Name: Key WS #3 Date: ______ Period: ____ ⑪

BC Calculus Unit 10 "Tests for Convergence" Review WS #3

Calculators Allowed:

1) The infinite series $\sum_{n=1}^{\infty}\frac{3}{4^{n+1}}$ has nth partial sum $S_n=\frac{1}{4}-\frac{1}{4^{n+1}}$. What is the sum of the series?

*Sum of series is $\lim_{n\to\infty} S_n$

$\lim_{n\to\infty}\frac{1}{4}-\frac{1}{4^{n+1}}\to\frac{1}{4}-0=\boxed{\frac{1}{4}}$

(find sum if possible)

Use the nth-Term Test for Divergence to determine if the series diverges. If inconclusive, find appropriate test.

2. $\sum_{n=0}^{\infty}\frac{\pi^{n+1}}{7^n}\to\sum_{n=0}^{\infty}\pi\left(\frac{\pi}{7}\right)^n$

$\lim_{n\to\infty}\pi\left(\frac{\pi}{7}\right)^n=0$ (n^{th} term inconclusive)

$r=\frac{\pi}{7}<1$

Converges by GST

$S=\frac{a_1}{1-r}$

$a_1=\pi \quad r=\pi/7$

$S=\frac{\pi}{1-\pi/7}\to\frac{\pi}{\frac{7-\pi}{7}}\to\pi\cdot\frac{7}{7-\pi}\to\boxed{\frac{7\pi}{7-\pi}}$

3. $\sum_{n=1}^{\infty}\frac{2(n-2)^2}{3(n+4)^2}$

$\lim_{n\to\infty}\frac{2(n-2)^2}{3(n+4)^2}=\frac{2}{3}\neq 0$

Diverges by n^{th} term test.

4. $\sum_{n=1}^{\infty}\frac{1}{e^n}\to\lim_{n\to\infty}\frac{1}{e^n}=0$ (Inconclusive)

$r=\frac{1}{e}<1$

$a_1=\frac{1}{e}$

$S=\frac{a_1}{1-r}$

$S=\frac{1/e}{1-1/e}$

$S=\frac{\frac{1}{e}}{\frac{e-1}{e}}$

$S=\frac{1}{e}\cdot\frac{e}{e-1}\to\boxed{\frac{1}{e-1}}$

Converges by GST

5) If the infinite series $\sum_{n=1}^{\infty}a^n$ has nth partial sum $S_n=\frac{4}{3}(4^n-1)$ for $n\geq 1$. What is the sum of the series?

$\lim_{n\to\infty}\frac{4}{3}(4^n-1)=\infty$ $\boxed{\text{Series Diverges}}$

6) Does the series $\sum_{n=1}^{\infty}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$ converge or diverge? If it converges find its sum.

$\overset{n=1}{\left(\frac{1}{2-1}-\cancel{\frac{1}{3}}\right)}+\overset{n=2}{\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{5}}\right)}+\overset{n=3}{\left(\cancel{\frac{1}{5}}-\cancel{\frac{1}{7}}\right)}+\overset{n=4}{\left(\cancel{\frac{1}{7}}-\cancel{\frac{1}{9}}\right)}+\cancel{\ }\ldots$

$=\frac{1}{1}=\boxed{1}$

7) What is the sum of the infinite geometric series $11+-\frac{11}{3}+\frac{11}{9}+-\frac{11}{27}+\cdots$?

$r=\frac{-\frac{11}{3}}{11}=\frac{-11}{3}\cdot\frac{1}{11}=\frac{-1}{3}$

$S=\frac{a_1}{1-r}$ $\quad a_1=11$

$S=\frac{11}{1-(-1/3)}\to\frac{11}{\frac{3}{3}+\frac{1}{3}}\to\frac{11}{4/3}$

$\to 11\cdot\frac{3}{4}=\boxed{\frac{33}{4}}$

* Geometric Series

8) What is the value of $\sum_{n=1}^{\infty} \frac{(-e)^{n+1}}{9^n}$?

