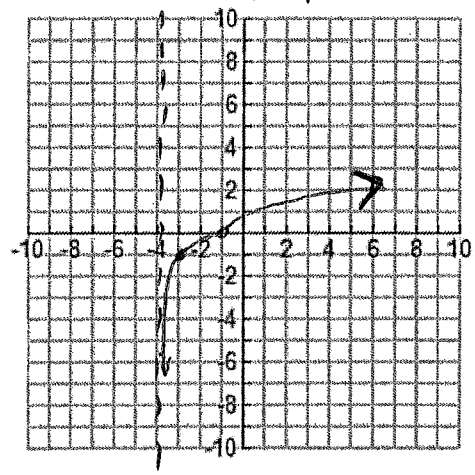


Key

Logs and Exponentials Test Review WS #4

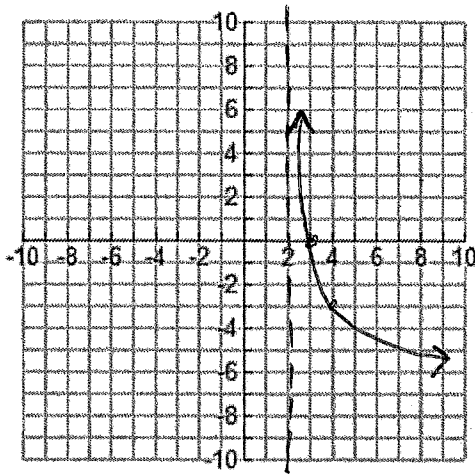
Graph each function and find the following characteristics.

1) $f(x) = \log_3(x+4) - 1$
 Parent function: $\log_3(x)$
 Transformations: translated 4 units left, 1 down
 Domain: $(-4, \infty)$ Range: $(-\infty, \infty)$
 Asymptote: $x = -4$ Intercepts: $(-1, 0)$
 $\lim_{x \rightarrow -\infty} f(x) = -\infty$ $\lim_{x \rightarrow -4^+} f(x) = -\infty$



x	y
-4	VA
-3	-1
-1	0

2) $g(x) = -3\log_2(x-2)$
 Parent function: $\log_2(x)$
 Transformations: reflection (x-axis), vertical stretch by 3, right 2 units
 Domain: $(2, \infty)$ Range: $(-\infty, \infty)$
 Asymptote: $x = 2$ Intercepts: $(3, 0)$
 $\lim_{x \rightarrow \infty} f(x) = -\infty$ $\lim_{x \rightarrow 2^+} f(x) = +\infty$



x	y
2	VA
3	0
4	-3

Rewrite each equation in logarithmic form.

3) $3^4 = 81$
 $\log_3 81 = 4$

4) $4^{-3} = \frac{1}{64}$
 $\log_4 \left(\frac{1}{64}\right) = -3$

Rewrite each equation in exponential form.

5) $\log_5 125 = 3$
 $5^3 = 125$

6) $\ln 10 \approx 2.303$
 $\log_e 10 = 2.303$
 $e^{2.303} = 10$

Evaluate.

7) $\log_3 81 = x$
 $3^x = 81 \rightarrow 3^x = 3^4$
 $x = 4$

8) $\log_4 \frac{1}{64} = x$
 $4^x = \frac{1}{64}$
 $4^x = 4^{-3}$
 $x = -3$

9) $\ln e^5$
 $= 5$

10) $\log 100,000$
 $\log_{10} 100,000 = x$
 $10^x = 100,000$
 $x = 5$

11) $\log_{16} 8 = x$
 $16^x = 8$
 $2^{4x} = 2^3$
 $4x = 3$
 $x = \frac{3}{4}$

12) $\log_9 \frac{1}{27} = x$
 $9^x = \frac{1}{27}$
 $3^{2x} = 3^{-3}$
 $2x = -3$
 $x = -\frac{3}{2}$

13) $5^{\log_5 2}$
 $= 2$

14) $\log_6 6^{17}$
 $= 17$

15) $\log_8 16 = x$
 $8^x = 16$
 $2^{3x} = 2^4$
 $3x = 4$
 $x = \frac{4}{3}$

16) $\log_{\frac{1}{27}} 9 = x$
 $\left(\frac{1}{27}\right)^x = 9$
 $3^{-3x} = 3^2$
 $-3x = 2$
 $x = -\frac{2}{3}$

17) $\log 0.0001 = x$
 $10^x = \frac{1}{10000}$
 $10^x = 10^{-4}$
 $x = -4$

18) $\log_{49} 7 = x$
 $49^x = 7$
 $7^{2x} = 7^1$
 $2x = 1$
 $x = \frac{1}{2}$

Expand fully.

19) $\ln \frac{\sqrt{e}}{y^2}$

$$\ln e^{1/2} - \ln y^2$$

$$\frac{1}{2} \ln e - 2 \ln y$$

$$\boxed{\frac{1}{2} - 2 \ln y}$$

20) $\log(200a^2b^3)^4$

$$4 \log(200a^2b^3)$$

$$4 \log 200 + 4 \log a^2 + 4 \log b^3$$

$$\boxed{4 \log 200 + 8 \log a + 12 \log b}$$

21) $\log_2 \frac{48m}{25n}$

$$\boxed{\log_2 48 + \log_2 m - \log_2 25 - \log_2 n}$$

Condense into a single logarithmic expression.

