Name: Key Review WS #2 Date: ______________ Period: ______ 6

BC Calculus Unit 10 "Tests for Convergence" Quiz Review WS #2 (10.1 - 10.6)

Calculators Allowed:

1. The infinite series $\sum_{n=1}^{\infty} a_n$ has nth partial sum $S_n = \frac{4^n - 1}{4^{n+1}}$ for $n \geq 1$. What is the sum of the series?

* Sum of Series is $\lim_{n\to\infty} S_n$

$$\lim_{n\to\infty} \frac{4^n-1}{4^{n+1}} \to \lim_{n\to\infty} \frac{4^n - 1}{4(4^n)} = \boxed{\frac{1}{4}}$$

2. Which of the following series diverge?

Limit Comparison Test

LCT

I. $\sum_{n=1}^{\infty} \frac{1}{n^2(n+3)} \to \frac{1}{n^3+3n^2}$

Compare with $\frac{1}{n^3}$ (convergent p-series)

$\lim_{n\to\infty} \frac{1}{n^3+3n^2} \cdot \frac{n^3}{1} \to \boxed{1}$ converges by LCT

Ratio Test

II. $\sum_{n=1}^{\infty} \frac{n^2 2^{n+1}}{3^n}$

$\lim_{n\to\infty} \left| \frac{(n+1)^2 \cdot 2^{n+2}}{3^{n+1}} \cdot \frac{3^n}{n^2(2^{n+1})} \right|$

$\lim_{n\to\infty} \left| \frac{\cancel{2^n} \cdot 2^2 \cdot \cancel{3^n}}{\cancel{2^n} \cdot 2 \cdot \cancel{3^n} \cdot 3} \cdot \frac{(n+1)^2}{n^2} \right| = \frac{2}{3} < 1$ (converges)

Ratio Test:

III. $\sum_{n=1}^{\infty} \frac{n!}{n4^n}$

$\lim_{n\to\infty} \left| \frac{(n+1)!}{(n+1)4^{n+1}} \cdot \frac{n \cdot 4^n}{n!} \right|$

$\lim_{n\to\infty} \left| \frac{n \cdot \cancel{4^n} \cdot \cancel{(n+1)} \cdot \cancel{n!}}{\cancel{4^n} \cdot 4 \cdot \cancel{n!} \cdot \cancel{(n+1)}} \right|$

$\lim_{n\to\infty} \left(\frac{n}{4}\right) = \infty > 1$

Diverges

(A) I only (B) II only $\boxed{\text{(C) III only}}$ (D) I and II only (E) I, II, and III

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3. The nth-Term Test can be used to determine divergence for which of the following series?

I. $\sum_{n=1}^{\infty} \frac{2n+1}{1-n}$

$\lim_{n\to\infty} \frac{2n+1}{1-n} = \frac{2}{-1} = -2 \neq 0$

(Diverges)

II. $\sum_{n=0}^{\infty} 5\left(\frac{2}{3}\right)^n$

Geometric Series

$r = \frac{2}{3} < 1$

Converges by GST

III. $\sum_{n=1}^{\infty} \frac{2n(n-1)^2}{4n^2 - 3n^3} \to \frac{2n(n^2-2n+1)}{4n^2-3n^3}$

$\lim_{n\to\infty} \frac{2n^3 - 4n^2 + 2n}{4n^2 - 3n^3} \to \frac{-2}{3} \neq 0$

(Diverges)

(A) I and II only (B) II and III only $\boxed{\text{(C) I and III only}}$ (D) I, II, and III

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4. If b and t are real numbers such that $0 < |t| < |b|$, what is the sum of $b^2 \sum_{n=0}^{\infty} \left(\frac{t^2}{b^2}\right)^n$?

Geometric Series

$\sum_{n=0}^{\infty} a(r)^n$

$r = \frac{t^2}{b^2} < 1$

(converging Geometric Series)

$a_1 = b^2\left(\frac{t^2}{b^2}\right)^0 = b^2$

$S = \frac{a_1}{1-r} \to \frac{b^2}{1 - \frac{t^2}{b^2}}$

$\frac{b^2}{\frac{b^2-t^2}{b^2}}$

$\to b^2 \cdot \frac{b^2}{b^2-t^2}$

$$\boxed{S = \frac{b^4}{b^2 - t^2}}$$

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5. Explain why the Integral Test does not apply to the series $\sum_{n=1}^{\infty} \frac{3}{n^{-2}}$. $\rightarrow \sum_{n=1}^{\infty} 3n^2 \leftarrow$ $\boxed{3n^2 \text{ is not decreasing for } n \geq 1}$

Series does not fulfill condition of function decreasing.

