8.10 Log Test Review

Date: ______________

Complete each problem without using a calculator.

Write each logarithm in exponential form.

Write each exponential in logarithmic form.

1. $log_{49} 7 = \frac{1}{2}$

$49^{1/2} = 7$

2. $log_{10} x = 4.5$

$10^{4.5} = x$

3. $3^{-4} = \frac{1}{81}$

$\log_3\left(\frac{1}{81}\right) = -4$

4. $e^5 = 148.413$

$\log_e 148.413 = 5$

$\boxed{\ln 148.413 = 5}$

Evaluate.

5. $log_5 \sqrt[3]{25} = x$

$5^x = \sqrt[3]{25} = \sqrt[3]{5^2}$

$5^x = 5^{2/3}$

$\boxed{x = 2/3}$

6. $log_2 \frac{1}{16} = x$

$2^x = \frac{1}{16} = 2^{-4}$

$\boxed{x = -4}$

7. $log_6(-36) = x$

$6^{-x} = -36$

$\boxed{\text{No solution}}$

8. $log_{10} 0.01 = x$

$10^x = 0.01 = \frac{1}{100}$

$10^x = 10^{-2}$

$\boxed{x = -2}$

9. $ln\, e^4 = x$

$\log_e e^4 = x$

$4\log_e e = x$

$\boxed{x = 4}$

10. $log_9 \sqrt[4]{3} = x$

$9^x = \sqrt[4]{3}$

$3^{2x} = 3^{1/4}$

$2x = \frac{1}{4}$

$x = \frac{1}{2} \cdot \frac{1}{4}$

$\boxed{x = \frac{1}{8}}$

11. $log_3 27 + 2\, log_5 25$

$\log_3 3^3 + 2\log_5 5^2$

$3\cancel{\log_3 3} + 2 \cdot 2\cancel{\log_5 5}$

$3 + 2(2) = \boxed{7}$

12. $8\, log_2 \sqrt{32}$

$8\log_2 \sqrt{2^5}$

$8\log_2 2^{5/2}$

$8 \cdot \frac{5}{2}\cancel{\log_2 2}$

$8\left(\frac{5}{2}\right) = \boxed{20}$

Use properties of logs to expand. Simplify, if possible.

13. $log_9 \frac{3x^4}{y}$

$\log_9 3x^4 - \log_9 y$

$\boxed{\log_9 3 + 4\log_9 x - \log_9 y}$

14. $log_3 \sqrt[5]{x^2y^3z^4}$

$\log_3 \left(x^2y^3z^4\right)^{1/5}$

$\frac{1}{5}\log_3\left(x^2y^3z^4\right) \rightarrow \frac{1}{5}\cdot 2\log_3 x + \frac{1}{5}\cdot 3\log_3 y + \frac{1}{5}\cdot 4\log_3 z$

$\rightarrow \boxed{\frac{2}{5}\log_3 x + \frac{3}{5}\log_3 y + \frac{4}{5}\log_3 z}$

15. $ln \sqrt[5]{x^3(x+1)}$

$\ln\left[x^3(x+1)\right]^{1/5} \rightarrow \frac{1}{5}\ln x^3 + \frac{1}{5}\ln(x+1)$

$\boxed{\frac{3}{5}\ln x + \frac{1}{5}\ln(x+1)}$

Use properties of logs to condense. Simplify, if possible.

16. $5log_4 a + 6log_4 b - \frac{1}{3}log_4 7c$

$\log_4 a^5 + \log_4 b^6 - \log_4 (7c)^{1/3} \rightarrow \boxed{\log_4\left(\frac{a^5 b^6}{\sqrt[3]{7c}}\right)}$

17. $2\log(x+1) - \log(x^2-1)$

$\rightarrow \log(x+1)^2 - \log(x^2-1)$

$\rightarrow \log\left(\frac{(x+1)^2}{x^2-1}\right) \rightarrow \log\left(\frac{\cancel{(x+1)}(x+1)}{\cancel{(x+1)}(x-1)}\right) \rightarrow \boxed{\log\left(\frac{x+1}{x-1}\right)}$

18. $\frac{5}{2}lnx + \frac{1}{2}\ln(y+8) - 3lny - \ln(10-x)$

$\ln x^{5/2} + \ln(y+8)^{1/2} - \ln y^3 - \ln(10-x) \rightarrow \boxed{\ln\left(\frac{x^{5/2}(y+8)^{1/2}}{y^3(10-x)}\right)}$

Solve. Write answers in simplest form.

