10.2 Graphing Parametric Equations, Eliminating the Parameter

p. 716 #3–53 odd

Parametric Equations: writing both x and y as functions of t

$x = f(t)$ and $y = g(t)$, t is the parameter.

Eliminating Parameter

1) Parametric equation
2) Solve for t
3) substitute into 2nd equation
4) write rectangular equation

a) Sketch curve b) Write corresponding rectangular equation.

3) $x = 3t - 1 \rightarrow t = \frac{x+1}{3}$

$y = 2t + 1$

$y = 2\left(\frac{x+1}{3}\right) + 1$

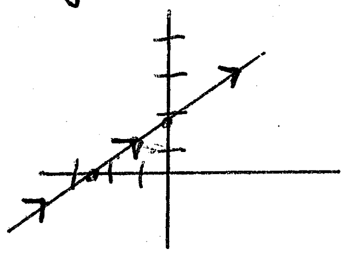
$y = \frac{2x+2}{3} + 1$

$y - 1 = \frac{2x+2}{3}$

$3y - 3 = 2x + 2$

$\boxed{2x - 3y = -5}$

$y = \frac{2}{3}x + \frac{5}{3}$

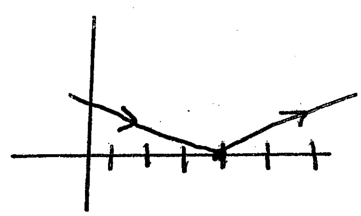


t	-1	0	1	2	3
x	-4	-1	2	5	8
y	-1	1	3	5	7

13) $x = 2t$

$y = |t - 2|$

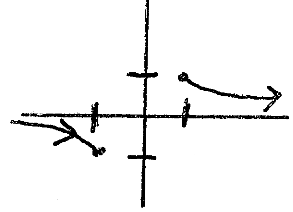
$y = \left|\frac{x}{2} - 2\right| = \frac{|x-4|}{2}$



17) $x = \sec\theta \longrightarrow x = \frac{1}{\cos\theta}$ $y = \cos\theta$

$0 \le \theta \le \pi/2$ $\pi/2 < \theta \le \pi$

$xy = 1$ $y = \frac{1}{x}$, $|x| \ge 1$, $|y| \le 1$

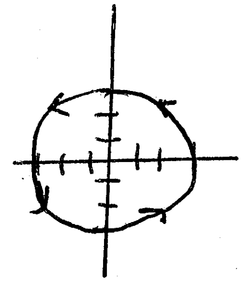


t	0	$\pi/4$	$\pi/2^{+}$	$\pi/2^{-}$	$3\pi/4$	π
x	1	$\frac{2}{\sqrt{2}}$	$+\infty$	$-\infty$	$-\frac{2}{\sqrt{2}}$	-1
y	1	$\frac{\sqrt{2}}{2}$	0	0	$-\frac{\sqrt{2}}{2}$	-1

19) $x = 3\cos\theta$ $\quad (3\cos\theta)^2 + (3\sin\theta)^2 =$

$y = 3\sin\theta$ $\quad 9\cos^2\theta + 9\sin^2\theta = 9$

$\boxed{x^2 + y^2 = 9}$



21) $x = 4\sin 2\theta$ $\quad x^2 = 16\sin^2 2\theta$ $\quad \frac{x^2}{16} = \sin^2 2\theta$

$y = 2\cos 2\theta$ $\quad y^2 = 4\cos^2 2\theta$ $\quad \frac{y^2}{4} = \cos^2 2\theta$

$\sin^2 2\theta + \cos^2 2\theta = 1$

$\boxed{\frac{x^2}{16} + \frac{y^2}{4} = 1}$

23) $x = 4 + 2\cos\theta$ $\quad \frac{x-4}{2} = \cos\theta$ $\quad \frac{(x-4)^2}{4} = \cos^2\theta$

$y = -1 + \sin\theta$ $\quad y+1 = \sin\theta$ $\quad (y+1)^2 = \sin^2\theta$

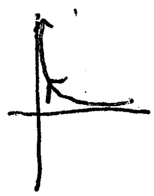
$\frac{(x-4)^2}{4} + \frac{(y+1)^2}{1} = 1$

27) $x = 4\sec\theta$ $\quad y = 3\tan\theta$ $\quad \boxed{\frac{x^2}{16} - \frac{y^2}{9} = 1}$ $\quad \sec^2\theta = 1 + \tan^2\theta$

$\left(\frac{x}{4}\right)^2 = \sec^2\theta$ $\quad \left(\frac{y}{3}\right)^2 = \tan^2\theta$ $\quad \sec^2\theta - \tan^2\theta = 1$

