BC Calculus Unit 10 "Tests for Convergence" Quiz Review WS #2

Calculators Allowed:

1. The infinite series $\sum_{n=0}^{\infty} a_n$ has nth partial sum $S_n = \frac{4^n - 1}{4^{n+1}}$ for $n \ge 1$. What is the sum of the series?

2. Which of the following series diverge?

$$I. \sum_{n=1}^{\infty} \frac{1}{n^2(n+3)}$$

II.
$$\sum_{n=1}^{\infty} \frac{n^2 2^{n+1}}{3^n}$$

III.
$$\sum_{n=1}^{\infty} \frac{n!}{n4^n}$$

- (A) I only
- (B) II only
- (C) III only
- (D) I and II only
- (E) I, II, and III
- 3. The nth-Term Test can be used to determine divergence for which of the following series?

$$I. \quad \sum_{n=1}^{\infty} \frac{2n+1}{1-n}$$

II.
$$\sum_{n=0}^{\infty} 5 \left(\frac{2}{3}\right)^n$$

III.
$$\sum_{n=1}^{\infty} \frac{2n(n-1)^2}{4n^2 - 3n^3}$$

- (A) I and II only
- (B) II and III only
- (C) I and III only
- (D) I, II, and III
- 4. If b and t are real numbers such that 0 < |t| < |b|, what is the sum of $b^2 \sum_{n=1}^{\infty} \left(\frac{t^2}{b^2}\right)^n$?

- 5. Explain why the Integral Test does not apply to the series $\sum_{n=2}^{\infty} \frac{3}{n^{-2}}$.
- 6. For what values of p will the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^{3p+1}}$ converge?

7. Calculator active. Which of the following series matches the following sequence of partial sums 0.1667, 0.3333, 0.4833, 0.6167, 0.7357, ...?

(A)
$$\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$$

(B)
$$\sum_{n=1}^{\infty} \frac{n}{(n+1)(n+2)}$$

$$(C) \sum_{n=1}^{\infty} \frac{n+1}{n+2}$$

(A)
$$\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$$
 (B) $\sum_{n=1}^{\infty} \frac{n}{(n+1)(n+2)}$ (C) $\sum_{n=1}^{\infty} \frac{n+1}{n+2}$ (D) $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+3)}$

8. For what values of x is the series $\sum_{n=1}^{\infty} \frac{(7x-3)^n}{n}$ conditionally convergent?

(A)
$$x = \frac{2}{7}$$
 (B) $x = \frac{4}{7}$ (C) $x > \frac{4}{7}$ (D) $x < \frac{2}{7}$

(B)
$$x = \frac{4}{7}$$

(C)
$$x > \frac{4}{7}$$

(D)
$$x < \frac{2}{7}$$

- 9. Which of the following series can be used with the Limit Comparison Test to determine whether the series $\sum_{n=1}^{\infty} \frac{3n+2}{n^3-2n}$ converges or diverges?
- (A) $\sum_{n=1}^{\infty} \frac{1}{n}$
- (B) $\sum_{n=1}^{\infty} \frac{1}{n^2}$
- (C) $\sum_{n=1}^{\infty} \frac{1}{n^3}$
- (D) $\sum_{n=1}^{\infty} \frac{1}{n^3 2n}$
- 10. Verify that the infinite series $\sum_{n=1}^{\infty} \frac{7^{n+1}-2}{7^{n+2}}$ diverges by using the *n*th-Term Test for Divergence. Show the value of the limit.
- 11. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{2^n}{9^n+n}$ is true?
 - (A) The series diverges by the nth Term Test.
 - (B) The series diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.
 - (C) The series converges by comparison with $\sum_{n=1}^{\infty} \frac{2^n}{9^n}$.
 - (D) The series converges by comparison with $\sum_{n=1}^{\infty} \frac{1}{9^n}$.
- 12. Which of the following series converge by the Alternating Series Test?

I.
$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$$

$$II. \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$$

III.
$$\sum_{n=1}^{\infty} (-1)^n \left(\frac{\pi}{e}\right)^n$$

13. Which of the following series is absolutely convergent?

I.
$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n^4}}$$

II.
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$$

III.
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

- (A) I only
- (B) I and II only
- (C) I and III only
- (D) I, II, and III
- 14. Use the Integral test to determine if the series $\sum_{n=1}^{\infty} \frac{3n^2}{n^3+1}$ converges or diverges.

- 15. Which of the following statements are true about the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n+1}{n}$?

 I. $a_{n+1} \le a_n$ for all $n \ge 1$.

 II. $\lim_{n \to \infty} a_n \ne 0$

 - III. The series converges by the Alternating Series Test

- A. I only
- B. I and II only
- C. II and III only
- D. I, II, and III



16. What are all values of x > 0 for which the series $\sum_{n=1}^{\infty} \frac{n^2 x^{n+1}}{7^n}$ converges.

17. Which of the following is a convergent *p*-series?

(A)
$$\sum_{n=1}^{\infty} n^4$$

(B)
$$\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$$

(C)
$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2}}$$

(D)
$$\sum_{n=1}^{\infty} \left(\frac{1}{n^3}\right)^{\frac{1}{2}}$$

18. Consider the series $\sum_{n=1}^{\infty} a_n$. If $\frac{a_{n+1}}{a_n} = \frac{1}{2}$ for all integers $n \ge 1$, and $\sum_{n=1}^{\infty} a_n = 64$, then $a_1 = ?$

Answers to Mid-Unit 10 WS#2

1. $\frac{1}{4}$ 2. C 3. C 4. $\frac{b^4}{b^2-t^2}$ 5. $f(n)$ is not a decreasing function for $n \ge 1$.						
6. $p > 0$ 7. B		8	. A		9. B	
10. Diverges by <i>n</i> th-Term Test, $\lim_{n\to\infty} a_n$	$=\frac{1}{7}$	11. C		12.]	3	13. B
14. $\int_1^\infty f(x) dx = \infty$, Series Diverges	15. B	•	16. :	<i>x</i> < 7	17. D	18. 32