CHAPTER 10
Conics, Parametric Equations, and Polar Coordinates

Section 10.1 Conics and Calculus

1. $y^2 = 4x$ Parabola

Vertex: $(0, 0)$

$p = 1 > 0$

Opens to the right

Matches (a).

2. $(x + 4)^2 = -2(y - 2)$ Parabola

Vertex: $(-4, 2)$

Opens downward

Matches (e).

3. $\dfrac{y^2}{16} - \dfrac{x^2}{1} = 1$ Hyperbola

Vertices: $(0, \pm 4)$

Matches (c).

4. $\dfrac{(x - 2)^2}{16} + \dfrac{(y + 1)^2}{4} = 1$ Ellipse

Center: $(2, -1)$

Matches (b).

5. $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ Ellipse

Center: $(0, 0)$

Vertices: $(0, \pm 3)$

Matches (f).

6. $\dfrac{(x - 2)^2}{9} - \dfrac{y^2}{4} = 1$ Hyperbola

Vertices: $(5, 0), (-1, 0)$

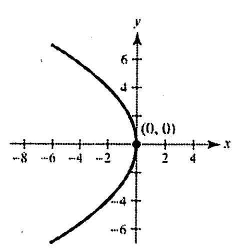
Matches (d).

7 $y^2 = -8x = 4(-2)x$

Vertex: $(0, 0)$

Focus: $(-2, 0)$

Directrix: $x = 2$



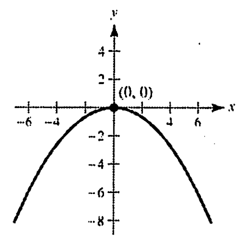
8. $x^2 + 6y = 0$

$$x^2 = -6y = 4\left(-\frac{3}{2}\right)y$$

Vertex: $(0, 0)$

Focus: $\left(0, -\dfrac{3}{2}\right)$

Directrix: $y = \dfrac{3}{2}$



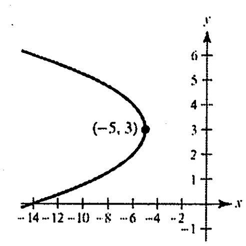
9. $(x + 5) + (y - 3)^2 = 0$

$$(y - 3)^2 = -(x + 5) = 4\left(-\tfrac{1}{4}\right)(x + 5)$$

Vertex: $(-5, 3)$

Focus: $\left(-\tfrac{21}{4}, 3\right)$

Directrix: $x = -\tfrac{19}{4}$



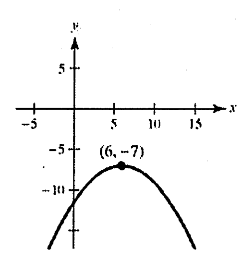
10. $(x - 6)^2 + 8(y + 7) = 0$

$$(x - 6)^2 = -8(y + 7) = 4(-2)(y + 7)$$

Vertex: $(6, -7)$

Focus: $(6, -9)$

Directrix: $y = -5$



11. $y^2 - 4y - 4x = 0$

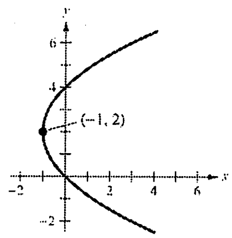
$y^2 - 4y + 4 = 4x + 4$

$(y - 2)^2 = 4(1)(x + 1)$

Vertex: $(-1, 2)$

