges find its sum.

$\underset{n=1}{\left(\frac{1}{2-1} - \cancel{\frac{1}{3}}\right)} + \underset{n=2}{\left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right)} + \underset{n=3}{\left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{7}}\right)} + \underset{n=4}{\left(\cancel{\frac{1}{7}} - \cancel{\frac{1}{9}}\right)} + \cancel{\ }\ldots$

$= \frac{1}{1} = \boxed{1}$

7) What is the sum of the infinite geometric series $11 + -\frac{11}{3} + \frac{11}{9} + -\frac{11}{27} + \cdots$?

$r = \frac{-\frac{11}{3}}{11} = -\frac{11}{3} \cdot \frac{1}{11} = -\frac{1}{3}$

$S = \frac{a_1}{1-r} \quad a_1 = 11$

$S = \frac{11}{1-(-1/3)} \to \frac{11}{\frac{3}{3}+\frac{1}{3}} \to \frac{11}{4/3}$

$\to 11 \cdot \frac{3}{4} = \boxed{\frac{33}{4}}$

$*$ Geometric Series

8) What is the value of $\sum_{n=1}^{\infty} \frac{(-e)^{n+1}}{9^n}$?

$\rightarrow \frac{(-e)^n \cdot (-e)^1}{9^n} \rightarrow -e\left(\frac{-e}{9}\right)^n$

$a_1 = \frac{(-e)^2}{9} = \frac{e^2}{9}$

$r = \frac{-e}{9}$

$S = \frac{a_1}{1-r}$

$S = \frac{e^2/9}{1-\left(\frac{-e}{9}\right)}$

$S = \frac{\frac{e^2}{9}}{\frac{9+e}{9}} \rightarrow \frac{e^2}{\cancel{9}} \cdot \frac{\cancel{9}}{9+e}$

$\boxed{S = \frac{e^2}{9+e}}$

9) For what value of $a$ does the infinite series $\sum_{n=0}^{\infty} a\left(-\frac{3}{5}\right)^n$ equal 15?

$r = \frac{-3}{5}$

$S = 15$

$S = \frac{a_1}{1-r}$

$15 = \frac{a_1}{1-(-3/5)}$

$15 = \frac{a_1}{8/5}$

$a_1 = \frac{8}{5}(15)$

$\boxed{a_1 = 24}$

10) The $n$th-Term Test can be used to determine divergence for which of the following series?

I. $\sum_{n=1}^{\infty} \frac{(n+1)^3}{3n^3 - 2n + 1}$

$\lim_{n \to \infty} \frac{(n+1)^3}{3n^3 - 2n + 1} = \frac{1}{3} \neq 0$

(Diverges by $n^{th}$ term test)

II. $\sum_{n=1}^{\infty} \frac{(n+1)^2}{2n^2 - 3n^3 + 1}$

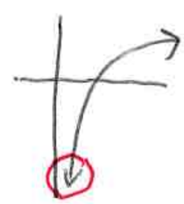
$\lim_{n \to \infty} \frac{(n+1)^2}{2n^2 - 3n^3 + 1} = 0$

Inconclusive by $n^{th}$ term test

III. $\sum_{n=1}^{\infty} \ln\frac{1}{n}$

$\lim_{n \to \infty} \ln\left(\frac{1}{n}\right) \rightarrow \ln\left(\frac{1}{100000\ldots}\right)$

$\rightarrow \ln(0.00000\ldots1)$



$\lim_{n \to \infty} \ln\left(\frac{1}{n}\right) = -\infty \neq 0$

Diverges by $n^{th}$ term test

A. III only

$\boxed{\text{B. I and III only}}$

C. II and III only

D. I, II, and III

11) Verify that the infinite series $\sum_{n=1}^{\infty} \frac{6^n + 1}{6^{n+1}}$ diverges by using the $n$th-Term Test for Divergence. Show the value of the limit.