13. $9^{3x+1} = 81$

$3^{2(3x+1)} = 3^4$

$2(3x+1) = 4$

$6x + 2 = 4$

$6x = 2$ $\boxed{x = 1/3}$

14. $125^{x-2} = 25^{2x+1}$

$5^{3(x-2)} = 5^{2(2x+1)}$

$3x - 6 = 4x + 2$

$-8 = 1x$

$\boxed{x = -8}$

15. $4e^{2x} - 13 = 5$

$4e^{2x} = 18$

$e^{2x} = \frac{18}{4} = 4.5$

$\ln e^{2x} = \ln 4.5$

$2x \cancel{\ln e} = \ln 4.5$

$2x = \ln 4.5$

$\boxed{x = \frac{\ln 4.5}{2}}$

16. $4^{x-1} = 12$

$\log 4^{x-1} = \log 12$

$(x-1)\log 4 = \log 12$

$x - 1 = \frac{\log 12}{\log 4}$

$\boxed{x = \frac{\log 12}{\log 4} + 1}$

17. $e^{2x} - 3e^x = 10$ $x = e^x$

$e^{2x} - 3e^x - 10 = 0$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$\downarrow$

$(e^x - 5)(e^x + 2) = 0$

$e^x - 5 = 0$

$e^x = 5$

$\ln e^x = \ln 5$

$\boxed{x = \ln 5}$

$e^x + 2 = 0$

$e^x = -2$

$\ln e^x = \ln(-2)$

$\boxed{\text{No solution}}$

18. $3e^{4x} - 5e^{2x} - 2 = 0$

$*x = e^x$

-6, -6, 1, 3, 3, -5

$3x^4 - 5x^2 - 2 = 0$

$(x^2 - 2)(x^2 + \frac{1}{3})$

$(x^2 - 2)(3x^2 + 1)$

$(e^{2x} - 2)(3e^{2x} + 1) = 0$

$e^{2x} - 2 = 0$

$e^{2x} = 2$

$\ln e^{2x} = \ln 2$

$2x \cancel{\ln e} = \ln 2$

$\boxed{x = \frac{\ln 2}{2}}$

$3e^{2x} + 1 = 0$

$3e^{2x} = -1$

$e^{2x} = -1/3$

$\ln e^{2x} = \ln(-1/3)$

$\boxed{\text{No solution}}$

19. $4^{2x+3} = 11^{2-x}$

$\log 4^{2x+3} = \log 11^{2-x}$

$(2x+3)\log 4 = (2-x)\log 11$

$2x\log 4 + 3\log 4 = 2\log 11 - x\log 11$

$2x\log 4 + x\log 11 = 2\log 11 - 3\log 4$

$x(2\log 4 + \log 11) = 2\log 11 - 3\log 4$

$\boxed{x = \frac{2\log 11 - 3\log 4}{2\log 4 + \log 11}}$

Solve.

20. $log_8(3x+1) = 2$

$8^2 = 3x+1$

$64 = 3x+1$

$\frac{63}{3} = \frac{3x}{3}$

$\boxed{21 = x}$ ✓

21. $ln(3x) + 5 = 5$

$ln(3x) = 0$

$log_e(3x) = 0$

$e^0 = 3x$

$1 = 3x$

$\boxed{\frac{1}{3} = x}$ ✓

22. $log_4 x + log_4(x+6) = 2$

$log_4 x(x+6) = 2$

$log_4(x^2+6x) = 2$

$4^2 = x^2+6x$

$0 = x^2+6x-16$

$0 = (x+8)(x-2)$

$x = \cancel{-8}$, $\boxed{x = 2}$

extraneous solution.

23. $ln(4x^2) = 2\,ln\,(x+4)$

$ln(4x^2) = ln(x+4)^2$

$4x^2 = (x+4)^2$

$4x^2 = (x+4)(x+4)$

$4x^2 = x^2+8x+16$

$3x^2-8x-16 = 0$

$(3x+4)(x-4) = 0$

$3x+4 = 0$ | $x-4 = 0$

$\boxed{x = \frac{-4}{3}}$ | $\boxed{x = 4}$

24. $log_7(x+6) - log_7(2x) = log_7(x+1)$

$log_7\left(\frac{x+6}{2x}\right) = log_7(x+1)$

$\frac{x+6}{2x} = \frac{x+1}{1}$

$2x(x+1) = x+6$

$2x^2+2x = x+6$

$2x^2+1x-6 = 0$

$(2x-3)(x+2) = 0$

$2x-3 = 0$ | $x+2 = 0$

$\boxed{x = \frac{3}{2}}$ | $x = -2$

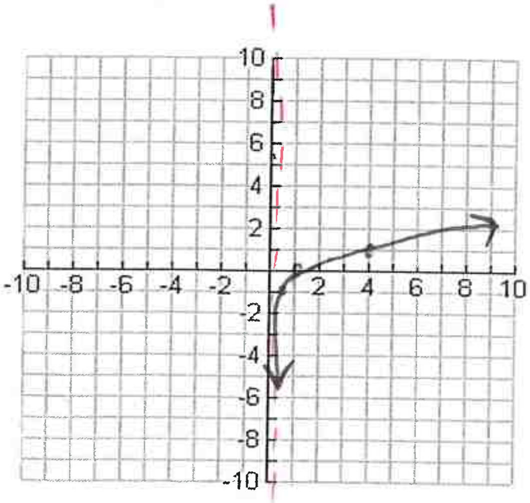
(extraneous solution)