Focus: $(0, 2)$

Directrix: $x = -2$



12. $y^2 + 6y + 8x + 25 = 0$

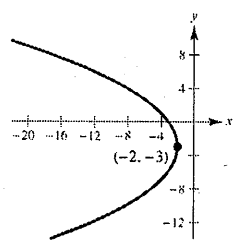
$y^2 + 6y + 9 = -8x - 25 + 9$

$(y + 3)^2 = 4(-2)(x + 2)$

Vertex: $(-2, -3)$

Focus: $(-4, -3)$

Directrix: $x = 0$



13. $x^2 + 4x + 4y - 4 = 0$

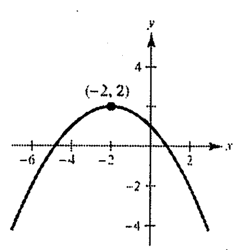
$x^2 + 4x + 4 = -4y + 4 + 4$

$(x + 2)^2 = 4(-1)(y - 2)$

Vertex: $(-2, 2)$

Focus: $(-2, 1)$

Directrix: $y = 3$



14. $y^2 + 4y + 8x - 12 = 0$

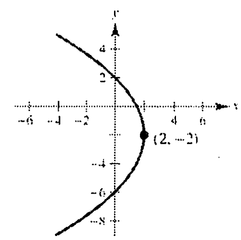
$y^2 + 4y + 4 = -8x + 12 + 4$

$(y + 2)^2 = 4(-2)(x - 2)$

Vertex: $(2, -2)$

Focus: $(0, -2)$

Directrix: $x = 4$



15. $(y - 4)^2 = 4(-2)(x - 5)$

$y^2 - 8y + 16 = -8x + 40$

$y^2 - 8y + 8x - 24 = 0$

16. $(x + 2)^2 = 4(-2)(y - 1)$

$x^2 + 4x + 8y - 4 = 0$

17. $(x - 0)^2 = 4(8)(y - 5)$

$x^2 = 4(8)(y - 5)$

$x^2 - 32y + 160 = 0$

18. Vertex: $(0, 2)$

$(y - 2)^2 = 4(2)(x - 0)$

$y^2 - 8x - 4y + 4 = 0$

19. Vertex: $(0, 4)$, vertical axis

$(x - 0)^2 = 4p(y - 4)$

$(-2, 0)$ on parabola: $(-2)^2 = 4p(-4)$

$4 = -16p$

$p = -\frac{1}{4}$

$x^2 = 4\left(-\frac{1}{4}\right)(y - 4)$

$x^2 = -(y - 4)$

$x^2 + y - 4 = 0$

20. Vertex: $(2, 4)$, vertical axis

$$(x - 2)^2 = 4p(y - 4)$$

$(0, 0)$ on parabola: $(-2)^2 = 4p(0 - 4)$

$$4 = -16p$$

$$p = -\frac{1}{4}$$

$$(x - 2)^2 = 4\left(-\frac{1}{4}\right)(y - 4)$$

$$x^2 - 4x + 4 = -y + 4$$

$$x^2 - 4x + y = 0$$

21. Because the axis of the parabola is vertical, the form of the equation is $y = ax^2 + bx + c$. Now, substituting the values of the given coordinates into this equation, you obtain