31) $x = e^{-t}$ $\quad y = e^{3t}$

$\frac{1}{x} = e^t$ $\quad y = (e^t)^3$



$y = \left(\frac{1}{x}\right)^3$

$y = \frac{1}{x^3}$

$x > 0$

$y > 0$

33) Determine differences b/t curves of parametric equation.

a) $x = t \quad y = 2t + 1 \rightarrow y = 2x + 1 \quad x, y \in (-\infty, \infty)$

b) $x = \cos\theta \quad y = 2\cos\theta + 1 \rightarrow -1 \leq x \leq 1 \quad -1 \leq y \leq 3$

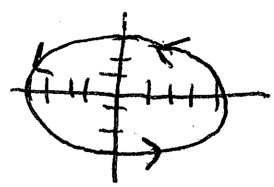
c) $x = e^{-t} \quad y = 2e^{-t} + 1 \quad x > 0$
$x = \frac{1}{e^t} \quad y > 1$

d) $x = e^{t} \quad y = 2e^{t} + 1 \quad x > 0$
$y > 1$

35) a) $x = \cos\theta \quad y = 2\sin^2\theta \quad 0 < \theta < \pi$

b) $x = \cos(-\theta) \quad y = 2\sin^2(-\theta) \quad 0 < \theta < \pi$

37) $x = 4\cos t$
$y = 3\sin t$



b) orientation of 2^{nd} curve is reversed.

d) $x = 1 + t \quad y = 1 + 2t$
$x = 1 - t \quad y = 1 - 2t$

39) Line:

$x = x_1 + t(x_2 - x_1)$ $\qquad t = \frac{x - x_1}{x_2 - x_1}$

$y = y_1 + t(y_2 - y_1)$

$y = y_1 + \frac{x - x_1}{x_2 - x_1}(y_2 - y_1)$ $\qquad \boxed{y - y_1 = m(x - x_1)}$

$y = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

41) Ellipse: $x = h + a\cos\theta$ $\qquad \frac{x-h}{a} = \cos\theta$ $\qquad \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$y = k + b\sin\theta$ $\qquad \frac{y-k}{b} = \sin\theta$

43) Line: $(0,0)$ $\qquad m = \frac{-2}{5}$ $\frac{y}{x}$

$(5,-2)$

$y = -2t \qquad x = 5t$

45) circle: center $(2,1)$ radius: 4 $\qquad (x-h)^2 + (y-k)^2 = r^2$

$(x-2)^2 + (y-1)^2 = 16$

$(4\sin\theta)^2 + (4\cos\theta)^2 = (4)^2$ $\qquad x = 2 + 4\cos\theta$ $\qquad \frac{x-2}{4} = \cos\theta$

$16\sin^2\theta + 16\cos^2\theta = 16$ $\qquad y = 1 + 4\sin\theta$

$\frac{\sin^2\theta}{16} + \frac{\cos^2\theta}{16} = 1$ $\qquad \frac{y-1}{4} = \sin\theta$

$\frac{(x-2)^2}{16} + \frac{(y-1)^2}{16} = 1$

$\frac{(x-2)^2}{16} + \frac{(y-1)^2}{16} = \frac{16}{16}$

$\frac{(x-2)^2}{16} + \frac{(y-1)^2}{16} = 1$ $\qquad \frac{x-2}{4} = \cos\theta$ $\qquad \boxed{x = 4\cos\theta + 2}$

$\left(\frac{x-2}{4}\right)^2 + \left(\frac{y-1}{4}\right)^2 = 1$ $\qquad \frac{y-1}{4} = \sin\theta$ $\qquad \boxed{y = 4\sin\theta + 1}$

47) Ellipse: vertices: $(\pm 5, 0)$ foci: $(\pm 4, 0)$

$a = 5$

$b = 3$

$c = 4$

$$\frac{(x)^2}{25} + \frac{(y^2)}{9} = 1$$

$\cos\theta = \frac{x}{5}$ $\sin\theta = \frac{y}{3}$

$x = 5\cos\theta$ $y = 3\sin\theta$

center: $(0, 0)$

49) Hyperbola: vertices: $(\pm 4, 0)$ foci $(\pm 5, 0)$

$a = 4$

$b = 3$

$c = 5$

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

$\sec^2\theta - \tan^2\theta = 1$

$\frac{x^2}{16} = \sec^2\theta$ $\frac{y^2}{9} = \tan^2\theta$

$\frac{x}{4} = \sec\theta$ $\frac{y}{3} = \tan\theta$

$x = 4\sec\theta$ $y = 3\tan\theta$

51) $y = 3x - 2$

$x = t$ $y = 3t - 2$

$x = t + 1$ $y = 3t + 1$

53) $y = x^3$

$x = t$ $y = t^3$

$x = \sqrt[3]{t}$ $y = t$

$x = t^3$ $y = t^9$