$\lim_{n \to \infty} \frac{6^n + 1}{6(6^n)} = \frac{1}{6} \neq 0$

$\boxed{\text{Diverges by } n^{th} \text{ term test}}$

12) Use the Integral Test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^5}$.

let $f(x) = \frac{1}{x^5}$ $f(x)$ is positive, decreasing, continuous

$\lim_{b \to \infty} \int_1^b \frac{1}{x^5}dx \rightarrow \int x^{-5}dx \rightarrow \frac{x^{-4}}{-4} \rightarrow \left.\frac{-1}{4x^4}\right]_1^b \rightarrow \lim_{b \to \infty} \frac{-1}{4b^4} - \frac{-1}{4(1)^4} = \boxed{\frac{1}{4}}$

$\boxed{\text{Series Converge by Integral Test}}$

13) Prove the Integral Test applies to the series $\sum_{n=1}^{\infty} \frac{1}{(n+1)^3}$. Determine the convergence or divergence of the series.

f(x) continuous, positive, decreasing

$\lim_{n\to\infty} \int_1^b \frac{1}{(x+1)^3}dx$

$u = x+1$

$\frac{du}{dx} = 1$

$\int u^{-3} du$

$\frac{u^{-2}}{-2} \to \frac{-1}{2u^2} \to \frac{-1}{2(x+1)^2}\Big]_1^b$

$\lim_{b\to\infty} \frac{-1}{2(1+b)^2} - \frac{-1}{2(2)^2} = \boxed{\frac{1}{8}}$

Series converge by Integral Test

14) Use the Integral Test to determine if the series $\sum_{n=1}^{\infty} \frac{4n}{2n^2+1}$ converges or diverges.

$f(x) = \frac{4x}{2x^2+1}$

f(x) positive, decreasing, continuous

$\lim_{b\to\infty} \int_1^b \frac{4x}{2x^2+1}dx$

$u = 2x^2+1$

$\frac{du}{dx} = 4x$

$dx = \frac{du}{4x}$

$\int \frac{\cancel{4x}}{u} \cdot \frac{du}{\cancel{4x}}$

$\int \frac{1}{u} du$

$\ln|u| \to \ln|2x^2+1|\Big]_1^b$

$\lim_{b\to\infty} \ln|2b^2+1| - \ln|2+1| = \boxed{\infty}$

Diverges by Integral Test

15) For what values of $p$ will the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^{1-p}}$ converge?

$1-p > 1$

$0 > p$

$\boxed{p < 0}$

16) For what values of $p$ will both infinite series $\sum_{n=1}^{\infty} \left(\frac{3}{p}\right)^n$ and $\sum_{n=1}^{\infty} \frac{1}{n^{5-p}}$ converge?

$\sum_{n=1}^{\infty} \left(\frac{3}{p}\right)^n$

Geometric series converge if

$\frac{3}{p} < 1$ so $\boxed{p > 3}$

$\sum_{n=1}^{\infty} \frac{1}{n^{5-p}}$

converges if the exponent in the p-series is greater than 1

$5-p > 1$

$4 > p$

$\boxed{p < 4}$

$\boxed{3 < p < 4}$

17) Which of the following is a divergent $p$-series?

A. $\sum_{n=1}^{\infty} n^{-\pi}$

$\left(\frac{1}{n}\right)^{\pi}$

$\frac{1}{n^{\pi}}$

$p = \pi > 1$

$\boxed{\text{Converges}}$

$\boxed{\text{B.}}$ $\sum_{n=1}^{\infty} \frac{1}{n}$

harmonic p-series

$\boxed{\text{diverges}}$

C. $\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^n$

geometric series $\boxed{\text{converge}}$ since

$r = \frac{e}{\pi} < 1$

D. $\sum_{n=1}^{\infty} \frac{1}{n^3}$

$p = 3 > 1$

$\boxed{\text{Converges}}$

18) **Which of the following series converges?**

Diverges by LCT, use $\frac{1}{n}$

(A) $\sum_{n=1}^{\infty} \frac{3n}{2n^2+1}$ $\lim_{n\to\infty} \frac{3n}{2n^2+1} \cdot \frac{n}{1} \to \frac{3n^2}{2n^2} \to \frac{3}{2} = L$

Diverges by $n^{th}$ term test

(B) $\sum_{n=1}^{\infty} \frac{3n^2}{n+2n^2}$ $\lim_{n\to\infty} \frac{3n^2}{2n^2} = \frac{3}{2} \neq 0$

(C) $\sum_{n=1}^{\infty} \left(\frac{\pi}{e}\right)^n$

$r = \frac{\pi}{e} > 1$

Diverges by $n^{th}$ term or GST

(D) $\sum_{n=1}^{\infty} \frac{3n^2}{2n^3+3n}$

Diverges by LCT, use $\frac{1}{n}$

$\lim_{n\to\infty} \frac{3n^2}{2n^3+3n} \cdot \frac{n}{1} = \frac{3n^2}{2n^3} = \frac{3}{2} = L$

