Name: Key Review WS #1 Date: Units 10.1-10.6 Period: ____ **Review**

# Mid-Unit 10 Review – Infinite Sequences and Series

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 10.

1. Which of the following series converges?

(A) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{n+3}{3n}\right)$ (B) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{n^2}{2\sqrt{n}}\right)$ (C) $\boxed{\sum_{n=1}^{\infty}(-1)^n\left(\frac{2\sqrt{n}}{n}\right)}$ (D) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{4-n}{n}\right)$

$\lim_{n\to\infty}\frac{n+3}{3n}=\frac{1}{3}\neq 0$ Diverges by nth term test

$\lim_{n\to\infty}\left(\frac{n^2}{2\sqrt{n}}\right)=\infty$

$\lim_{n\to\infty}\frac{2\sqrt{n}}{n}\to\frac{2}{\sqrt{n}}=0$ Converges by AST

$\lim_{n\to\infty}\left(\frac{4-n}{n}\right)=1\neq 0$

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2. What is the value of $\sum_{n=1}^{\infty}\frac{2^{n+1}}{7^n}$? $\to \frac{2^n\cdot 2^1}{7^n}\to\sum_{n=1}^{\infty}2\left(\frac{2}{7}\right)^n$ Geometric Series, $r=2/7$ (converges)

$a_1=2\left(\frac{2}{7}\right)=\frac{4}{7}$

$S=\frac{a_1}{1-r}\to\frac{4/7}{1-2/7}\to\frac{4/7}{5/7}=\boxed{\frac{4}{5}}$

3. Which of the following series can be used with the Limit Comparison Test to determine whether the series $\sum_{n=1}^{\infty}\frac{2^n}{3^n-n^2}$ converges or diverges?

$\frac{2^n}{3^n}\to\left(\frac{2}{3}\right)^n$

(A) $\sum_{n=1}^{\infty}\left(\frac{3}{2}\right)^n$ (B) $\sum_{n=1}^{\infty}\frac{1}{3^n}$ (C) $\boxed{\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n}$ (D) $\sum_{n=1}^{\infty}\frac{1}{n}$

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4. **Calculator active.** Find the sequence of partial sums $S_1, S_2, S_3, S_4$, and $S_5$ for the infinite series $\sum_{n=1}^{\infty}\frac{3}{2^{n-1}}$.

$S_1=\frac{3}{2^0}=\boxed{3}$

$S_2=3+\frac{3}{2}=\boxed{4.5}$

$S_3=4.5+\frac{3}{2^2}=\boxed{5.25}$

$S_4=5.25+\frac{3}{2^3}=\boxed{5.625}$

$S_5=5.625+\frac{3}{2^4}=\boxed{5.8125}$

5. Verify that the infinite series $\sum_{n=1}^{\infty} \frac{3^n + 1}{3^{n+2}}$ diverges by using the $n$th-Term Test for Divergence. Show the value of the limit.

$$\lim_{n\to\infty} \frac{3^n+1}{3^2(3^n)} \to \boxed{\frac{1}{9} \neq 0}$$

Diverges by $n^{th}$-Term Test

6. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{3^n}{n!}$ II. $\sum_{n=1}^{\infty} \frac{n}{8^n}$ III. $\sum_{n=1}^{\infty} \frac{2}{n\sqrt{n}}$

Ratio Test

$$\lim_{n\to\infty} \frac{3^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n}$$

$$\lim_{n\to\infty} \frac{\cancel{3^n} \cdot 3 \cdot \cancel{n!}}{(n+1)\cdot\cancel{n!}\cdot\cancel{3^n}} \to \lim_{n\to\infty} \frac{3}{(n+1)} = 0 < 1$$

Converges by Ratio Test

Ratio Test

$$\lim_{n\to\infty} \frac{n+1}{8^{n+1}} \cdot \frac{8^n}{n} \to \frac{n+1}{n} \cdot \frac{\cancel{8^n}}{\cancel{8^n} \cdot 8}$$

$$\lim_{n\to\infty} \frac{n+1}{8n} = \frac{1}{8} < 1$$

Converges by Ratio Test

$$\to \sum \frac{2}{n \cdot n^{1/2}}$$

$$2 \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$

p-series $p = 3/2 > 1$

Converges

(A) I only (B) I and II only (C) I and III only $\boxed{\text{(D) I, II, and III}}$

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7. For what values of $x$ is the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n}$ conditionally convergent?

