

Exponentials and Logs Test Review WS #2

Key

Solve each of the following exponential equations. Round to three decimals when necessary.

1. $2^x = 7$

$\log 2^x = \log 7$

$x \log 2 = \log 7$

$x = \frac{\log 7}{\log 2} = \boxed{2.807}$

2. $4^{x+1} = 3$

$\log 4^{x+1} = \log 3$

$(x+1) \log 4 = \log 3$

$(x+1) = \frac{\log 3}{\log 4}$

$x+1 = 0.792$

$x = \boxed{-0.208}$

3. $7 \cdot e^{x-3} = 57$

$e^{x-3} = \frac{57}{7} = 8.143$

$e^{x-3} = 8.143$

$\ln e^{x-3} = \ln 8.143$

$(x-3) \ln e = \ln 8.143$

$x = 3 + 2.097$

$x = \boxed{5.097}$

4. $8e^{2x} = 20$

$e^{2x} = 2.5$

$\ln e^{2x} = \ln 2.5$

$2x \ln e = \ln 2.5$

$x = \frac{\ln 2.5}{2} =$

$x = \boxed{0.458}$

5. $e^{3-2x} = 4$

$\ln e^{3-2x} = \ln 4$

$(3-2x) \ln e = \ln 4$

$3-2x = 1.386$

$-2x = -3 - 1.386$

$\frac{-2x}{-2} = \frac{-1.6137}{-2}$

$x = \boxed{0.807}$

6) $5^{2x-1} = 7^{1-x}$

$\log 5^{2x-1} = \log 7^{1-x}$

$(2x-1) \log 5 = (1-x) \log 7$

$2x \log 5 - \log 5 = \log 7 - x \log 7$

$2x \log 5 + x \log 7 = \log 7 + \log 5$

$x(2 \log 5 + \log 7) = \log 7 + \log 5$

$x = \frac{\log 7 + \log 5}{2 \log 5 + \log 7} \approx \boxed{0.688}$

#7 Method 2

$4^x = 8$
 $2^{2x} = 2^3$
 $2x = 3$
 $x = \boxed{3/2}$

7. $4^x - 5 = 3$

Method 1

$4^x = 8$

$\log 4^x = \log 8$

$x \log 4 = \log 8$

$x = \frac{\log 8}{\log 4} \approx \frac{3}{2} = 1.5$

8. $4 - 2e^x = -23$

$\frac{-2e^x}{-2} = \frac{-27}{-2}$

$e^x = 13.5$

$\ln e^x = \ln 13.5$

$x \ln e = \ln 13.5$

$x = \boxed{2.603}$

9. $3^{x+1} = 3^2$

$x+1 = 2$

$x = \boxed{1}$

Solve the following logarithmic equations. Round to three decimals when necessary. Check your answer