$3 = c, 4 = 9a + 3b + c, 11 = 16a + 4b + c.$

Solving this system, you have $a = \frac{5}{3}, b = -\frac{14}{3}, c = 3.$

So,

$y = \frac{5}{3}x^2 - \frac{14}{3}x + 3$ or $5x^2 - 14x - 3y + 9 = 0.$

22. From Example 2: $4p = 8$ or $p = 2$

Vertex: $(4, 0)$

$$(x - 4)^2 = 8(y - 0)$$

$$x^2 - 8x - 8y + 16 = 0$$

23. $16x^2 + y^2 = 16$

$$x^2 + \frac{y^2}{16} = 1$$

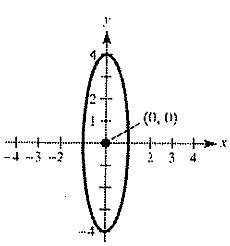
$a^2 = 16, b^2 = 1, c^2 = 16 - 1 = 15$

Center: $(0, 0)$

Foci: $\left(0, \pm\sqrt{15}\right)$

Vertices: $(0, \pm 4)$

$$e = \frac{c}{a} = \frac{\sqrt{15}}{4}$$



24. $3x^2 + 7y^2 = 63$

$$\frac{x^2}{21} + \frac{y^2}{9} = 1$$

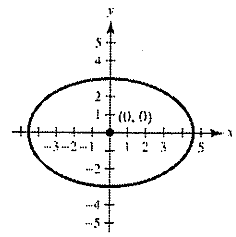
$a^2 = 21, b^2 = 9, c^2 = 21 - 9 = 12$

Center: $(0, 0)$

Foci: $\left(\pm 2\sqrt{3}, 0\right)$

Vertices: $\left(\pm\sqrt{21}, 0\right)$

$$e = \frac{c}{a} = \frac{2\sqrt{3}}{\sqrt{21}} = \frac{2\sqrt{7}}{7}$$



25. $\dfrac{(x - 3)^2}{16} + \dfrac{(y - 1)^2}{25} = 1$

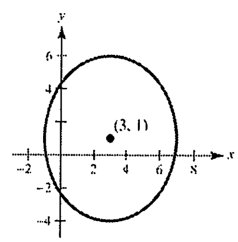
$a^2 = 25, b^2 = 16, c^2 = 25 - 16 = 9$

Center: $(3, 1)$

Foci: $(3, 1 + 3) = (3, 4), (3, 1 - 3) = (3, -2)$

Vertices: $(3, 6), (3, -4)$

$$e = \frac{c}{a} = \frac{3}{5}$$



26. $(x + 4)^2 + \dfrac{(y + 6)^2}{1/4} = 1$

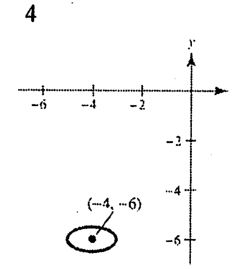
$a^2 = 1, b^2 = \frac{1}{4}, c^2 = 1 - \frac{1}{4} = \frac{3}{4}$

Center: $(-4, -6)$

Foci: $\left(-4 \pm \frac{\sqrt{3}}{2}, -6\right)$

Vertices: $(-5, -6), (-3, -6)$

$$e = \frac{c}{a} = \frac{\sqrt{3}}{2}$$



27.
$$9x^2 + 4y^2 + 36x - 24y + 36 = 0$$
$$9(x^2 + 4x + 4) + 4(y^2 - 6y + 9) = -36 + 36 + 36 = 36$$
$$\frac{(x+2)^2}{4} + \frac{(y-3)^2}{9} = 1$$

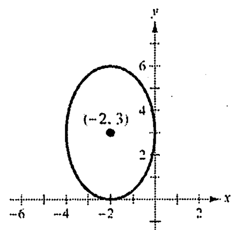
$a^2 = 9, b^2 = 4, c^2 = 5$

Center: $(-2, 3)$

Foci: $(-2, 3 \pm \sqrt{5})$

Vertices: $(-2, 6), (-2, 0)$

$e = \frac{\sqrt{5}}{3}$



28.
$$16x^2 + 25y^2 - 64x + 150y + 279 = 0$$
$$16(x^2 - 4x + 4) + 25(y^2 + 6y + 0) = -279 + 64 + 225 = 10$$
$$\frac{(x-2)^2}{(5/8)} + \frac{(y+3)^2}{(2/5)} = 1$$

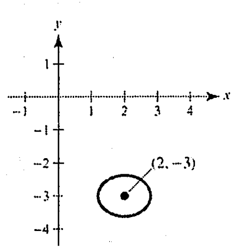
$a^2 = \frac{5}{8}, b^2 = \frac{2}{5}, c^2 = a^2 - b^2 = \frac{9}{40}$