6. For what values of p will the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^{3p+1}}$ converge?

*converges if $3p+1 > 1$

$3p > 0$

$\boxed{p > 0}$

7. **Calculator active.** Which of the following series matches the following sequence of partial sums 0.1667, 0.3333, 0.4833, 0.6167, 0.7357, ...?

(A) $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$ $\boxed{\text{(B)}}$ $\sum_{n=1}^{\infty} \frac{n}{(n+1)(n+2)}$ (C) $\sum_{n=1}^{\infty} \frac{n+1}{n+2}$ (D) $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+3)}$

$S_1 = \frac{1}{6}$

$S_2 = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

$S_3 = \frac{1}{3} + \frac{3}{20} = .4833$

$S_4 = 0.4833 + \frac{2}{15} \approx 0.6167$

$S_5 = 0.616 + \frac{5}{42} \approx 0.7357$

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8. For what values of x is the series $\sum_{n=1}^{\infty} \frac{(7x-3)^n}{n}$ conditionally convergent?

*Ratio Test

$\lim_{n\to\infty} \left| \frac{(7x-3)^{n+1}}{n+1} \cdot \frac{n}{(7x-3)^n} \right| < 1$

$\lim_{n\to\infty} \left| \frac{\cancel{(7x-3)^n}(7x-3)}{\cancel{(7x-3)^n}} \cdot \frac{n}{n+1} \right| < 1$

$|7x-3| < 1$

$-1 < 7x-3 < 1$

$2 < 7x < 4$

$\frac{2}{7} < x < \frac{4}{7}$

test $x = 2/7$

$\frac{[7(2/7)-3]^n}{n} \rightarrow \frac{(2-3)^n}{n} \rightarrow \frac{(-1)^n}{n}$

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ is alternating harmonic series

$\hookrightarrow$ conditionally convergent

$\boxed{\text{(A) } x = \frac{2}{7}}$ (B) $x = \frac{4}{7}$ (C) $x > \frac{4}{7}$ (D) $x < \frac{2}{7}$

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9. Which of the following series can be used with the Limit Comparison Test to determine whether the series $\sum_{n=1}^{\infty} \frac{3n+2}{n^3-2n}$ converges or diverges? $\frac{3n}{n^3} \rightarrow \frac{1}{n^2}$ (Comparison partner for series)

(A) $\sum_{n=1}^{\infty} \frac{1}{n}$ $\boxed{\text{(B)}}$ $\sum_{n=1}^{\infty} \frac{1}{n^2}$ (C) $\sum_{n=1}^{\infty} \frac{1}{n^3}$ (D) $\sum_{n=1}^{\infty} \frac{1}{n^3-2n}$

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10. Verify that the infinite series $\sum_{n=1}^{\infty} \frac{7^{n+1}-2}{7^{n+2}}$ diverges by using the nth-Term Test for Divergence. Show the value of the limit.

$\lim_{n\to\infty} \frac{7(7^n)-2}{7^2(7^n)} \rightarrow \frac{7}{49} \rightarrow \frac{1}{7} \neq 0$ Diverges by n^{th} term test.

11. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{2^n}{9^n+n}$ is true? $\rightarrow$ comparison partner $\frac{2^n}{9^n}$

$\left(\frac{2}{9}\right)^n$ converges by GST since $r = \frac{2}{9} < 1$

(A) The series diverges by the nth Term Test.

(B) The series diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

$\boxed{\text{(C)}}$ The series converges by comparison with $\sum_{n=1}^{\infty} \frac{2^n}{9^n}$.

(D) The series converges by comparison with $\sum_{n=1}^{\infty} \frac{1}{9^n}$.

$0 \le \frac{2^n}{9^n+n} < \frac{2^n}{9^n}$

series converges by Direct Comparison Test

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12. Which of the following series converge by the Alternating Series Test? (AST)

I. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ II. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ III. $\sum_{n=1}^{\infty} (-1)^n \left(\frac{\pi}{e}\right)^n$

I: $\lim_{n\to\infty} \frac{1}{n} = 0$ ✓

$\frac{1}{n}$ is decreasing ✓

converges by (AST)

II: $\lim_{n\to\infty} \frac{1}{n^{1/2}} = 0$ ✓

$\frac{1}{n^{1/2}}$ is decreasing ✓

converges by AST

III: $\lim_{n\to\infty} \left(\frac{\pi}{e}\right)^n = \infty$

Diverges by n^{th} term test

A. I only $\boxed{\text{B.}}$ I and II only C. I and III only D. I, II, and III

13. Which of the following series is absolutely convergent?

I. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n^4}}$

$\sum_{n=1}^{\infty} \frac{1}{n^{4/3}}$ converges by p-series test

$p = 4/3 > 1$

(Absolute Convergence)