*Ratio Test

$$\lim_{n\to\infty} \left| \frac{(x+2)^{n+1}}{n+1} \cdot \frac{n}{(x+2)^n} \right| < 1$$

$$\lim_{n\to\infty} \left| \frac{\cancel{(x+2)^n}\cdot(x+2)}{\cancel{(x+2)^n}} \cdot \frac{n}{(n+1)} \right| < 1$$

$$|x+2| < 1$$

$$-1 < x+2 < 1$$

$$-3 < x < -1$$

$\boxed{\text{If } x = -3}$

$$\sum_{n=1}^{\infty} \frac{(-3+2)^n}{n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

Alternating harmonic series converges conditionally

$\boxed{\text{If } x = -1}$

$$\sum_{n=1}^{\infty} \frac{(1)^n}{n}$$

harmonic series diverges

(A) $x > -1$ $\boxed{\text{(B) } x = -3}$ (C) $x = -1$ (D) $x = 3$

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8. Use the Integral Test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^2}$.

$$\text{Let } f(x) = \frac{e^{1/x}}{x^2}$$

$\to f(x)$ is continuous, positive, decreasing

$$\lim_{b\to\infty} \int_1^b \frac{e^{x^{-1}}}{x^2} dx \to \lim_{b\to\infty} \int_1^b \frac{e^u}{\cancel{x^2}} \cdot -\cancel{x^2} du$$

$u = x^{-1}$

$\frac{du}{dx} = -1x^{-2}$

$\frac{du}{dx} = \frac{-1}{x^2}$

$-dx = x^2 du$

$dx = -x^2 du$

$$-\int e^u du \to -e^u \to -e^{1/x} \Big]_1^b$$

$$\lim_{b\to\infty} -e^{1/b} - (-e^{1/1})$$

$$-e^0 + e = \boxed{-1 + e}$$

Series Converges by Integral Test

③

9. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n^{-1}}{\sqrt{n}}$ II. $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$ III. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

$\sum \frac{1}{n^{1/2} \cdot n} \rightarrow \frac{1}{n^{3/2}}$

$p = 3/2 > 1$

converges by p-series test

$r = \frac{2}{3} < 1$

converges by Geometric Series Test

Integral Test

let $f(x) = \frac{1}{x \ln x}$

$f(x)$ is positive, continuous, decreasing

$\lim_{b \to \infty} \int_2^b \frac{1}{x \ln x} dx$

$u = \ln x$ | $dx = x\,du$

$\frac{du}{dx} = \frac{1}{x}$ | $\int \frac{1}{\cancel{x} \cdot u} \cdot \cancel{x} du$

$\int \frac{1}{u} du \rightarrow \ln|u| \rightarrow \ln|\ln x| \Big|_2^b$

$\lim_{b \to \infty} \ln|\ln b| - \ln|\ln 2|$

$\infty - \ln|\ln 2|$

Diverges by Integral Test

(A) I only (B) II only (C) III only $\boxed{\text{(D) I and II only}}$ (E) I, II, and III

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10. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{3}{1+2^n}$ is true?

$n^{th}$ term test inconclusive

$\lim_{n \to \infty} \frac{3}{1+2^n} = 0$

(A) Diverges by the $n$th Term test.

compare with $\frac{1}{2^n}$ (converges by Geometric Series) $\rightarrow \left(\frac{1}{2}\right)^n$

$r = \frac{1}{2} < 1$

(B) Diverges by comparison to $\sum_{n=1}^{\infty} \frac{1}{2^n}$.

$\boxed{\text{(C)}}$ Converges by comparison to $\sum_{n=1}^{\infty} \frac{1}{2^n}$.

$0 \leq \frac{3}{1+2^n} < \frac{1}{2^n}$

By Direct Comparison Test, series also converge.

(D) Diverges by comparison to $\sum_{n=1}^{\infty} \frac{1}{n}$.

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11. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{n^2+3}$ is true?

✓ $\lim_{n \to \infty} \frac{n}{n^2+3} = 0$

✓ series is decreasing

(A) The series diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(B) The series diverges by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(C) The series converges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

$\boxed{\text{(D)}}$ The series converges by the Alternating Series Test.

12. Which of the following is required in order to apply the Integral Test to the series $\sum_{n=1}^{\infty} a_n$?

(A) $\lim_{n\to\infty} a_n = 0$ and $\sum_{n=1}^{\infty} a_n$ is a positive series.

(B) $\lim_{n\to\infty} a_n \neq 0$ and $\sum_{n=1}^{\infty} a_n$ is a convergent series.

(C) $a_n = f(n)$ and $f(x)$ is positive, continuous, and increasing on $[1, \infty)$.

$\boxed{\text{(D)}}$ $a_n = f(n)$ and $f(x)$ is positive, continuous, and decreasing on $[1, \infty)$.