$\rightarrow \frac{(-e)^n \cdot (-e)^1}{9^n} \rightarrow -e\left(\frac{-e}{9}\right)^n$

$a_1 = \frac{(-e)^2}{9} = \frac{e^2}{9}$

$r = \frac{-e}{9}$

$S = \frac{a_1}{1-r}$

$S = \frac{e^2/9}{1-\left(\frac{-e}{9}\right)}$

$S = \frac{\frac{e^2}{9}}{\frac{9+e}{9}} \rightarrow \frac{e^2}{\cancel{9}} \cdot \frac{\cancel{9}}{9+e}$

$\boxed{S = \frac{e^2}{9+e}}$

9) For what value of a does the infinite series $\sum_{n=0}^{\infty} a\left(-\frac{3}{5}\right)^n$ equal 15?

$r = \frac{-3}{5}$

$S = 15$

$S = \frac{a_1}{1-r}$

$15 = \frac{a_1}{1-(-3/5)}$

$15 = \frac{a_1}{8/5}$

$a_1 = \frac{8}{5}(15)$

$\boxed{a_1 = 24}$

10) The nth-Term Test can be used to determine divergence for which of the following series?

I. $\sum_{n=1}^{\infty} \frac{(n+1)^3}{3n^3 - 2n + 1}$

$\lim_{n\to\infty} \frac{(n+1)^3}{3n^3-2n+1} = \frac{1}{3} \neq 0$

(Diverges by nth term test)

II. $\sum_{n=1}^{\infty} \frac{(n+1)^2}{2n^2 - 3n^3 + 1}$

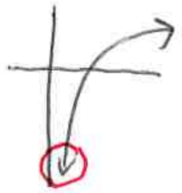
$\lim_{n\to\infty} \frac{(n+1)^2}{2n^2-3n^{③}+1} = 0$

Inconclusive by nth term test

III. $\sum_{n=1}^{\infty} \ln\frac{1}{n}$

$\lim_{n\to\infty} \ln\left(\frac{1}{n}\right) \rightarrow \ln\left(\frac{1}{100000\ldots}\right)$

$\rightarrow \ln(0.00000\ldots1)$



$\lim_{n\to\infty} \ln\left(\frac{1}{n}\right) = -\infty \neq 0$

Diverges by nth term test

A. III only

$\boxed{\text{B. I and III only}}$

C. II and III only

D. I, II, and III

11) Verify that the infinite series $\sum_{n=1}^{\infty} \frac{6^n + 1}{6^{n+1}}$ diverges by using the nth-Term Test for Divergence. Show the value of the limit.

$\lim_{n\to\infty} \frac{6^n + 1}{6(6^n)} = \frac{1}{6} \neq 0$

$\boxed{\text{Diverges by nth term test}}$

12) Use the Integral Test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^5}$.

let $f(x) = \frac{1}{x^5}$ f(x) is positive, decreasing, continuous

$\lim_{b\to\infty} \int_1^b \frac{1}{x^5}dx \rightarrow \int x^{-5}dx \rightarrow \frac{x^{-4}}{-4} \rightarrow \left.\frac{-1}{4x^4}\right]_1^b \rightarrow \lim_{b\to\infty} \frac{-1}{4b^4} - \frac{-1}{4(1)^4} = \boxed{\frac{1}{4}}$

$\boxed{\text{Series Converge by Integral Test}}$

13) Prove the Integral Test applies to the series $\sum_{n=1}^{\infty} \frac{1}{(n+1)^3}$. Determine the convergence or divergence of the series.

f(x) continuous, positive, decreasing

$$\lim_{n \to \infty} \int_1^b \frac{1}{(x+1)^3} dx$$

$u = x+1$

$\frac{du}{dx} = 1$

$$\int u^{-3} du$$

$$\frac{u^{-2}}{-2} \to \frac{-1}{2u^2} \to \frac{-1}{2(x+1)^2}\Bigg]_1^b$$

$$\lim_{b \to \infty} \frac{-1}{2(1+b)^2} - \frac{-1}{2(2)^2} = \boxed{\frac{1}{8}}$$