Center: $(2, -3)$

Foci: $\left(2 \pm \frac{3\sqrt{10}}{20}, -3\right)$

Vertices: $\left(2 \pm \frac{\sqrt{10}}{4}, -3\right)$

$e = \frac{c}{a} = \frac{3}{5}$



29. Center: $(0, 0)$

Focus: $(5, 0)$

Vertex: $(6, 0)$

Horizontal major axis

$a = 6, c = 5 \Rightarrow b = \sqrt{a^2 - c^2} = \sqrt{11}$

$$\frac{x^2}{36} + \frac{y^2}{11} = 1$$

30. Vertices: $(0, 3), (8, 3)$

Eccentricity: $\frac{3}{4}$

Horizontal major axis

Center: $(4, 3)$

$a = 4, e = \frac{c}{a} \Rightarrow c = 4\left(\frac{3}{4}\right) = 3$

$\Rightarrow b = \sqrt{16 - 9} = \sqrt{7}$

$$\frac{(x-4)^2}{16} + \frac{(y-3)^2}{7} = 1$$

31. Vertices: $(3, 1), (3, 9)$

Minor axis length: 6

Vertical major axis

Center: $(3, 5)$

$a = 4, b = 3$

$$\frac{(x-3)^2}{9} + \frac{(y-5)^2}{16} = 1$$

32. Foci: $(0, \pm 9)$

Major axis length: 22

Vertical major axis

Center: $(0, 0)$

$c = 9, a = 11 \Rightarrow b = \sqrt{40}$

$$\frac{x^2}{40} + \frac{y^2}{121} = 1$$

33. Center: $(0, 0)$

Horizontal major axis

Points on ellipse: $(3, 1), (4, 0)$

Because the major axis is horizontal,

$$\left(\frac{x^2}{a^2}\right) + \left(\frac{y^2}{b^2}\right) = 1.$$

Substituting the values of the coordinates of the given points into this equation, you have

$$\left(\frac{9}{a^2}\right) + \left(\frac{1}{b^2}\right) = 1, \text{ and } \frac{16}{a^2} = 1.$$

The solution to this system is $a^2 = 16, b^2 = \frac{16}{7}$.

So,

$$\frac{x^2}{16} + \frac{y^2}{16/7} = 1, \frac{x^2}{16} + \frac{7y^2}{16} = 1.$$

34. Center: $(1, 2)$

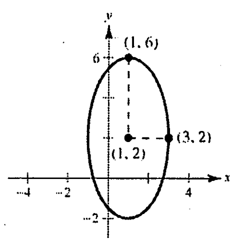
Vertical major axis

Points on ellipse: $(1, 6), (3, 2)$

From the sketch, you can see that

$h = 1, k = 2, a = 4, b = 2$

$$\frac{(x-1)^2}{4} + \frac{(y-2)^2}{16} = 1.$$



35. $\frac{x^2}{25} - \frac{y^2}{16} = 1$

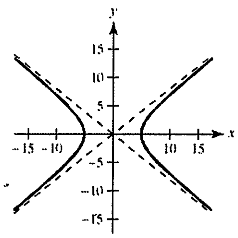
$a = 5, b = 4, c = \sqrt{25 + 16} = \sqrt{41}$

Center: $(0, 0)$

Vertices: $(\pm 5, 0)$

Foci: $\left(\pm\sqrt{41}, 0\right)$

Asymptotes: $y = \pm\frac{b}{a}x = \pm\frac{4}{5}x$



36. $\frac{(y+3)^2}{225} - \frac{(x-5)^2}{64} = 1$

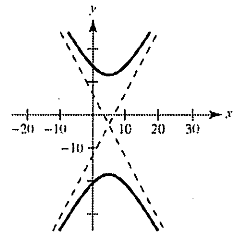
$a = 15, b = 8, c = \sqrt{225 + 64} = 17$

Center: $(5, -3)$

Vertices: $(5, 12), (5, -18)$

Foci: $(5, 14), (5, -20)$

Asymptotes: $y = k \pm \frac{a}{b}(x - h) = -3 \pm \frac{15}{8}(x - 5)$



37.

$$9x^2 - y^2 - 36x - 6y + 18 = 0$$
$$9\left(x^2 - 4x + 4\right) - \left(y^2 + 6y + 9\right) = -18 + 36 - 9$$
$$\frac{(x-2)^2}{1} - \frac{(y+3)^2}{9} = 1$$