22) $\log_3 x - 3$

$$\log_3 x - \log_3 3^3$$

$$\log_3 x - \log_3 27$$

$$\boxed{\log_3 \left(\frac{x}{27} \right)}$$

23) $\ln b + 3 \ln c - 2 \ln a$

$$\ln b + \ln c^3 - \ln a^2$$

$$\boxed{\ln \left(\frac{bc^3}{a^2} \right)}$$

24) $\frac{1}{2} - 2(3 \log m + 4 \log n)$

$$\log_{10} 10^{1/2} - 6 \log m - 8 \log n$$

$$\log_{10} 10^{1/2} - \log m^6 - \log n^8$$

$$\boxed{\log \left(\frac{10^{1/2}}{m^6 n^8} \right)}$$

Solve each equation.

25) $\log x = 4$

$$\log_{10} x = 4$$

$$10^4 = x$$

$$\boxed{x = 10000}$$

26) $\log_x 3 = \frac{1}{2}$

$$x^{1/2} = 3$$

$$(x^{1/2})^2 = (3)^2$$

$$\boxed{x = 9}$$

27) $4^x = 3$

$$\log 4^x = \log 3$$

$$x \log 4 = \log 3$$

$$x = \frac{\log 3}{\log 4}$$

$$\boxed{x = 0.792}$$

28) $\ln x = 1.8$

$$\log_e x = 1.8$$

$$e^{1.8} = x$$

$$\boxed{x = e^{1.8}}$$

29) $\left(\frac{1}{2}\right)^x = 32^{x-1}$

$$2^{-x} = 2^{5(x-1)}$$

$$-x = 5x - 5$$

$$-6x = -5$$

$$x = \frac{-5}{-6}$$

$$\boxed{x = 5/6}$$

30) $4(2^{3x-1}) - 3 = 0$

$$4(2^{3x-1}) = 3$$

$$2^{3x-1} = \frac{3}{4}$$

$$2^{3x-1} = 0.75$$

$$\log 2^{(3x-1)} = \log 0.75$$

$$(3x-1) \log 2 = \log 0.75$$

$$3x \log 2 - \log 2 = \log 0.75$$

$$3x \log 2 = \log 2 + \log 0.75$$

$$x = \frac{\log 2 + \log 0.75}{3 \log 2} = \boxed{0.195}$$

* check for extraneous solutions

31) $9 = 4 + \log_2(x + 5)$

$5 = \log_2(x + 5)$

$2^5 = x + 5$

$32 = x + 5$

$x = 27$ ✓

32) $\log x + \log(x - 2) = \log 8$

$\log_{10}(x)(x-2) = \log_{10} 8$ $x = 4, x = -2$

$x^2 - 2x = 8$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4$

Extraneous solution

33) $7^{x-1} \cdot 7^{x+3} = 14$

$7^{x-1+x+3} = 14$

$7^{2x+2} = 14$

$\log 7^{2x+2} = \log 14$

$(2x+2)\log 7 = \log 14$

35) $6^{2x} = 30$

$\log 6^{2x} = \log 30$

$2x \log 6 = \log 30$

$x = \frac{\log 30}{2 \log 6} \approx 0.949$

$2x \log 7 + 2 \log 7 = \log 14$

$2x \log 7 = \log 14 - 2 \log 7$

$x = \frac{\log 14 - 2 \log 7}{2 \log 7}$

$x \approx -0.322$

34) $\log_2(x + 1) - \log_2(x - 5) = 3$

$\log_2\left(\frac{x+1}{x-5}\right) = 3$ $8x - 40 = x + 1$

$2^3 = \frac{x+1}{x-5}$

$\frac{8}{1} = \frac{x+1}{x-5}$

$7x = 41$

$x = \frac{41}{7}$ ✓

36) $\log_3(16x + 1) = 4$

$3^4 = 16x + 1$

$81 = 16x + 1$

$80 = 16x$

$x = 5$ ✓

37) $\log_7(3x^2 + 8) - \log_7 8 = 4$

$\log_7\left(\frac{3x^2+8}{8}\right) = 4$

$7^4 = \frac{3x^2+8}{8}$

$3x^2 = 19200$

$x^2 = 6400$

$x = \pm 80$

$x = 80, x = -80$

$3x^2 + 8 = 7^4 \cdot 8$

$3x^2 + 8 = 19208$

38) $\frac{5^{6x+7}}{5^{3x+5}} = 6$

$5^{6x+7-(3x+5)} = 6$

$5^{3x+2} = 6$

$\log 5^{3x+2} = \log 6$

$(3x+2)\log 5 = \log 6$

$3x \log 5 + 2 \log 5 = \log 6$

$3x \log 5 = \log 6 - 2 \log 5$

$x = \frac{\log 6 - 2 \log 5}{3 \log 5}$

$x \approx -0.296$

Log Properties:

- 1) Product Property: $\log uv = \log u + \log v$
- 2) Quotient Property: $\log\left(\frac{u}{v}\right) = \log u - \log v$
- 3) Power Property: $\log u^n = n \cdot \log u$
- 4) $\log\left(\frac{ab}{cde}\right) = \log a + \log b - \log c - \log d - \log e$
- 5) $\log(u+v) \neq \log u + \log v$
- 6) $\log_e e^x = x \rightarrow \ln e^x = x$
- 7) $e^{\log_e x} = x$

Graphing Log Functions:

Helpful Log Characteristics

- 1) $\log_b(x)$ * set the log argument = 0 to find the Vertical Asymptote
 - 2) $\log_b(1) = 0$
 - 3) $\log_b(b) = 1$
 - * 4) $\log_b\left(\frac{1}{b}\right) = -1$
-
- * $\log_b(b^x) = x$
- * $b^{\log_b(x)} = x$