Key

BC Calculus – 10.3 Notes – *n*th term, p-series, and Integral Test

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots + a_n$$

If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$

If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ may either converge or diverge.

Nth Term Test for Divergence

If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges (the series diverges!)

Use the Nth term test to make a conclusion about divergence for each series.

1. $\sum_{n=1}^{\infty} \frac{3n^3+1}{5n^3-2n^2+1}$

$\lim_{n\to\infty} a_n = \frac{3}{5} \neq 0$

Since the sequence of values do not approach zero, the series diverge by n^{th} term test (even though the sequence converges to $3/5$)

2. $\sum_{n=0}^{\infty} 3\left(\frac{1}{2}\right)^n$

$\lim_{n\to\infty} a_n = 0$. N^{th} term test for divergence is inconclusive

* converging series by GST since $|r|<1$

3. $\sum_{n=1}^{\infty} \frac{1}{n}$

$\lim_{n\to\infty} \frac{1}{n} = 0$

N^{th} term test is inconclusive.

4. $\sum_{n=1}^{\infty} \frac{2^{n+2}}{2^{n+3}+1} \rightarrow \frac{2^n \cdot 2^2}{2^n \cdot 2^3 + 1}$

$\lim_{n\to\infty} a_n = \frac{1}{2} \neq 0$

Diverges by n^{th} term test.

5. $\sum_{n=1}^{\infty} \frac{e^{4n}}{3n} \rightarrow$ L'Hopital's Rule $\rightarrow \lim_{n\to\infty} \frac{4e^{4n}}{3} = \infty$

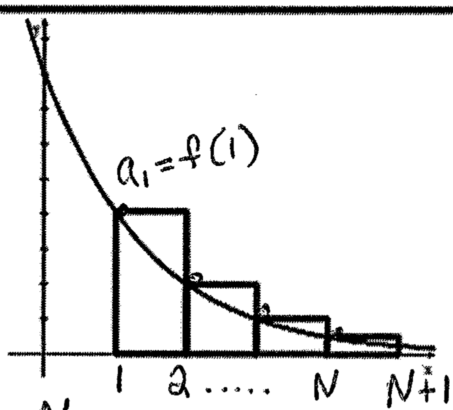
Diverges by the n^{th} term test.

Integral Test for Convergence

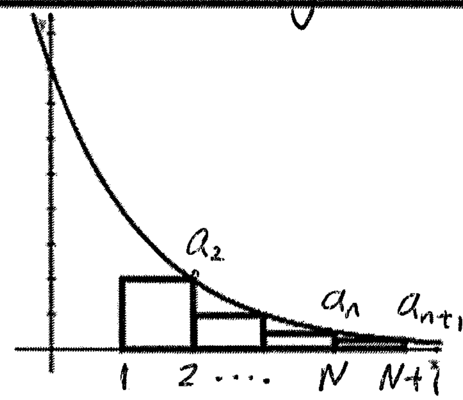
If f is a positive, continuous, and decreasing function for $x \geq k$, and $a_n = f(x)$, then

$$\sum_{n=k}^{\infty} a_n \quad \text{and} \quad \int_k^{\infty} f(x)\,dx$$

both converge or both diverge



$$\sum_{n=1}^{N} a_1 \geq \int_1^{N+1} f(x)\,dx$$



$$\sum_{n=2}^{N+1} a_n \leq \int_1^{N+1} f(x)\,dx$$

*Add a_1 to both sides

$$\sum_{n=1}^{N+1} a_n \leq \int_1^{N+1} f(x)\,dx + a_1$$

*As $n \to \infty$, $\int_1^{\infty} f(x)\,dx \leq \sum_{n=1}^{\infty} a_n \leq \int_1^{\infty} f(x)\,dx + \text{constant}$

Determine the convergence or divergence of the series

1. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

*check conditions
i) positive
ii) decreasing
iii) continuous

$\lim_{b \to \infty} \int_2^b \frac{1}{x \ln x}\,dx$

$u = \ln x$ | $dx = x\,du$

$\frac{du}{dx} = \frac{1}{x}$

$\lim_{b \to \infty} \int_2^b \frac{1}{x \cdot u} \cdot x\,du$

$\lim_{b \to \infty} \int_2^b \frac{1}{u}\,du$

$\lim_{b \to \infty} \ln|u|$

$\lim_{b \to \infty} \ln|\ln x| \Big]_2^b$

$\lim_{b \to \infty} \ln|\ln b| - \ln|\ln 2|$

$\infty - \ln|\ln 2| \to \boxed{\text{diverges}}$

2. $\sum_{n=1}^{\infty} \frac{1}{n^2}$

positive ✓
continuous ✓
decreasing ✓

$\int_1^b \frac{1}{x^2}\,dx$

$\lim_{b \to \infty} \int_1^b x^{-2}\,dx \to \frac{x^{-1}}{-1} \to \frac{-1}{x}\Big]_1^b$