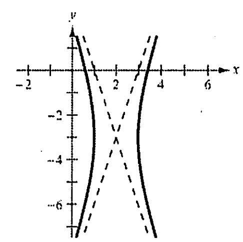
$a = 1, b = 3, c = \sqrt{10}$

Center: $(2, -3)$

Vertices: $(1, -3), (3, -3)$

Foci: $\left(2 \pm \sqrt{10}, -3\right)$

Asymptotes: $y = -3 \pm 3(x - 2)$



38. $y^2 - 16x^2 + 64x - 208 = 0$

$$y^2 - 16\left(x^2 - 4x + 4\right) = 208 - 64 = 144$$

$$\frac{y^2}{144} - \frac{(x-2)^2}{9} = 1$$

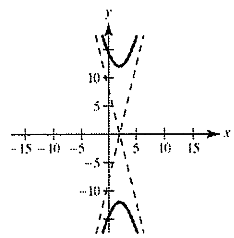
$a = 12, b = 3, c = \sqrt{144 + 9} = \sqrt{153}$

Center: $(2, 0)$

Vertices: $(2, 12), (2, -12)$

Foci: $\left(2, \pm\sqrt{153}\right)$

Asymptotes: $y = \pm\frac{12}{3}(x - 2) = \pm 4(x - 2)$



39. $x^2 - 9y^2 + 2x - 54y - 80 = 0$

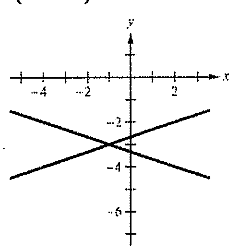
$$\left(x^2 + 2x + 1\right) - 9\left(y^2 + 6y + 9\right) = 80 + 1 - 81 = 0$$

$$(x+1)^2 - 9(y+3)^2 = 0$$

$$y + 3 = \pm\frac{1}{3}(x + 1)$$

$$y = -3 \pm \frac{1}{3}(x + 1)$$

Degenerate hyperbola is two lines intersecting at $(-1, -3)$.



40. $9\left(x^2 + 6x + 9\right) - 4\left(y^2 - 2y + 1\right) = -78 + 81 - 4 = -1$

$$9(x+3)^2 - 4(y-1)^2 = -1$$

$$\frac{(y-1)^2}{1/4} - \frac{(x+3)^2}{1/9} = 1$$

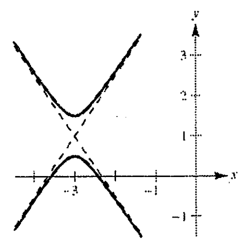
$a = \frac{1}{2}, b = \frac{1}{3}, c = \frac{\sqrt{13}}{6}$