$*\log_b \frac{1}{b} = -1$ $\quad *\log_b b = 1$ $\quad *\log_b(0) \rightarrow$ Vertical Asymptote

$*\log_b 1 = 0$

25. $f(x) = \log_4 x$ Graph the parent function, *f(x)*. State its asymptote, domain, range, and x-intercept

$y = \log_4 x$



x	y
0	VA: x = 0
1/4	-1
1	0
4	1

Asymptote: VA: x = 0

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

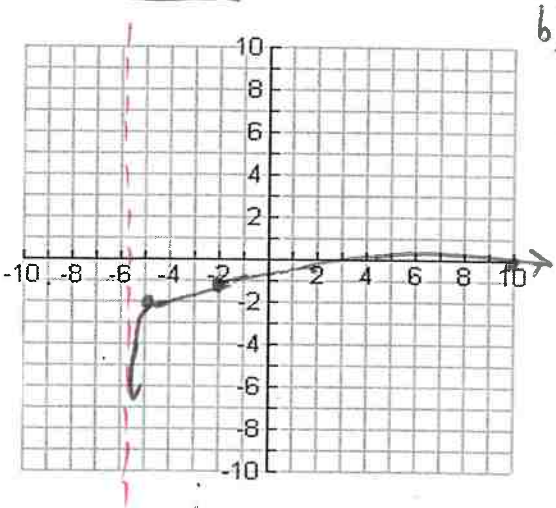
X-Intercept: $(1, 0)$

Next, analyze the other functions as transformations of *f(x) from #25 above*. Graph each. Then state its asymptote, domain and range.

26. $g(x) = \log_4(x+6) - 2$

Transformations to map *f(x)* onto *g(x)*: a) translated (shift) left 6 units

b) shift down 2 units



x	y
-6	VA
-5	-2
-2	-1

Asymptote: X = -6

Domain: $(-6, \infty)$

Range: $(-\infty, \infty)$

X-int: $(10, 0)$

VA: $x + 6 = 0$

$x = -6$

$\boxed{* \text{ to find x-int, set } y = 0}$

$0 = \log_4(x+6) - 2$

$2 = \log_4(x+6)$

$4^2 = x + 6$

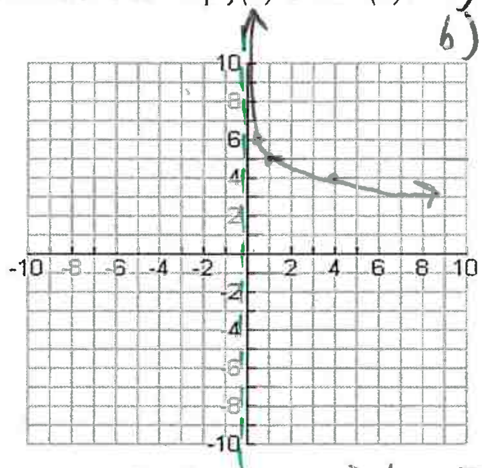
$16 = x + 6$

$\boxed{10 = x}$

27. $h(x) = -log_4(x) + 5$

Transformations to map *f(x)* onto *h(x)*: a) reflection over x-axis
b) vertical translation (shift up) 5 units

x	y
0	V.A.
1	5
4	4
$\frac{1}{4}$	6



Asymptote: VA: $x=0$

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

x-int: $(4^5, 0)$

x-int: (set $y=0$)

$0 = -\log_4(x) + 5$

$-5 = -\log_4(x)$

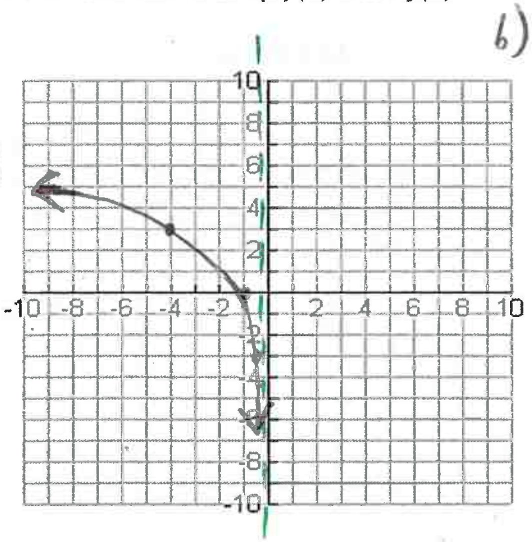
$5 = \log_4 x$

$\boxed{4^5 = x}$

28. $j(x) = 3log_4(-x)$

Transformations to map *f(x)* onto *j(x)*: a) Vertical stretch, factor of 3
b) reflection over y-axis

x	y
0	VA
-1	0
-4	3
$-\frac{1}{4}$	-3



Asymptote: $x=0$

Domain: $(-\infty, 0)$

Range: $(-\infty, \infty)$

x-int: $(-1, 0)$