$\boxed{\text{(E)}}$ $\sum_{n=1}^{\infty} \frac{n-1}{n4^n}$

LCT, use $\frac{1}{4^n} \to \left(\frac{1}{4}\right)^n$ converge by GST

$\lim_{n\to\infty} \frac{n-1}{n\cdot 4^n} \cdot \frac{4^n}{1}$

$= \frac{n}{n-1} \to 1 = L$ $\boxed{\text{Converge by LCT}}$

19) **Which of the following series can be used with the Limit Comparison Test to determine whether the series $\sum_{n=1}^{\infty} \frac{5^n}{7^n - n^2}$ diverges or converges?** $\to$ comparison partner $\frac{5^n}{7^n} \to \left(\frac{5}{7}\right)^n$ converge by GST

$r = 5/7 < 1$

(A) $\sum_{n=1}^{\infty} \frac{1}{n}$

$\lim_{n\to\infty} \frac{5^n}{7^n - n^2} \cdot \frac{7^n}{5^n} = 1 = L$

$\boxed{\text{converges by LCT}}$

(B) $\sum_{n=1}^{\infty} \frac{1}{5^n}$

(C) $\sum_{n=1}^{\infty} \frac{1}{7^n}$

$\boxed{\text{(D)}}$ $\sum_{n=1}^{\infty} \left(\frac{5}{7}\right)^n$

20) **Use the Comparison Test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n-2}{n5^n}$. You must identify the series you are using for comparison.**

compare with $\sum \frac{1}{5^n} \to \left(\frac{1}{5}\right)^n$

converges by GST

since $r = \frac{1}{5} < 1$

$\lim_{n\to\infty} \frac{n-2}{n\cancel{5^n}} \cdot \frac{\cancel{5^n}}{1} = 1 = L$

$\boxed{\text{Series converge by LCT.}}$

21) **Determine whether the series $\sum_{n=1}^{\infty} \frac{n5^n}{4n^4-3}$ converges or diverges. Identify the test for convergence used.**

compare with $\frac{5^n}{n^3}$

diverges by $n^{th}$ term test since $\lim_{n\to\infty} \frac{5^n}{n^3} = \infty \neq 0$

$\lim_{n\to\infty} \frac{n5^n}{4n^4-3} \cdot \frac{n^3}{5^n} \to \frac{1}{4} = L$

$\boxed{\text{Series diverge by LCT}}$

22) Explain why the Alternating Series Test does not apply to the series $\sum_{n=1}^{\infty} \frac{(-1)^n \cos(n\pi)}{n^2}$.

$n=1 \to (-1)^1\cos(\pi) = (-1)(-1) = 1$
$n=2 \to (-1)^2\cos(2\pi) = (1)(1) = 1$
$n=3 \to (-1)^3\cos(3\pi) = (-1)(-1) = 1$

* series does not alternate in signs

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (absolutely) since $p = 2 > 1$

23) Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n n}{\ln(n+1)}$.

$\boxed{LRPE}$

$\lim_{n\to\infty} \frac{n}{\ln(n+1)} = \infty \neq 0$

* comparative growth rate ($\lim_{n\to\infty}$) Logs < Radicals < Polynomial < Exponent

series diverge by $n^{th}$ term test.

24) Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{(-1)^n}{4^n}$ II. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\pi^n}$ III. $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{1+n^2}$

A. I only
$\boxed{\text{B. I and II only}}$
C. I and III only
D. I, II, and III

i) $\lim_{n\to\infty} \frac{1}{4^n} = 0$
ii) $|a_{n+1}| \leq |a_n|$

i) $\lim_{n\to\infty} \left(\frac{1}{\pi}\right)^n = 0$
ii) $|a_{n+1}| \leq |a_n|$

$\lim_{n\to\infty} \frac{n^2}{1+n^2} = 1 \neq 0$
Diverges by $n^{th}$ term test

25) Which of the following statements is true?

