

Exponentials and Logs Test Review WS #3

Key

Write the equation in logarithmic form.

1) $3^6 = 729$

$\log_3 729 = 6$

2) $8^{\frac{2}{3}} = 4$

$\log_8 4 = \frac{2}{3}$

Write the equation in exponential form.

3) $\log_5 \frac{1}{25} = -2$

$5^{-2} = \frac{1}{25}$

4) $\log_{x+y} z = 3$

$(x+y)^3 = z$

Evaluate the logarithm.

5) $\log_6 \frac{1}{216} = x$

$6^x = \frac{1}{216} \rightarrow 6^x = 6^{-3}$

$x = -3$

6) $\log_9 729 = x$

$9^x = 729$

$x = 3$ ← $9^x = 9^3$

7) $\log 0.1 = x$

$\log_{10} \left(\frac{1}{10}\right) = x$

$10^x = \frac{1}{10} \rightarrow 10^x = 10^{-1}$

$x = -1$ ←

Condense the expression as a single logarithm.

8) $4 \log_2 x - 6 \log_2 y$

$\log_2 x^4 - \log_2 y^6$

$\log_2 \left(\frac{x^4}{y^6}\right)$

9) $2 \log x + \log(x+2)$

$\log_6 x^2 + \log_{10}(x+2)$

$\log_{10} x^2(x+2) \rightarrow \log(x^3 + 2x^2)$

Expand the logarithmic expression.

10) $\log_3(12b^4)$

$\log_3 12 + \log_3 b^4$

$\log_3 12 + 4 \log_3 b$

11) $\log_2 \left(\frac{c^3}{d}\right)$

$3 \log_2 c - \log_2 d$

Solve the exponential equation.

$$12) \frac{1}{25} = 5^{x+2}$$

$$5^{-2} = 5^{x+2}$$

$$-2 = x+2$$

$$\boxed{-4 = x}$$

$$13) 4^{5x-1} = 256$$

$$4^{5x-1} = 4^4$$

$$5x-1=4$$

$$5x=5$$

$$\boxed{x=1}$$

Solve the logarithmic equation.

$$14) \log(x+3) - \log x = 1$$

$$\log\left(\frac{x+3}{x}\right) = 1$$

$$\frac{10}{1} = \frac{x+3}{x}$$

$$\boxed{x = \frac{1}{3}} \checkmark$$

$$\log_{10}\left(\frac{x+3}{x}\right) = 1$$

$$10x = x+3$$

$$10^1 = \frac{x+3}{x}$$

$$9x = 3$$

$$15) 3 \log_2 2 + \log_2 x = 6$$

$$\log_2 2^3 + \log_2 x = 6$$

$$\log_2 8x = 6$$

$$2^6 = 8x$$

$$\frac{2^6}{8} = x$$

$$\boxed{x = 8} \checkmark$$

Solve for x:

$$16) 5^{3x-1} \cdot 5^{2x-5} = 5^{x+6}$$

$$5^{3x-1+2x-5} = 5^{x+6}$$

$$5x-6 = x+6$$

$$4x = 12$$

$$\boxed{x = 3}$$

$$17) 5^{x-2} = 10^{2x+1}$$

$$\log 5^{x-2} = \log 10^{2x+1}$$

$$(x-2) \log 5 = (2x+1) \log 10$$

$$x \log 5 - 2 \log 5 = 2x + 1$$

$$x \log 5 - 2x = 2 \log 5 + 1$$

$$x(\log 5 - 2) = 2 \log 5 + 1$$

$$\boxed{x = \frac{2 \log 5 + 1}{\log 5 - 2}}$$

Solve the natural logarithmic equation. Round to nearest hundredth.

$$18) \ln(3x+2) = 5$$

$$\log_e(3x+2) = 5$$

$$e^5 = 3x+2$$

$$e^5 - 2 = 3x$$

$$\boxed{\frac{e^5 - 2}{3} = x} \checkmark$$

$$19) \ln x - \ln 3 = 0$$

$$\log_e\left(\frac{x}{3}\right) = 0$$

$$1 = \frac{x}{3}$$

$$e^0 = \frac{x}{3}$$

$$\boxed{x = 3} \checkmark$$

Use natural logarithms to solve the equations. Round to the nearest hundredth.

$$20) e^x = \frac{5}{7}$$

$$\ln e^x = \ln\left(\frac{5}{7}\right)$$

$$x \ln e = \ln\left(\frac{5}{7}\right)$$

$$\boxed{x = \ln\left(\frac{5}{7}\right)}$$

$$21) 3e^{-x} + 1 = 7$$

$$\frac{3e^{-x}}{3} = \frac{6}{3}$$

$$e^{-x} = 2$$

$$\ln e^{-x} = \ln 2$$

$$-x \ln e = \ln 2$$

$$\boxed{x = -\ln 2}$$

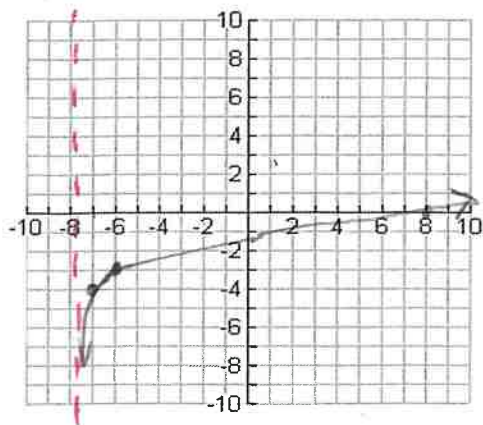
Graph Log functions. Identify ordered pairs, VA, Domain, Range, Asymptote, x-intercept