Center: $(-3, 1)$

Vertices: $\left(-3, \frac{1}{2}\right), \left(-3, \frac{3}{2}\right)$

Foci: $\left(-3, 1 \pm \frac{1}{6}\sqrt{13}\right)$

Asymptotes: $y = 1 \pm \frac{3}{2}(x + 3)$



41. Vertices: $(\pm 1, 0)$

Asymptotes: $y = \pm 5x$

Horizontal transverse axis

Center: $(0, 0)$

$a = 1, \frac{b}{a} = 5 \Rightarrow b = 5$

$\frac{x^2}{1} - \frac{y^2}{25} = 1$

42. Vertices: $(0, \pm 4)$

Asymptotes: $y = \pm 2x$

Vertical transverse axis

$a = 4, \frac{a}{b} = 2 \Rightarrow b = 2$

$\frac{y^2}{16} - \frac{x^2}{4} = 1$

43. Vertices: $(2, \pm 3)$

Point on graph: $(0, 5)$

Vertical transverse axis

Center: $(2, 0)$

$a = 3$

So, the equation is of the form

$$\frac{y^2}{9} - \frac{(x-2)^2}{b^2} = 1.$$

Substituting the coordinates of the point $(0, 5)$, you have

$$\frac{25}{9} - \frac{4}{b^2} = 1 \quad \text{or} \quad b^2 = \frac{9}{4}.$$

So, the equation is $\frac{y^2}{9} - \frac{(x-2)^2}{9/4} = 1.$

44. Vertices: $(2, \pm 3)$

Foci: $(2, \pm 5)$

Vertical transverse axis

Center: $(2, 0)$

$a = 3, c = 5, b^2 = c^2 - a^2 = 16$

So, $\frac{y^2}{9} - \frac{(x-2)^2}{16} = 1.$

45. Center: $(0, 0)$

Vertex: $(0, 2)$

Focus: $(0, 4)$

Vertical transverse axis

$a = 2, c = 4, b^2 = c^2 - a^2 = 12$

So, $\frac{y^2}{4} - \frac{x^2}{12} = 1.$

46. Center: $(0, 0)$

Vertex: $(6, 0)$

Focus: $(10, 0)$

Horizontal transverse axis

$a = 6, c = 10, b^2 = c^2 - a^2 = 100 - 36 = 64$

$$\frac{x^2}{36} - \frac{y^2}{64} = 1$$

47. Vertices: $(0, 2), (6, 2)$

Asymptotes: $y = \frac{2}{3}x, y = 4 - \frac{2}{3}x$

Horizontal transverse axis

Center: $(3, 2)$

$a = 3$

Slopes of asymptotes: $\pm\frac{b}{a} = \pm\frac{2}{3}$

So, $b = 2$. Therefore,

$$\frac{(x-3)^2}{9} - \frac{(y-2)^2}{4} = 1.$$

48. Focus: $(20, 0)$

Asymptotes: $y = \pm\frac{3}{4}x$

Horizontal transverse axis

Center: $(0, 0)$

$c = 20$

$\frac{b}{a} = \frac{3}{4} \Rightarrow b = \frac{3}{4}a$

$c^2 = 400 = a^2 + b^2 = a^2 + \frac{9}{16}a^2 = \frac{25}{16}a^2$

$\Rightarrow a^2 = 256 \quad \text{and} \quad b^2 = 144$

$$\frac{x^2}{256} - \frac{y^2}{144} = 1$$

49. (a) $\frac{x^2}{9} - y^2 = 1, \frac{2x}{9} - 2yy' = 0, \frac{x}{9y} = y'$

At $x = 6$: $y = \pm\sqrt{3}, y' = \frac{\pm 6}{9\sqrt{3}} = \frac{\pm 2\sqrt{3}}{9}$

At $(6, \sqrt{3})$: $y - \sqrt{3} = \frac{2\sqrt{3}}{9}(x - 6)$

or $2x - 3\sqrt{3}y - 3 = 0$

At $(6, -\sqrt{3})$: $y + \sqrt{3} = \frac{-2\sqrt{3}}{9}(x - 6)$

or $2x + 3\sqrt{3}y - 3 = 0$

(b) From part (a) you know that the slopes of the normal lines must be $\mp 9/(2\sqrt{3})$.

At $(6, \sqrt{3})$: $y - \sqrt{3} = -\frac{9}{2\sqrt{3}}(x - 6)$

or $9x + 2\sqrt{3}y - 60 = 0$

At $(6, -\sqrt{3})$: $y + \sqrt{3} = \frac{9}{2\sqrt{3}}(x - 6)$

or $9x - 2\sqrt{3}y - 60 = 0$

50. (a) $\frac{y^2}{4} - \frac{x^2}{2} = 1, y^2 - 2x^2 = 4, 2yy' - 4x = 0,$

$y' = \frac{4x}{2y} = \frac{2x}{y}$

At $x = 4$: $y = \pm 6, y' = \frac{\pm 2(4)}{6} = \pm\frac{4}{3}$

At $(4, 6)$: $y - 6 = -\frac{4}{3}(x - 4)$ or $4x + 3y - 34 = 0$

At $(4, -6)$: $y + 6 = -\frac{4}{3}(x - 4)$ or $4x + 3y + 2 = 0$

(b) From part (a) you know that the slopes of the normal lines must be $\mp 3/4$.

