10.2 Exercises

Using Parametric Equations **In Exercises 1–18, sketch the curve represented by the parametric equations (indicate the orientation of the curve), and write the corresponding rectangular equation by eliminating the parameter.**

1. $x = 2t - 3, \quad y = 3t + 1$

2. $x = 5 - 4t, \quad y = 2 + 5t$

3. $x = t + 1, \quad y = t^2$

4. $x = 2t^2, \quad y = t^4 + 1$

5. $x = t^3, \quad y = \dfrac{t^2}{2}$

6. $x = t^2 + t, \quad y = t^2 - t$

7. $x = \sqrt{t}, \quad y = t - 5$

8. $x = \sqrt[4]{t}, \quad y = 8 - t$

9. $x = t - 3, \quad y = \dfrac{t}{t - 3}$

10. $x = 1 + \dfrac{1}{t}, \quad y = t - 1$

11. $x = 2t, \quad y = |t - 2|$

12. $x = |t - 1|, \quad y = t + 2$

13. $x = e^t, \quad y = e^{3t} + 1$

14. $x = e^{-t}, \quad y = e^{2t} - 1$

15. $x = \sec\theta, \quad y = \cos\theta, \quad 0 \le \theta < \pi/2, \quad \pi/2 < \theta \le \pi$

16. $x = \tan^2\theta, \quad y = \sec^2\theta$

17. $x = 8\cos\theta, \quad y = 8\sin\theta$

18. $x = 3\cos\theta, \quad y = 7\sin\theta$

Using Parametric Equations **In Exercises 19–30, use a graphing utility to graph the curve represented by the parametric equations (indicate the orientation of the curve). Eliminate the parameter and write the corresponding rectangular equation.**

19. $x = 6\sin 2\theta$, $y = 4\cos 2\theta$

20. $x = \cos\theta$, $y = 2\sin 2\theta$

21. $x = 4 + 2\cos\theta$, $y = -1 + \sin\theta$

22. $x = -2 + 3\cos\theta$, $y = -5 + 3\sin\theta$

23. $x = -3 + 4\cos\theta$, $y = 2 + 5\sin\theta$

24. $x = \sec\theta$, $y = \tan\theta$

25. $x = 4\sec\theta$, $y = 3\tan\theta$

26. $x = \cos^3\theta$, $y = \sin^3\theta$

27. $x = t^3, \quad y = 3\ln t$

28. $x = \ln 2t, \quad y = t^2$

29. $x = e^{-t}, \quad y = e^{3t}$

30. $x = e^{2t}, \quad y = e^t$

Comparing Plane Curves **In Exercises 31–34, determine any differences between the curves of the parametric equations. Are the graphs the same? Are the orientations the same? Are the curves smooth? Explain.**

31. (a) $x = t$, $y = 2t + 1$ (b) $x = \cos\theta$, $y = 2\cos\theta + 1$

(c) $x = e^{-t}$, $y = 2e^{-t} + 1$ (d) $x = e^t$, $y = 2e^t + 1$

32. (a) $x = 2\cos\theta$, $y = 2\sin\theta$ (b) $x = \sqrt{4t^2 - 1}/|t|$, $y = 1/t$

(c) $x = \sqrt{t}$, $y = \sqrt{4 - t}$ (d) $x = -\sqrt{4 - e^{2t}}$, $y = e^t$

33. (a) $x = \cos\theta$, $y = 2\sin^2\theta$, $0 < \theta < \pi$ (b) $x = \cos(-\theta)$, $y = 2\sin^2(-\theta)$, $0 < \theta < \pi$

34. (a) $x = t + 1, y = t^3$ (b) $x = -t + 1, y = (-t)^3$

35. Conjecture

(a) Use a graphing utility to graph the curves represented by the two sets of parametric equations.

$x = 4\cos t$, $y = 3\sin t$ $x = 4\cos(-t)$, $y = 3\sin(-t)$

(b) Describe the change in the graph when the sign of the parameter is changed.

(c) Make a conjecture about the change in the graph of parametric equations when the sign of the parameter is changed.

(d) Test your conjecture with another set of parametric equations.