$x-1=0$

22) $f(x) = \log_2(x+8) - 4$

* shifts left 8 units
* shift down 4 units

$x+8=0$
 $x=-8$ (VA)



x	y
-8	VA
-7	-4
-6	-3

Domain: $(-8, \infty)$ Range: $(-\infty, \infty)$

Asymptote: $x=-8$ x-int: $(8, 0)$

$0 = \log_2(x+8) - 4$
 $4 = \log_2(x+8)$

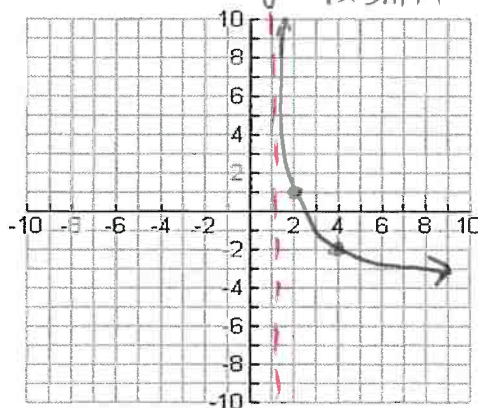
$2^4 = x+8$
 $16-8 = x$
 $8 = x$

(8, 0)

23) $f(x) = -3\log_3(x-1) + 1$

* Reflection (x-axis)
* vertical stretch by 3
* shift down 1
* shift up 1

$x=1$ (VA)



x	y
1	VA
2	1
4	-2

Domain: $(1, \infty)$ Range: $(-\infty, \infty)$

Asymptote: $x=1$ x-int: $(3^{1/3}+1, 0)$

$0 = -3\log_3(x-1) + 1$
 $-1 = -3\log_3(x-1)$

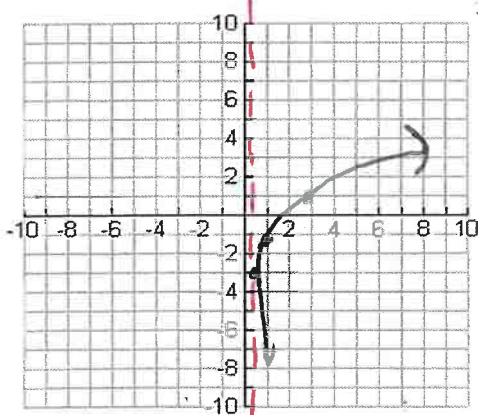
$\frac{1}{3} = \log_3(x-1)$
 $3^{1/3} = x-1$

$3^{1/3} + 1 = x$

24) $g(x) = 2\ln(x) - 1$

$x=0$ (VA)

* stretch by factor of 2
* shift down 1



x	y
0	VA
1	-1
e	1
$\frac{1}{e}$	-3

Domain: $(0, \infty)$ Range: $(-\infty, \infty)$

Asymptote: $x=0$ x-int: $(e^{0.5}, 0)$

$0 = 2\log_e(x) - 1$
 $1 = 2\log_e(x)$
 $\frac{1}{2} = \log_e(x)$

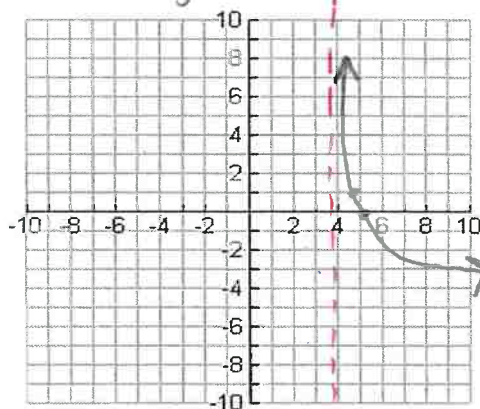
$e^{1/2} = x$

25) $f(x) = \log_{1/3}(x-4)$

$x-4=0$

$x=4$ (VA)

* shifts right 4



x	y
4	VA
5	0
$\frac{13}{3}$	1

Domain: $(4, \infty)$ Range: $(-\infty, \infty)$

Asymptote: $x=4$ x-int: $(5, 0)$

$x-4 = 1/3$
 $x = 4 + 1/3$
 $x = \frac{13}{3} \approx 4.33$

Log Properties:

- 1) Product Property: $\log uv = \log u + \log v$
- 2) Quotient Property: $\log\left(\frac{u}{v}\right) = \log u - \log v$
- 3) Power Property: $\log u^n = n \cdot \log u$
- 4) $\log\left(\frac{ab}{cde}\right) = \log a + \log b - \log c - \log d - \log e$
- 5) $\log(u+v) \neq \log u + \log v$
- 6) $\log_e e^x = x \rightarrow \ln e^x = x$
- 7) $e^{\log_e x} = x$

Graphing Log Functions:

Helpful Log Characteristics

- 1) $\log_b(x)$ * set the log argument = 0 to find the Vertical Asymptote
 - 2) $\log_b(1) = 0$
 - 3) $\log_b(b) = 1$
 - * 4) $\log_b\left(\frac{1}{b}\right) = -1$
-
- * $\log_b(b^x) = x$
- * $b^{\log_b(x)} = x$