At $(4, 6)$: $y - 6 = -\frac{3}{4}(x - 4)$ or $3x + 4y - 36 = 0$

At $(4, -6)$: $y + 6 = \frac{3}{4}(x - 4)$ or $3x - 4y - 36 = 0$

51.
$$x^2 + 4y^2 - 6x + 16y + 21 = 0$$
$$(x^2 - 6x + 9) + 4(y^2 + 4y + 4) = -21 + 9 + 16$$
$$(x - 3)^2 + 4(y + 2)^2 = 4$$

Ellipse

52.
$$4x^2 - y^2 - 4x - 3 = 0$$
$$4\left(x^2 - x + \tfrac{1}{4}\right) - y^2 = 3 + 1$$
$$4\left(x - \tfrac{1}{2}\right)^2 - y^2 = 4$$

Hyperbola

53.
$$25x^2 - 10x - 200y - 119 = 0$$
$$25\left(x^2 - \tfrac{2}{5}x + \tfrac{1}{25}\right) = 200y + 119 + 1$$
$$25\left(x - \tfrac{1}{5}\right)^2 = 200(y + 1)$$

Parabola

54.
$$y^2 - 4y = x + 5$$
$$y^2 - 4y + 4 = x + 5 + 4$$
$$(y - 2)^2 = x + 9$$

Parabola

55.
$$9x^2 + 9y^2 - 36x + 6y + 34 = 0$$
$$9(x^2 - 4x + 4) + 9\left(y^2 + \tfrac{2}{3}y + \tfrac{1}{9}\right) = -34 + 36 + 1$$
$$9(x - 2)^2 + 9\left(y + \tfrac{1}{3}\right)^2 = 3$$

Circle (Ellipse)

56.
$$2x(x - y) = y(3 - y - 2x)$$
$$2x^2 - 2xy = 3y - y^2 - 2xy$$
$$2x^2 + y^2 - 3y = 0$$
$$2x^2 + \left(y - \tfrac{3}{2}\right)^2 = \tfrac{9}{4}$$

Ellipse

57.
$$3(x - 1)^2 = 6 + 2(y + 1)^2$$
$$3(x - 1)^2 - 2(y + 1)^2 = 6$$
$$\frac{(x - 1)^2}{2} - \frac{(y + 1)^2}{3} = 1$$

Hyperbola

58.
$$9(x + 3)^2 = 36 - 4(y - 2)^2$$
$$9(x + 3)^2 + 4(y - 2)^2 = 36$$
$$\frac{(x + 3)^2}{4} + \frac{(y - 2)^2}{9} = 1$$

Ellipse

59. (a) A parabola is the set of all points (x, y) that are equidistant from a fixed line (directrix) and a fixed point (focus) not on the line.

(b) For directrix $y = k - p$: $(x - h)^2 = 4p(y - k)$

For directrix $x = h - p$: $(y - k)^2 = 4p(x - h)$

(c) If P is a point on a parabola, then the tangent line to the parabola at P makes equal angles with the line passing through P and the focus, and with the line passing through P parallel to the axis of the parabola.

60. (a) An ellipse is the set of all points (x, y), the sum of whose distance from two distinct fixed points (foci) is constant.

(b) $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$ or $\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1$

61. (a) A hyperbola is the set of all points (x, y) for which the absolute value of the difference between the distances from two distinct fixed points (foci) is constant.