36. Writing Review Exercises 31–34 and write a short paragraph describing how the graphs of curves represented by different sets of parametric equations can differ even though eliminating the parameter from each yields the same rectangular equation.

Eliminating a Parameter **In Exercises 37–40, eliminate the parameter and obtain the standard form of the rectangular equation.**

37. Line through (x_1, y_1) and (x_2, y_2):

$x = x_1 + t(x_2 - x_1), \quad y = y_1 + t(y_2 - y_1)$

38. Circle: $x = h + r\cos\theta, \quad y = k + r\sin\theta$

39. Ellipse: $x = h + a\cos\theta, \quad y = k + b\sin\theta$

40. Hyperbola: $x = h + a\sec\theta, \quad y = k + b\tan\theta$

Writing a Set of Parametric Equations **In Exercises 41–48, use the results of Exercises 37–40 to find a set of parametric equations for the line or conic.**

41. Line: passes through $(0, 0)$ and $(4, -7)$

42. Line: passes through $(1, 4)$ and $(5, -2)$

43. Circle: center: $(3, 1)$; radius: 2

44. Circle: center: $(-6, 2)$; radius: 4

45. Ellipse: vertices: $(\pm 10, 0)$; foci: $(\pm 8, 0)$

46. Ellipse: vertices: $(4, 7), (4, -3)$; foci: $(4, 5), (4, -1)$

47. Hyperbola: vertices: $(\pm 4, 0)$; foci: $(\pm 5, 0)$

48. Hyperbola: vertices: $(0, \pm 1)$; foci: $(0, \pm 2)$

Finding Parametric Equations **In Exercises 49–52, find two different sets of parametric equations for the rectangular equation.**

49. $y = 6x - 5$

50. $y = 4/(x - 1)$

51. $y = x^3$

52. $y = x^2$

Finding Parametric Equations **In Exercises 53–56, find a set of parametric equations for the rectangular equation that satisfies the given condition.**

53. $y = 2x - 5$, $t = 0$ at the point $(3, 1)$

54. $y = 4x + 1$, $t = -1$ at the point $(-2, -7)$

55. $y = x^2$, $t = 4$ at the point $(4, 16)$

56. $y = 4 - x^2$, $t = 1$ at the point $(1, 3)$

Graphing a Plane Curve **In Exercises 57–64, use a graphing utility to graph the curve represented by the parametric equations. Indicate the direction of the curve. Identify any points at which the curve is not smooth.**

57. Cycloid: $x = 2(\theta - \sin\theta)$, $y = 2(1 - \cos\theta)$

58. Cycloid: $x = \theta + \sin\theta$, $y = 1 - \cos\theta$

59. Prolate cycloid: $x = \theta - \frac{3}{2}\sin\theta$, $y = 1 - \frac{3}{2}\cos\theta$

60. Prolate cycloid: $x = 2\theta - 4\sin\theta$, $y = 2 - 4\cos\theta$

61. Hypocycloid: $x = 3\cos^3\theta$, $y = 3\sin^3\theta$

62. Curtate cycloid: $x = 2\theta - \sin\theta$, $y = 2 - \cos\theta$

63. Witch of Agnesi: $x = 2\cot\theta$, $y = 2\sin^2\theta$

64. Folium of Descartes: $x = 3t/(1 + t^3)$, $y = 3t^2/(1 + t^3)$

WRITING ABOUT CONCEPTS

65. Plane Curve State the definition of a plane curve given by parametric equations.

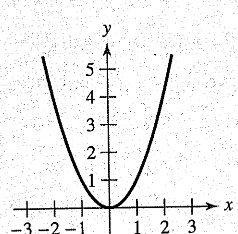
66. Plane Curve Explain the process of sketching a plane curve given by parametric equations. What is meant by the orientation of the curve?