Series converge by Integral Test

14) Use the Integral Test to determine if the series $\sum_{n=1}^{\infty} \frac{4n}{2n^2+1}$ converges or diverges.

$$f(x) = \frac{4x}{2x^2+1}$$

f(x) positive, decreasing, continuous

$$\lim_{b \to \infty} \int_1^b \frac{4x}{2x^2+1} dx$$

$u = 2x^2+1$

$\frac{du}{dx} = 4x$

$dx = \frac{du}{4x}$

$$\int \frac{\cancel{4x}}{u} \cdot \frac{du}{\cancel{4x}}$$

$$\int \frac{1}{u} du$$

$$\ln|u| \to \ln|2x^2+1|\Big]_1^b$$

$$\lim_{b \to \infty} \ln|2b^2+1| - \ln|2+1| = \boxed{\infty}$$

Diverges by Integral Test

15) For what values of p will the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^{1-p}}$ converge?

$1 - p > 1$

$0 > p$

$\boxed{p < 0}$

16) For what values of p will both infinite series $\sum_{n=1}^{\infty} \left(\frac{3}{p}\right)^n$ and $\sum_{n=1}^{\infty} \frac{1}{n^{5-p}}$ converge?

$$\sum_{n=1}^{\infty} \left(\frac{3}{p}\right)^n$$

Geometric series converge if

$\frac{3}{p} < 1$ so $\boxed{p > 3}$

$$\sum_{n=1}^{\infty} \frac{1}{n^{5-p}}$$

converges if the exponent in the p-series is greater than 1

$5 - p > 1$

$4 > p$

$\boxed{p < 4}$

$\boxed{3 < p < 4}$

17) Which of the following is a divergent p-series?

A. $\sum_{n=1}^{\infty} n^{-\pi}$

$\left(\frac{1}{n}\right)^{\pi}$

$\frac{1}{n^{\pi}}$

$p = \pi > 1$

converges

[✓ B.] $\sum_{n=1}^{\infty} \frac{1}{n}$

harmonic p-series

diverges

C. $\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^n$

geometric series converge since

$r = \frac{e}{\pi} < 1$

D. $\sum_{n=1}^{\infty} \frac{1}{n^3}$

$p = 3 > 1$

converges

18) Which of the following series converges?

Diverges by LCT, use $\frac{1}{n}$

(A) $\sum_{n=1}^{\infty}\frac{3n}{2n^2+1}$ $\lim_{n\to\infty}\frac{3n}{2n^2+1}\cdot\frac{n}{1}\to\frac{3n^2}{2n^2}\to\frac{3}{2}=L$

(B) $\sum_{n=1}^{\infty}\frac{3n^2}{n+2n^2}$

Diverges by n^{th} term test $\lim_{n\to\infty}\frac{3n^2}{2n^2}=\frac{3}{2}\neq 0$

(C) $\sum_{n=1}^{\infty}\left(\frac{\pi}{e}\right)^n$

$r=\frac{\pi}{e}>1$

Diverges by n^{th} term or GST

(D) $\sum_{n=1}^{\infty}\frac{3n^2}{2n^3+3n}$

Diverges by LCT, use $\frac{1}{n}$

$\lim_{n\to\infty}\frac{3n^2}{2n^3+3n}\cdot\frac{n}{1}=\frac{3n^2}{2n^3}=\frac{3}{2}=L$

$\boxed{\text{(E)}}$ $\sum_{n=1}^{\infty}\frac{n-1}{n4^n}$

LCT, use $\frac{1}{4^n}\to\left(\frac{1}{4}\right)^n$ Converge by GST

$\lim_{n\to\infty}\frac{n-1}{n\cdot 4^n}\cdot\frac{4^n}{1}$

$=\frac{n}{n-1}\to 1=L$ $\boxed{\text{Converge by LCT}}$

19) Which of the following series can be used with the Limit Comparison Test to determine whether the series $\sum_{n=1}^{\infty}\frac{5^n}{7^n-n^2}$ diverges or converges? → comparison partner $\frac{5^n}{7^n}\to\left(\frac{5}{7}\right)^n$ converge by GST