10. $\ln x = 8$

$$\log_e x = 8$$

$$e^8 = x$$

$$\boxed{x = e^8} \checkmark$$

11. $\log_2(x + 2) = 5$

$$2^5 = x + 2$$

$$32 = x + 2$$

$$\boxed{30 = x} \checkmark$$

12. $\log_7(25 - x) = 3$

$$7^3 = 25 - x$$

$$x = 25 - 7^3$$

$$\boxed{x = -318} \checkmark$$

13. $4 + 3 \log(2x) = 16$

$$\frac{3 \log_{10}(2x)}{3} = \frac{12}{3}$$

$$\log_{10}(2x) = 4$$

$$10^4 = 2x$$

$$\frac{10^4}{2} = x$$

$$\boxed{x = 5000} \checkmark$$

14. $\log(x + 2) + \log(x - 1) = 1$

$$\log(x+2)(x-1) = 1$$

$$\log_{10}(x^2 + x - 2) = 1$$

$$10^1 = x^2 + x - 2$$

$$0 = x^2 + x - 12$$

$$0 = (x+4)(x-3)$$

$$x = -4 \quad \boxed{x = 3} \checkmark$$

extraneous solution

15. $5 \ln(3 - x) = 4$

$$\log_e(3-x) = \frac{4}{5}$$

$$\log_e(3-x) = 0.8$$

$$e^{0.8} = 3 - x$$

$$x = 3 - e^{0.8}$$

$$\boxed{x = 0.774} \checkmark$$

16. $\log_2(x + 2) = \log_2 x^2$

$$x + 2 = x^2$$

$$0 = x^2 - x - 2$$

$$0 = (x-2)(x+1)$$

$$\boxed{x = 2} \quad \boxed{x = -1} \checkmark$$

17. $\ln(x + 5) = \ln(x - 1) - \ln(x + 1)$

$$\ln(x+5) = \ln\left(\frac{x-1}{x+1}\right)$$

$$\frac{x+5}{1} = \frac{x-1}{x+1}$$

$$x-1 = (x+5)(x+1)$$

$$x-1 = x^2 + 5x + 1x + 5$$

$$0 = x^2 + 5x + 6$$

$$0 = (x+3)(x+2)$$

$$x = -3, x = -2$$

No solution

18. $-5 + 2 \ln 3x = 5$

$$\frac{2 \ln 3x}{2} = \frac{10}{2}$$

$$\ln(3x) = 5$$

$$\log_e(3x) = 5$$

$$e^5 = 3x$$

$$\frac{e^5}{3} = x$$

$$\boxed{x = \frac{e^5}{3}} \checkmark$$

19. $\log_5(-4r - 8) = \log_5(r + 7)$

$$-4r - 8 = r + 7$$

$$-15 = 5r$$

$$\frac{-15}{5} = r$$

$$\boxed{r = -3} \checkmark$$

Condense each expression to a single logarithm.

24. $2 \log_7 x - 4 \log_7 y$

$$\log_7 x^2 - \log_7 y^4$$

$$\boxed{\log_7 \left(\frac{x^2}{y^4} \right)}$$

25. $5 \log_9 a + 15 \log_9 b$

$$\log_9 a^5 + \log_9 b^{15}$$

$$\boxed{\log_9 a^5 b^{15}}$$

26. $3 \log_2 x - 4 \log_2 (x+3)$

$$\log_2 x^3 - \log_2 (x+3)^4$$

$$\boxed{\log_2 \left(\frac{x^3}{(x+3)^4} \right)}$$

Expand each logarithm.

27. $\log_2 (x^2 y)$

$$\boxed{2 \log_2 x + \log_2 y}$$

28. $\log_6 \left(\frac{a^4}{b} \right)$

$$\boxed{4 \log_6 a - \log_6 b}$$

29. $\log_2 \left(\frac{8x^4}{5} \right)$

$$\log_2 8 + \log_2 x^4 - \log_2 5$$

$$\boxed{\log_2 8 + 4 \log_2 x - \log_2 5}$$

Rewrite each into logarithmic form:

33. $3^x = 12$

$$\boxed{\log_3 12 = x}$$

34. $2^{-1} = \frac{1}{2}$

$$\boxed{\log_2 \left(\frac{1}{2} \right) = -1}$$

35. $e^x = 15$

$$\log_e 15 = x$$

$$\boxed{\ln 15 = x}$$

OR:

$$\ln e^x = \ln 15$$

$$x \ln e = \ln 15$$

$$\boxed{x = \ln 15}$$

~~$\log 3^x = \log 12$~~
 ~~$x \log 3 = \log 12$~~
 ~~$x = \frac{\log 12}{\log 3} = 2.262$~~

No need to solve

Rewrite each into exponential form:

36. $\log_{49} 7 = \frac{1}{2}$

$$\boxed{49^{1/2} = 7}$$

37. $\ln 14 = x$

$$\log_e 14 = x$$

$$\boxed{e^x = 14}$$

38. $\log_2 \frac{1}{4} = -2$

$$\boxed{2^{-2} = \frac{1}{4}}$$

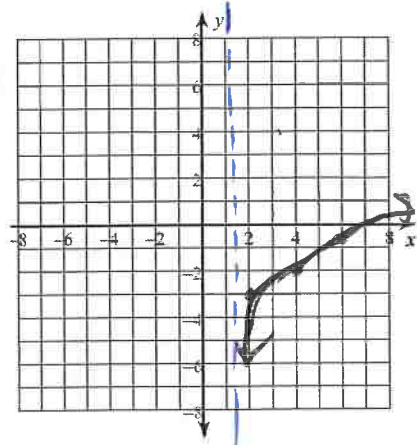
Graph Log functions. Identify ordered pairs, VA, Domain, Range, Asymptote, x-intercept