67. Smooth Curve State the definition of a smooth curve.

68. HOW DO YOU SEE IT? Which set of parametric equations is shown in the graph below? Explain your reasoning.

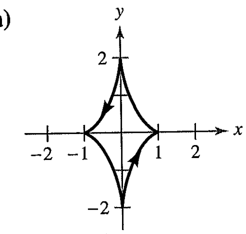
(a) $x = t$, $y = t^2$

(b) $x = t^2$, $y = t$

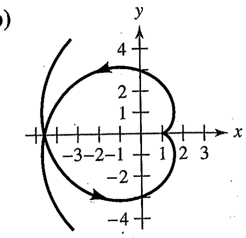


Matching **In Exercises 69–72, match each set of parametric equations with the correct graph. [The graphs are labeled (a), (b), (c), and (d).] Explain your reasoning.**

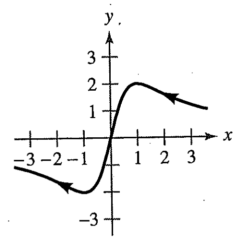
(a)



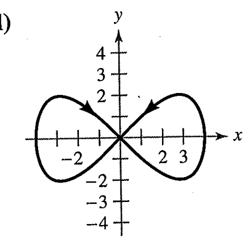
(b)



(c)



(d)



69. Lissajous curve: $x = 4\cos\theta$, $y = 2\sin 2\theta$

70. Evolute of ellipse: $x = \cos^3\theta$, $y = 2\sin^3\theta$

71. Involute of circle: $x = \cos\theta + \theta\sin\theta$, $y = \sin\theta - \theta\cos\theta$

72. Serpentine curve: $x = \cot\theta$, $y = 4\sin\theta\cos\theta$

73. Curtate Cycloid A wheel of radius a rolls along a line without slipping. The curve traced by a point P that is b units from the center $(b < a)$ is called a **curtate cycloid** (see figure). Use the angle θ to find a set of parametric equations for this curve.

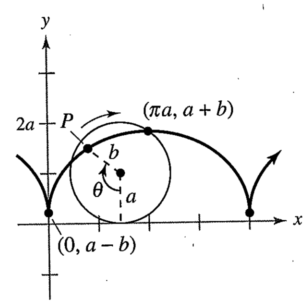


Figure for 73

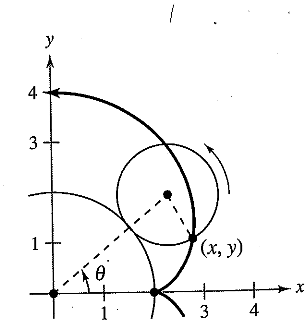


Figure for 74

74. Epicycloid A circle of radius 1 rolls around the outside of a circle of radius 2 without slipping. The curve traced by a point on the circumference of the smaller circle is called an epicycloid (see figure). Use the angle θ to find a set of parametric equations for this curve.

True or False? **In Exercises 75–77, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

75. The graph of the parametric equations $x = t^2$ and $y = t^2$ is the line $y = x$.

76. If y is a function of t and x is a function of t, then y is a function of x.

77. The curve represented by the parametric equations $x = t$ and $y = \cos t$ can be written as an equation of the form $y = f(x)$.

78. Translation of a Plane Curve Consider the parametric equations $x = 8\cos t$ and $y = 8\sin t$.

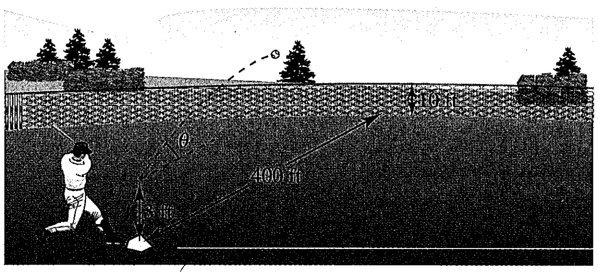
(a) Describe the curve represented by the parametric equations.

(b) How does the curve represented by the parametric equations $x = 8\cos t + 3$ and $y = 8\sin t + 6$ compare to the curve described in part (a)?

(c) How does the original curve change when cosine and sine are interchanged?

Projectile Motion In Exercises 79 and 80, consider a projectile launched at a height h feet above the ground and at an angle θ with the horizontal. When the initial velocity is v_0 feet per second, the path of the projectile is modeled by the parametric equations $x = (v_0 \cos\theta)t$ and $y = h + (v_0 \sin\theta)t - 16t^2$.