$r=5/7<1$

$\lim_{n\to\infty}\frac{5^n}{7^n-n^2}\cdot\frac{7^n}{5^n}=1=L$

$\boxed{\text{converges by LCT}}$

(A) $\sum_{n=1}^{\infty}\frac{1}{n}$

(B) $\sum_{n=1}^{\infty}\frac{1}{5^n}$

(C) $\sum_{n=1}^{\infty}\frac{1}{7^n}$

$\boxed{\text{(D)}}$ $\sum_{n=1}^{\infty}\left(\frac{5}{7}\right)^n$

20) Use the Comparison Test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty}\frac{n-2}{n5^n}$. You must identify the series you are using for comparison.

compare with $\sum\frac{1}{5^n}\to\left(\frac{1}{5}\right)^n$

converges by GST

since $r=\frac{1}{5}<1$

$\lim_{n\to\infty}\frac{n-2}{n\cancel{5^n}}\cdot\frac{\cancel{5^n}}{1}=1=L$

$\boxed{\text{Series converge by LCT.}}$

21) Determine whether the series $\sum_{n=1}^{\infty}\frac{n5^n}{4n^4-3}$ converges or diverges. Identify the test for convergence used.

compare with $\frac{5^n}{n^3}$

diverges by n^{th} term test since $\lim_{n\to\infty}\frac{5^n}{n^3}=\infty\neq 0$

$\lim_{n\to\infty}\frac{n5^n}{4n^4-3}\cdot\frac{n^3}{5^n}\to\frac{1}{4}=L$

$\boxed{\text{Series diverge by LCT}}$

22) Explain why the Alternating Series Test does not apply to the series $\sum_{n=1}^{\infty} \frac{(-1)^n \cos(n\pi)}{n^2}$.

$n=1 \rightarrow (-1)^1\cos(\pi) = (-1)(-1) = 1$

$n=2 \rightarrow (-1)^2\cos(2\pi) = (1)(1) = 1$

$n=3 \rightarrow (-1)^3\cos(3\pi) = (-1)(-1) = 1$

*series does not alternate in signs

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (absolutely) since $p = 2 > 1$

23) Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n n}{\ln(n+1)}$.

$\boxed{\text{LRPE}}$

$\lim_{n\to\infty} \frac{n}{\ln(n+1)} = \infty \neq 0$

*comparative growth rate $\left(\lim_{n\to\infty}\right)$

Logs < Radicals < Polynomial < Exponent

series diverge by n^{th} term test.

24) Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{(-1)^n}{4^n}$ II. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\pi^n}$ III. $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{1+n^2}$

A. I only [✓ B. I and II only] C. I and III only D. I, II, and III

i) $\lim_{n\to\infty} \frac{1}{4^n} = 0$

ii) $|a_{n+1}| \leq |a_n|$

i) $\lim_{n\to\infty} \left(\frac{1}{\pi}\right)^n = 0$

ii) $|a_{n+1}| \leq |a_n|$

$\lim_{n\to\infty} \frac{n^2}{1+n^2} = 1 \neq 0$

Diverges by n^{th} term test

25) Which of the following statements is true?

A. $\sum_{n=1}^{\infty} \frac{(-1)^n(1-n)}{n}$ converges by the Alternating Series Test. $\lim_{n\to\infty} \frac{1-n}{n} = -1 \neq 0$

B. $\sum_{n=1}^{\infty} \frac{(-1)^n(n+1)}{2n}$ converges by the Alternating Series Test. $\lim_{n\to\infty} \frac{n+1}{2n} = \frac{1}{2} \neq 0$

C. $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{4\sqrt{n}}$ converges by the Alternating Series Test. $\lim_{n\to\infty} \frac{n^2}{4\sqrt{n}} = \infty \neq 0$

Diverges by n^{th} term test

[✓ D.] $\sum_{n=1}^{\infty} \frac{(-1)^n 2\sqrt{n}}{n}$ converges by the Alternating Series Test. $\lim_{n\to\infty} \frac{2\sqrt{n}}{n} = 0$ ✓

$|a_{n+1}| \leq |a_n|$

26) Use the Ratio Test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty}\frac{n^4}{3^n}$.