(b) Transverse axis is horizontal:

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

Transverse axis is vertical:

$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$$

(c) Transverse axis is horizontal:

$y = k + (b/a)(x - h)$ and $y = k - (b/a)(x - h)$

Transverse axis is vertical:

$y = k + (a/b)(x - h)$ and $y = k - (a/b)(x - h)$

62. $e = \frac{c}{a}, c = \sqrt{a^2 - b^2},\ 0 < e < 1$

For $e \approx 0$, the ellipse is nearly circular.

For $e \approx 1$, the ellipse is elongated.

63. $9x^2 + 4y^2 - 36x - 24y - 36 = 0$

(a) $9(x^2 - 4x + 4) + 4(y^2 - 6y + 9) = 36 + 36 + 36$

$9(x - 2)^2 + 4(y - 3)^2 = 108$

$\dfrac{(x-2)^2}{12} + \dfrac{(y-3)^2}{27} = 1$

Ellipse

(b) $9x^2 - 4y^2 - 36x - 24y - 36 = 0$

$9(x^2 - 4x + 4) - 4(y^2 + 6y + 9) = 36 + 36 - 36$

$\dfrac{(x-2)^2}{4} - \dfrac{(y+3)^2}{9} = 1$

Hyperbola

(c) $4x^2 + 4y^2 - 36x - 24y - 36 = 0$

$4\left(x^2 - 9x + \dfrac{81}{4}\right) + 4(y^2 - 6y + 9) = 36 + 81 + 36$

$\left(x - \dfrac{9}{2}\right)^2 + (y - 3)^2 = \dfrac{153}{4}$

Circle

(d) *Sample answer:* Eliminate the y^2-term

64. (a) A circle is formed when a plane intersects the top or bottom half of a double-napped cone and is perpendicular to the axis of the cone.

(b) An ellipse is formed when a plane intersects only the top or bottom half of a double-napped cone but is not parallel or perpendicular to the axis of the cone, is not parallel to the side of the cone, and does not intersect the vertex.

(c) A parabola is formed when a plane intersects the top or bottom half of a double-napped cone, is parallel to the side of the cone, and does not intersect the vertex.

(d) A hyperbola is formed when a plane intersects both halves of a double-napped cone, is parallel to the axis of the cone, and does not intersect the vertex.

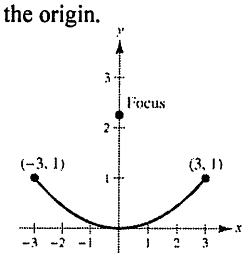
65. Assume that the vertex is at the origin.

$x^2 = 4py$

$(3)^2 = 4p(1)$

$\frac{9}{4} = p$

The pipe is located $\frac{9}{4}$ meters from the vertex.



66. Assume that the vertex is at the origin.

(a) $x^2 = 4py$

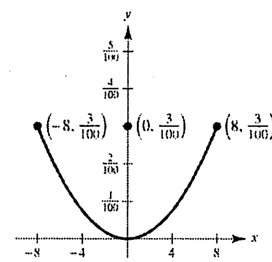
$8^2 = 4p\left(\dfrac{3}{100}\right)$

$\dfrac{1600}{3} = p$

$x^2 = 4\left(\dfrac{1600}{3}\right)y = \dfrac{6400}{3}y$

(b) The deflection is 1 cm when

$y = \dfrac{2}{100} \Rightarrow x = \pm\sqrt{\dfrac{128}{3}} \approx \pm 6.53$ meters.



67. (a) Without loss of generality, place the coordinate system so that the equation of the parabola is $x^2 = 4py$ and, so,

$y' = \left(\dfrac{1}{2p}\right)x.$

So, for distinct tangent lines, the slopes are unequal and the lines intersect.

(b) $x^2 - 4x - 4y = 0$

$2x - 4 - 4\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx} = \dfrac{1}{2}x - 1$

At $(0, 0)$, the slope is -1: $y = -x$. At $(6, 3)$, the slope is 2: $y = 2x - 9$. Solving for x,

$-x = 2x - 9$

$-3x = -9$

$x = 3$

$y = -3.$

Point of intersection: $(3, -3)$