39)
Transformations:
i) Right 1
ii) Down 3

$$y = \log_3(x-1) - 3$$

$$x-1=0$$

$$x=1 \text{ (VA)}$$



x	y
1	VA
2	-3
4	-2

Domain: $(1, \infty)$ Range: $(-\infty, \infty)$

Asymptote: $x=1$ x-int: $(28, 0)$

set $y=0$ ←

$$0 = \log_3(x-1) - 3$$

$$3 = \log_3(x-1)$$

$$3^3 = x-1$$

$$27 = x-1$$

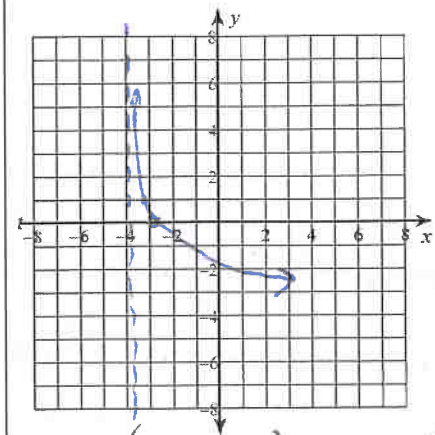
$$28 = x$$

$$x+4=0$$

$$x=-4 \text{ (VA)}$$

40)

$$y = \log_{\frac{1}{3}}(x+4)$$



x	y
-4	VA
-3	0
-11/3	1

Domain: $(-4, \infty)$ Range: $(-\infty, \infty)$

Asymptote: $x=-4$ x-int: $(-3, 0)$

Transformations:

i) left 4 units

$$x+4 = 1/3$$

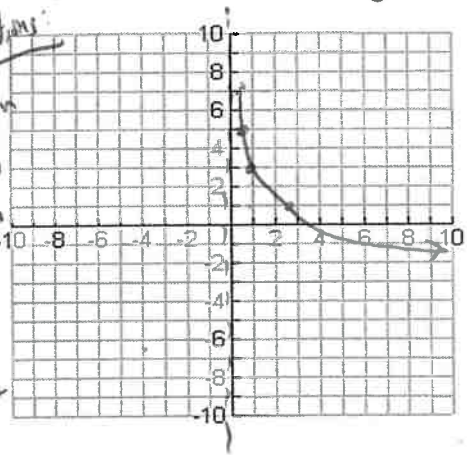
$$x = 1/3 - 4$$

$$x = -11/3$$

41) $g(x) = -2 \ln x + 3$

$$y = -2 \log_e(x) + 3$$

Transformations:
i) Reflection (x-axis)
ii) shift up 3 units
iii) vertical stretch by factor of 2



x	y
0	VA
1	3
e	1
e^{1.5}	0

Domain: $(0, \infty)$ Range: $(-\infty, \infty)$

Asymptote: $x=0$ x-int: $(e^{1.5}, 0)$

0 = -2 log_e(x) + 3

$$-3 = -2 \log_e x$$

$$\frac{3}{2} = \log_e x$$

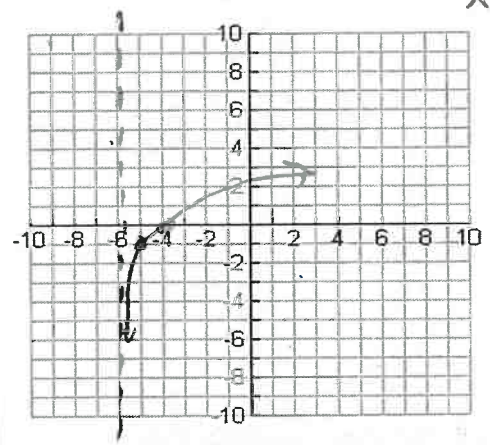
$$e^{1.5} = x$$

$$(e^{1.5}, 0)$$

42) $f(x) = \log_2(x+6) - 1$

$$x+6=0$$

$$x=-6 \text{ (VA)}$$



x	y
-6	VA
-5	-1
-4	0

Domain: $(-6, \infty)$ Range: $(-\infty, \infty)$

Asymptote: $x=-6$ x-int: $(-4, 0)$

Transformations:

i) shift left 6 units
ii) shift down 1 unit.