★ Ratio Test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$$

$$\lim_{n\to\infty}\left|\frac{(n+1)^4}{3^{n+1}}\cdot\frac{3^n}{n^4}\right| = \lim_{n\to\infty}\left|\frac{\cancel{3^n}\cdot(n+1)^4}{3\cdot\cancel{3^n}\cdot n^4}\right| = \frac{1}{3} < 1$$

$\boxed{\text{Converges by Ratio Test}}$

27) If the Ratio Test is applied to the series $\sum_{n=0}^{\infty}\frac{6^n}{(n+1)^n}$, which of the following inequalities results, implying that the series converges?

A. $\lim_{n\to\infty}\frac{6^n}{(n+1)^n} < 1$

$\boxed{\text{B.}}$ $\lim_{n\to\infty}\frac{6(n+1)^n}{(n+2)^{n+1}} < 1$

C. $\lim_{n\to\infty}\frac{6^{n+1}}{(n+1)^n} < 1$

D. $\lim_{n\to\infty}\frac{6^{n+1}}{(n+1)^{n+1}} < 1$

$$\lim_{n\to\infty}\left|\frac{6^{n+1}}{(n+2)^{n+1}}\cdot\frac{(n+1)^n}{6^n}\right| < 1$$

$$\lim_{n\to\infty}\left|\frac{\cancel{6^n}\cdot 6\cdot(n+1)^n}{(n+2)^{n+1}\cdot\cancel{6^n}}\right| < 1$$

$$\lim_{n\to\infty}\left|\frac{6(n+1)^n}{(n+2)^{n+1}}\right| < 1$$

$\boxed{\text{By Root test}}$

$$\lim_{n\to\infty}\sqrt[n]{\left(\frac{6}{n+1}\right)^n} = \lim_{n\to\infty}\frac{6}{n+1} = 0 < 1$$

Converges by Root test

28)

If $a_n > 0$ for all n and $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = 5$, which of the following series converges?

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{(n+1)^2}\cdot\frac{n^2}{a_n}\right|$$

$5\cdot 1 = 5 > 1$

A. $\sum_{n=1}^{\infty}\frac{a_n}{n^2}$

Diverges by Ratio Test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{2^{n+1}}\cdot\frac{2^n}{a_n}\right|$$

B. $\sum_{n=1}^{\infty}\frac{a_n}{2^n}$

$$\left|\frac{a_{n+1}}{a_n}\cdot\frac{1}{2}\right| = 5\left(\frac{1}{2}\right) = \frac{5}{2} > 1$$

Diverges by Ratio Test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{(n+1)^5}\cdot\frac{n^5}{a_n}\right| = 5(1) = 5 > 1$$

C. $\sum_{n=1}^{\infty}\frac{a_n}{n^5}$

Diverges by Ratio Test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{7^{n+1}}\cdot\frac{7^n}{a_n}\right| = \left|\frac{a_{n+1}}{a_n}\cdot\frac{1}{7}\right| = \frac{5}{7} < 1$$

$\boxed{\text{D.}}$ $\sum_{n=1}^{\infty}\frac{a_n}{7^n}$

Converges by Ratio Test

29) What are all values of $x > 0$ for which the series $\sum_{n=1}^{\infty} \frac{6n^3}{x^n}$ converges?

$$\lim_{n\to\infty} \left| \frac{6(n+1)^3}{x^{n+1}} \cdot \frac{x^n}{6n^3} \right| < 1$$

$$\left| \frac{1}{x} \right| < 1$$

$$\frac{1}{x} < \frac{1}{1}$$

$$1 < x$$

$$\boxed{x > 1}$$

30) Which of the following series converge?