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13. If $a_n > 0$ for all $n$ and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{2}{3}$, which of the following series converges? *Ratio Test

(A) $\sum_{}^{\infty} 3^n a_n$

$\lim_{n\to\infty} \left| \frac{3^{n+1} \cdot a_{n+1}}{3^n \cdot a_n} \right|$

$\lim_{n\to\infty} \left| \frac{\cancel{3^n} \cdot 3 \cdot a_{n+1}}{\cancel{3^n}\ a_n} \right|$

$\to 3 \cdot \left(\frac{2}{3}\right) = 2 > 1$ (Diverges)

(B) $\sum_{}^{\infty} \frac{2^n}{a_n}$

$\lim_{n\to\infty} \left| \frac{2^{n+1}}{a_{n+1}} \cdot \frac{a_n}{2^n} \right|$

$\lim_{n\to\infty} \left| \frac{\cancel{2^n} \cdot 2 \cdot a_n}{\cancel{2^n}\ a_{n+1}} \right|$

$2 \cdot \left(\frac{3}{2}\right) = 3 > 1$ (Diverges)

(C) $\sum_{}^{\infty} a_n \left(\frac{7}{2}\right)^n$

$\lim_{n\to\infty} \frac{a_{n+1} \cdot \left(\frac{7}{2}\right)^{n+1}}{a_n \cdot \left(\frac{7}{2}\right)^n}$

$= \frac{2}{3} \cdot \left(\frac{7}{2}\right) = 7/3 > 1$

Diverges

$\boxed{\text{(D) } \sum_{}^{\infty} \frac{(a_n)^2}{3^n}}$

$\lim_{n\to\infty} \frac{(a_{n+1})^2}{3^{n+1}} \cdot \frac{3^n}{(a_n)^2}$

$\to \frac{\cancel{3^n}}{\cancel{3^n} \cdot 3} \cdot \left(\frac{a_{n+1}}{a_n}\right)^2$

$\to \frac{1}{3} \cdot \left(\frac{2}{3}\right)^2 \to \frac{4}{27} < 1$ (Converges)

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14. The infinite series $\sum_{n=1}^{\infty} \frac{1}{7^{n+1}}$ has $n$th partial sum $S_n = \frac{1}{6}\left(\frac{1}{7} - \frac{1}{7^{n+1}}\right)$ for $n \geq 1$. What is the sum of the series?

*Sum of Series is $\lim_{n\to\infty} S_n$

$\lim_{n\to\infty} \frac{1}{6}\left[\frac{1}{7} - \frac{1}{7^{n+1}}\right] = \frac{1}{6}\left(\frac{1}{7} - 0\right) = \boxed{\frac{1}{42}}$

15. For what value of $r$ does the infinite series $\sum_{n=0}^{\infty} 10r^n$ equal 22? ← Geometric Series $S = \frac{a_1}{1-r}$

$a_1 = 10r^0 = 10$

$S = \frac{a_1}{1-r}$

$22 = \frac{10}{1-r}$

$10 = 22(1-r)$

$10 = 22 - 22r$

$22r = 12$

$r = \frac{12}{22}$

$\boxed{r = \frac{6}{11}}$

16. Determine whether the series $\sum_{n=1}^{\infty} \frac{\sin\left[\frac{(2n-1)\pi}{2}\right]}{n}$ converges absolutely, converges conditionally, or diverges.

$n=1 \rightarrow \sin\left(\frac{\pi}{2}\right)=1$

$n=2 \rightarrow \sin\left(\frac{3\pi}{2}\right)=-1$

$n=3 \rightarrow \sin\left(\frac{5\pi}{2}\right)=1$

$n=4 \rightarrow \sin\left(\frac{7\pi}{2}\right)=-1$

Same as $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ $\rightarrow$ Alternating Harmonic series

$\lim_{n\to\infty} \frac{1}{n} = 0$ ✓

$\frac{1}{n}$ is decreasing ✓

$\boxed{\text{Series converges conditionally}}$

17. Determine the convergence of the infinite $p$-series $\sum_{n=1}^{\infty} n^{-\pi}$.

$\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)^{\pi} \rightarrow \frac{1}{n^{\pi}}$ $\boxed{p=\pi>1}$

By p-series test, Series converges.

18. The $n$th-Term Test can be used to determine divergence for which of the following series?

I. $\sum_{n=1}^{\infty} \frac{2}{n+1}$

$\lim_{n\to\infty} \frac{2}{n+1} = 0$

(Inconclusive)

II. $\sum_{n=1}^{\infty} (-1)^{n+1}\left(\frac{n}{4n+1}\right)$

$\lim_{n\to\infty} \frac{n}{4n+1} = \frac{1}{4} \neq 0$

Diverges

III. $\sum_{n=1}^{\infty} \frac{n(n-2)^2}{3n^3+1} \rightarrow \frac{n(n^2-4n+4)}{3n^3+1}$

$\lim_{n\to\infty} \frac{n^3-4n^2+4n}{3n^3+1} = \frac{1}{3} \neq 0$

Diverges

(A) III only

$\boxed{\text{(B) II and III only}}$

(C) I and III only

(D) I, II, and III