A. $\sum_{n=1}^{\infty} \frac{(-1)^n(1-n)}{n}$ converges by the Alternating Series Test. $\lim_{n\to\infty} \frac{1-n}{n} = -1 \neq 0$

B. $\sum_{n=1}^{\infty} \frac{(-1)^n(n+1)}{2n}$ converges by the Alternating Series Test. $\lim_{n\to\infty} \frac{n+1}{2n} = \frac{1}{2} \neq 0$

C. $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{4\sqrt{n}}$ converges by the Alternating Series Test. $\lim_{n\to\infty} \frac{n^2}{4\sqrt{n}} = \infty \neq 0$

Diverges by $n^{th}$ term test

$\boxed{\text{D.}}$ $\sum_{n=1}^{\infty} \frac{(-1)^n 2\sqrt{n}}{n}$ converges by the Alternating Series Test. $\lim_{n\to\infty} \frac{2\sqrt{n}}{n} = 0$ ✓

$|a_{n+1}| \leq |a_n|$

26) Use the Ratio Test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n^4}{3^n}$.

* Ratio Test

$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$

$\lim_{n\to\infty} \left| \frac{(n+1)^4}{3^{n+1}} \cdot \frac{3^n}{n^4} \right| = \lim_{n\to\infty} \left| \frac{\cancel{3^n} \cdot (n+1)^4}{3 \cdot \cancel{3^n} \cdot n^4} \right| = \frac{1}{3} < 1$

Converges by Ratio Test

27) If the Ratio Test is applied to the series $\sum_{n=0}^{\infty} \frac{6^n}{(n+1)^n}$, which of the following inequalities results, implying that the series converges?

A. $\lim_{n\to\infty} \frac{6^n}{(n+1)^n} < 1$ $\quad$ $\boxed{\text{B.}}$ $\lim_{n\to\infty} \frac{6(n+1)^n}{(n+2)^{n+1}} < 1$ $\quad$ C. $\lim_{n\to\infty} \frac{6^{n+1}}{(n+1)^n} < 1$ $\quad$ D. $\lim_{n\to\infty} \frac{6^{n+1}}{(n+1)^{n+1}} < 1$

$\lim_{n\to\infty} \left| \frac{6^{n+1}}{(n+2)^{n+1}} \cdot \frac{(n+1)^n}{6^n} \right| < 1$

$\lim_{n\to\infty} \left| \frac{\cancel{6^n} \cdot 6 \cdot (n+1)^n}{(n+2)^{n+1} \cdot \cancel{6^n}} \right| < 1$

$\lim_{n\to\infty} \left| \frac{6(n+1)^n}{(n+2)^{n+1}} \right| < 1$

By Root test

$\lim_{n\to\infty} \sqrt[n]{\left(\frac{6}{n+1}\right)^n} = \lim_{n\to\infty} \frac{6}{n+1} = 0 < 1$

Converges by Root test

28)

If $a_n > 0$ for all $n$ and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 5$, which of the following series converges?

$\lim_{n\to\infty} \left| \frac{a_{n+1}}{(n+1)^2} \cdot \frac{n^2}{a_n} \right|$

$5 \cdot 1 = 5 > 1$

A. $\sum_{n=1}^{\infty} \frac{a_n}{n^2}$

Diverges by Ratio Test

$\lim_{n\to\infty} \left| \frac{a_{n+1}}{2^{n+1}} \cdot \frac{2^n}{a_n} \right|$

B. $\sum_{n=1}^{\infty} \frac{a_n}{2^n}$

$\left| \frac{a_{n+1}}{a_n} \cdot \frac{1}{2} \right| = 5\left(\frac{1}{2}\right)$

$= \frac{5}{2} > 1$

Diverges by Ratio Test

$\lim_{n\to\infty} \left| \frac{a_{n+1}}{(n+1)^5} \cdot \frac{n^5}{a_n} \right|$

$= 5(1) = 5 > 1$

C. $\sum_{n=1}^{\infty} \frac{a_n}{n^5}$

Diverges by Ratio Test

$\lim_{n\to\infty} \left| \frac{a_{n+1}}{7^{n+1}} \cdot \frac{7^n}{a_n} \right|$

$= \left| \frac{a_{n+1}}{a_n} \cdot \frac{1}{7} \right| = \frac{5}{7} < 1$

$\boxed{\text{D.}}$ $\sum_{n=1}^{\infty} \frac{a_n}{7^n}$

Converges by Ratio Test

29) What are all values of $x > 0$ for which the series $\sum_{n=1}^{\infty} \frac{6n^3}{x^n}$ converges?