I. $\sum^{\infty} \frac{1}{n!}$ II. $\sum^{\infty} \frac{9^n}{n^5}$ III. $\sum^{\infty} \frac{5n}{2n-1}$

save time by applying nth term test before Ratio Test

A. I only B. I and II only C. I and III only D. I, II, and III

$\boxed{\text{A. I only}}$

$$\lim_{n\to\infty} \left| \frac{1}{(n+1)!} \cdot \frac{n!}{1} \right|$$

$$\lim_{n\to\infty} \frac{\cancel{n!}}{(n+1)\cancel{n!}} = 0 < 1$$

converges by Ratio Test

$$\lim_{n\to\infty} \frac{9^n}{n^5} = \infty \neq 0$$

Diverges by nth term test

$$\lim_{n\to\infty} \frac{5n}{2n-1} = \frac{5}{2} \neq 0$$

Diverges by nth term test

31) For what values of x is the series $\sum_{n=0}^{\infty} (-1)^n (5x+1)^n$ absolutely convergent?

$$\lim_{n\to\infty} \left| \frac{(5x+1)^{n+1}}{(5x+1)^n} \right| < 1$$

$$|5x+1| < 1$$

$$-1 < 5x+1 < 1$$

$$-2 < 5x < 0$$

$$\frac{-2}{5} < \frac{5x}{5} < \frac{0}{5}$$

$$\boxed{-\frac{2}{5} < x < 0}$$

32) For what values of x is the series $\sum_{n=1}^{\infty} \frac{(5x-2)^n}{n}$ conditionally convergent?

A. $x > \frac{3}{5}$ B. $x = \frac{3}{5}$ C. $x = \frac{1}{5}$ D. $x < \frac{1}{5}$

$\boxed{\text{C.}}$

$$\lim_{n\to\infty} \left| \frac{(5x-2)^{n+1}}{n+1} \cdot \frac{n}{(5x-2)^n} \right| < 1$$

$$\lim_{n\to\infty} \left| \frac{5x-2}{1} \cdot \frac{n}{n+1} \right| < 1$$

$$|5x-2| < 1$$

$$-1 < 5x-2 < 1$$

$$1 < 5x < 3$$

$$\frac{1}{5} < x < \frac{3}{5}$$

test $x = 1/5$

$$\frac{(5(\frac{1}{5})-2)^n}{n}$$

$$\frac{(-1)^n}{n}$$

alternating harmonic series

test $x = 3/5$

$$(5(\tfrac{3}{5})-2)^n$$

$$\frac{(3-2)^n}{n} = \frac{1^n}{n}$$

diverges by p-series

⑱

33) Which of the following statements is true about the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2\sqrt{n}}$. $\to \frac{(-1)^{n+1}}{n^{5/2}}$

$p = 5/2 > 1$

Series converge
(Absolute convergence)

A. The series converges conditionally.

[✓ B.] The series converges absolutely.

C. The series converges but neither conditionally nor absolutely.

D. The series diverges.

34) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+5}$ converges absolutely, converges conditionally, or diverges.

$\frac{1}{n}$ diverges (p-series)

By LCT

$\lim_{n\to\infty} \frac{1}{n+5} \cdot \frac{n}{1} = 1 = L$

$\frac{1}{n+5}$ diverges by LCT

By AST,

i) $\lim_{n\to\infty} \frac{1}{n+5} = 0$ ✓

ii) $|a_{n+1}| \le |a_n|$

converges by AST

$\boxed{\text{Series converge conditionally}}$

35) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)!}$ converges absolutely, converges conditionally, or diverges.

$\lim_{n\to\infty} \left| \frac{1}{(n+2)!} \cdot \frac{(n+1)!}{1} \right|$

$\lim_{n\to\infty} \left| \frac{\cancel{(n+1)}\,\cancel{n!}}{(n+2)\cancel{(n+1)}\,\cancel{n!}} \right|$

$\lim_{n\to\infty} \left| \frac{1}{n+2} \right| = 0 < 1$

Converges by Ratio Test

$\boxed{\text{Series converge (Absolute Convergence)}}$