Ratio Test II. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

$\lim_{n\to\infty} \left| \frac{1}{(n+1)!} \cdot \frac{n!}{1} \right|$

$\lim_{n\to\infty} \left| \frac{\cancel{n!}}{(n+1)\cancel{n!}} \right| = 0 < 1$

Converges by Ratio Test (Absolute Convergence)

III. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

$\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (harmonic series)

(A) I only $\boxed{\text{(B)}}$ I and II only (C) I and III only (D) I, II, and III

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14. Use the Integral test to determine if the series $\sum_{n=1}^{\infty} \frac{3n^2}{n^3+1}$ converges or diverges.

let $f(x) = \frac{3x^2}{x^3+1}$

$\lim_{b\to\infty} \int_1^b \frac{3x^2}{x^3+1} dx$

$dx = \frac{du}{3x^2}$

$u = x^3+1$

$\frac{du}{dx} = 3x^2$

$\int \frac{\cancel{3x^2}}{u} \cdot \frac{du}{\cancel{3x^2}}$

$\int \frac{1}{u} du = \ln|u| \rightarrow \ln|x^3+1| \Big]_1^b$

$\lim_{b\to\infty} \ln|b^3+1| - \ln|1+1| = \infty$

$\boxed{\text{Series diverge by Integral Test}}$

15. Which of the following statements are true about the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n+1}{n}$?

✓ I. $a_{n+1} \le a_n$ for all $n \ge 1$.

✓ II. $\lim_{n\to\infty} a_n \neq 0$

X III. The series converges by the Alternating Series Test

$\lim_{n\to\infty} \frac{n+1}{n} = 1 \neq 0$ Series diverge by n^{th} term test.

A. I only $\boxed{\text{B.}}$ I and II only C. II and III only D. I, II, and III

16. What are all values of $x > 0$ for which the series $\sum_{n=1}^{\infty} \frac{n^2 x^{n+1}}{7^n}$ converges. *Ratio Test

$$\lim_{n\to\infty} \left| \frac{(n+1)^2 x^{n+2}}{7^{n+1}} \cdot \frac{7^n}{n^2 \cdot x^{n+1}} \right| < 1$$

$$\lim_{n\to\infty} \left| \frac{\cancel{x^n} \cdot x^2 \cdot \cancel{7^n}}{\cancel{x^n} \cdot x \cdot \cancel{7^n} \cdot 7} \cdot \frac{(n+1)^2}{n^2} \right| < 1$$

$$\left|\frac{x}{7}\right| < 1$$

$$|x| < 7$$

$$\boxed{0 < x < 7}$$

17. Which of the following is a convergent p-series?

Diverges nth term test

converges geometric series test $r = 1/2$

$\frac{1}{n^{2/3}}$, $p = 2/3 < 1$ diverges by p-series test.

$\sum \frac{1}{n^{3/2}}$ $p = 3/2 > 1$ converges by p-series test

(A) $\sum_{n=1}^{\infty} n^4$ (B) $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$ (C) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2}}$ $\boxed{\text{(D)}}$ $\sum_{n=1}^{\infty} \left(\frac{1}{n^3}\right)^{\frac{1}{2}}$

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18. Consider the series $\sum_{n=1}^{\infty} a_n$. If $\frac{a_{n+1}}{a_n} = \frac{1}{2}$ for all integers $n \geq 1$, and $\sum_{n=1}^{\infty} a_n = 64$, then $a_1 = ?$

*Geometric Series

$$S = \frac{a_1}{1-r}$$

$S = 64$

$r = 1/2$

$$64 = \frac{a_1}{1 - 1/2}$$

$$64 = \frac{a_1}{1/2} \rightarrow a_1 = \left(\tfrac{1}{2}\right)(64)$$

$$\boxed{a_1 = 32}$$

Answers to Mid-Unit 10 Corrective Assignment

1. $\frac{1}{4}$	2. C	3. C	4. $\frac{b^4}{b^2 - t^2}$	5. $f(n)$ is not a decreasing function for $n \geq 1$.
6. $p > 0$	7. B	8. A	9. B	
10. Diverges by nth-Term Test, $\lim_{n\to\infty} a_n = \frac{1}{7}$	11. C	12. B	13. B	
14. $\int_1^{\infty} f(x)\,dx = \infty$, Series Diverges	15. B	16. $x < 7$	17. D	18. 32