79. The center field fence in a ballpark is 10 feet high and 400 feet from home plate. The ball is hit 3 feet above the ground. It leaves the bat at an angle of θ degrees with the horizontal at a speed of 100 miles per hour (see figure).



(a) Write a set of parametric equations for the path of the ball.

(b) Use a graphing utility to graph the path of the ball when $\theta = 15°$. Is the hit a home run?

(c) Use a graphing utility to graph the path of the ball when $\theta = 23°$. Is the hit a home run?

(d) Find the minimum angle at which the ball must leave the bat in order for the hit to be a home run.

80. A rectangular equation for the path of a projectile is $y = 5 + x - 0.005x^2$.

(a) Eliminate the parameter t from the position function for the motion of a projectile to show that the rectangular equation is

$$y = -\frac{16\sec^2\theta}{v_0^2}x^2 + (\tan\theta)x + h.$$

(b) Use the result of part (a) to find h, v_0, and θ. Find the parametric equations of the path.

(c) Use a graphing utility to graph the rectangular equation for the path of the projectile. Confirm your answer in part (b) by sketching the curve represented by the parametric equations.

(d) Use a graphing utility to approximate the maximum height of the projectile and its range.

SECTION PROJECT

Cycloids

In Greek, the word *cycloid* means *wheel*, the word *hypocycloid* means *under the wheel*, and the word *epicycloid* means *upon the wheel*. Match the hypocycloid or epicycloid with its graph. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]

Hypocycloid, H(A, B)

The path traced by a fixed point on a circle of radius B as it rolls around the *inside* of a circle of radius A

$$x = (A - B)\cos t + B\cos\left(\frac{A - B}{B}\right)t$$

$$y = (A - B)\sin t - B\sin\left(\frac{A - B}{B}\right)t$$

Epicycloid, E(A, B)

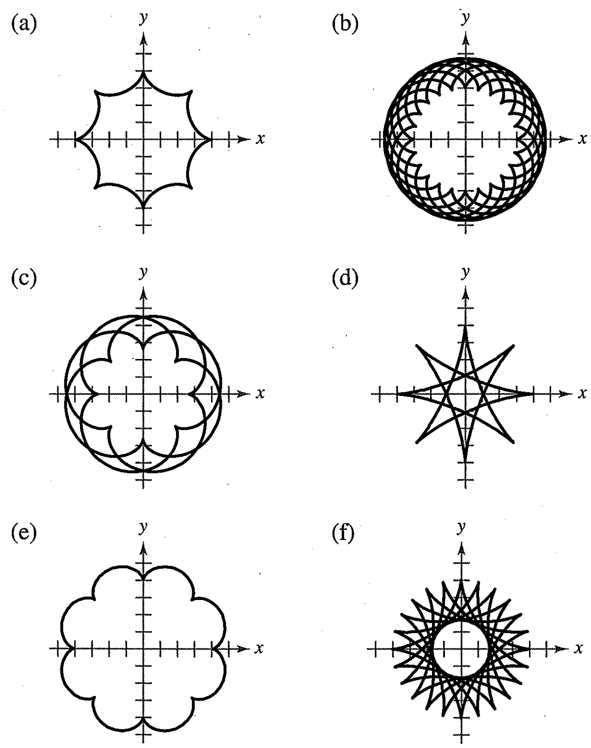
The path traced by a fixed point on a circle of radius B as it rolls around the *outside* of a circle of radius A

$$x = (A + B)\cos t - B\cos\left(\frac{A + B}{B}\right)t$$

$$y = (A + B)\sin t - B\sin\left(\frac{A + B}{B}\right)t$$

I. $H(8, 3)$ II. $E(8, 3)$ III. $H(8, 7)$

IV. $E(24, 3)$ V. $H(24, 7)$ VI. $E(24, 7)$



Exercises based on "Mathematical Discovery via Computer Graphics: Hypocycloids and Epicycloids" by Florence S. Gordon and Sheldon P. Gordon, *College Mathematics Journal*, November 1984, p. 441. Used by permission of the authors.