$\lim_{n\to\infty} \left| \frac{6(n+1)^3}{x^{n+1}} \cdot \frac{x^n}{6n^3} \right| < 1$

$\left| \frac{1}{x} \right| < 1$

$\frac{1}{x} < \frac{1}{1}$

$1 < x$

$\boxed{x > 1}$

30) Which of the following series converge?

I. $\sum_{}^{\infty} \frac{1}{n!}$ II. $\sum_{}^{\infty} \frac{9^n}{n^5}$ III. $\sum_{}^{\infty} \frac{5n}{2n-1}$

save time by applying nth term test before Ratio Test

$\boxed{\text{A. I only}}$ B. I and II only C. I and III only D. I, II, and III

$\lim_{n\to\infty} \left| \frac{1}{(n+1)!} \cdot \frac{n!}{1} \right|$

$\lim_{n\to\infty} \frac{\cancel{n!}}{(n+1)\cancel{n!}} = 0 < 1$

converges by Ratio Test

$\lim_{n\to\infty} \frac{9^n}{n^5} = \infty \neq 0$

Diverges by nth term test

$\lim_{n\to\infty} \frac{5n}{2n-1} = \frac{5}{2} \neq 0$

Diverges by nth term test

31) For what values of $x$ is the series $\sum_{n=0}^{\infty} (-1)^n (5x+1)^n$ absolutely convergent?

$\lim_{n\to\infty} \left| \frac{(5x+1)^{n+1}}{(5x+1)^n} \right| < 1$

$|5x+1| < 1$

$-1 < 5x+1 < 1$

$-2 < 5x < 0$

$\frac{-2}{5} < \frac{5x}{5} < \frac{0}{5}$

$\boxed{-\frac{2}{5} < x < 0}$

32) For what values of $x$ is the series $\sum_{n=1}^{\infty} \frac{(5x-2)^n}{n}$ conditionally convergent?

A. $x > \frac{3}{5}$ B. $x = \frac{3}{5}$ $\boxed{\text{C.}}$ $x = \frac{1}{5}$ D. $x < \frac{1}{5}$

$\lim_{n\to\infty} \left| \frac{(5x-2)^{n+1}}{n+1} \cdot \frac{n}{(5x-2)^n} \right| < 1$

$\lim_{n\to\infty} \left| \frac{5x-2}{1} \cdot \frac{n}{n+1} \right| < 1$

$|5x-2| < 1$

$-1 < 5x-2 < 1$

$1 < 5x < 3$

$1/5 < x < 3/5$

test $x = 1/5$

$\frac{(5(\frac{1}{5})-2)^n}{n}$

$\frac{(-1)^n}{n}$

alternating harmonic series

test $x = 3/5$

$(5(\frac{3}{5})-2)^n$

$\frac{(3-2)^n}{n} = \frac{1^n}{n}$

diverges by p-series

33) Which of the following statements is true about the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2\sqrt{n}}$. $\to \frac{(-1)^{n+1}}{n^{5/2}}$

$p = 5/2 > 1$

Series converge
(Absolute convergence)

A. The series converges conditionally.

$\boxed{\text{B.}}$ The series converges absolutely.

C. The series converges but neither conditionally nor absolutely.

D. The series diverges.

34) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+5}$ converges absolutely, converges conditionally, or diverges.

$\frac{1}{n}$ diverges (p-series)

By LCT

$\lim_{n\to\infty} \frac{1}{n+5} \cdot \frac{n}{1} = 1 = L$

$\frac{1}{n+5}$ diverges by LCT

By AST,

i) $\lim_{n\to\infty} \frac{1}{n+5} = 0$ ✓

ii) $|a_{n+1}| \le |a_n|$

converges by AST

$\boxed{\text{Series converge conditionally}}$

35) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)!}$ converges absolutely, converges conditionally, or diverges.

$\lim_{n\to\infty} \left| \frac{1}{(n+2)!} \cdot \frac{(n+1)!}{1} \right|$

$\lim_{n\to\infty} \left| \frac{\cancel{(n+1)}\cancel{n!}}{(n+2)\cancel{(n+1)}\cancel{n!}} \right|$

$\lim_{n\to\infty} \left| \frac{1}{n+2} \right| = 0 < 1$

Converges by Ratio Test

$\boxed{\text{Series converge (Absolute Convergence)}}$