$\lim_{b \to \infty} \frac{-1}{x}\Big]_1^b \to \lim_{b \to \infty} \frac{-1}{b} - \left(\frac{-1}{1}\right)$

$\to 0 + 1 = \boxed{1}$

Series converge by the Integral test (But series does not converge to 1)

p-Series

Let p be a positive constant of the series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots$

* p-value is positive in the denominator

The series converges if $p > 1$

The series diverges if $0 < p \leq 1$

Harmonic Series

*p-value = 1

*diverges

$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \ldots$ $\boxed{\text{Diverges}}$

Ex:

* $\frac{1}{3^n} = \left(\frac{1}{3}\right)^n$

This is geometric series

Do the following series converge or diverge?

1. $\sum_{n=1}^{\infty} \frac{1}{n^3}$ $\boxed{\text{Series converge}}$

$p = 3 > 1$

2. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \rightarrow \frac{1}{n^{1/2}}$ $p = 1/2$

$\boxed{\text{Series diverges}}$

For what values of k will the series converge?

3. $\sum_{n=1}^{\infty} \frac{1}{n^{2k-5}} \rightarrow \frac{1}{n^p}$

*series converge if

$p > 1$

$2k - 5 > 1$

$2k > 6$

$\boxed{k > 3}$

4. $\sum_{n=1}^{\infty} \frac{1}{n(n^{2k})} \rightarrow \frac{1}{n^1 \cdot n^{2k}}$

$\frac{1}{n^{1+2k}} \rightarrow \frac{1}{n^p}$

$1 + 2k > 1$

$2k > 0$

$\boxed{k > 0}$

5. $\sum_{n=1}^{\infty} \frac{n}{n^{4k} + 5}$ ← we want 1 up in the numerator

$\dfrac{\frac{n}{n}}{\frac{n^{4k}}{n} + \frac{5}{n}} \rightarrow \dfrac{1}{n^{4k-1} + \frac{5}{n}}$

$4k - 1 > 1$

$4k > 2$

$\boxed{k > 1/2}$

Things we should now recognize

Series

- Geometric
- Harmonic
- p-Series

Tests for convergence/divergence

- Nth Term Test for Divergence
- Integral Test

10.3 The nth Term Test for Divergence

Calculus

Practice

For each of the following series, determine the convergence or divergence of the given series. State the reasoning behind your answer.

1. $\sum_{n=1}^{\infty} \frac{3-2n}{5n+1}$

$\lim_{n\to\infty} \frac{3-2n}{5n+1} = -\frac{2}{5} \neq 0$

$\boxed{\text{Diverges by } n^{th} \text{ term test}}$

2. $\sum_{n=1}^{\infty} \frac{3^{n+1}}{5^n}$

$\sum \frac{3^n \cdot 3}{5^n} \to \sum 3\left(\frac{3}{5}\right)^n$

$r = \frac{3}{5} < 1$ * geometric series

$a_1 = 3\left(\frac{3}{5}\right) = \frac{9}{5}$

$S = \frac{9/5}{1-3/5} \to \frac{9/5}{2/5} \to \boxed{\frac{9}{2}}$

Series converge by Geometric Series Test. (GST)

3. $\sum_{n=1}^{\infty} \frac{2n}{\sqrt{n^2+1}}$

$\lim_{n\to\infty} \frac{2n}{\sqrt{n^2+1}} = 2$

$\boxed{\text{Diverges by } n^{th} \text{ term test}}$

4. $\sum_{n=1}^{\infty} \frac{e^{n+1}}{\pi^n}$

$\frac{e^n \cdot e}{\pi^n} \to e\left(\frac{e}{\pi}\right)^n$

$r = \frac{e}{\pi} < 1$

$a_1 = e\left(\frac{e}{\pi}\right) = \frac{e^2}{\pi}$

$S = \frac{\frac{e^2}{\pi}}{1-\frac{e}{\pi}}$

$S = \frac{\frac{e^2}{\pi}}{\frac{\pi - e}{\pi}}$

$S = \frac{e^2}{\pi} \cdot \frac{\pi}{\pi - e} = \boxed{\frac{e^2}{\pi - e}}$

Converges by GST

5. $\sum_{n=1}^{\infty} \frac{7^n+1}{7^{n+1}}$

$\lim_{n\to\infty} \left(\frac{7^n}{7^n \cdot 7} + \frac{1}{7^n \cdot 7} \right)$

$= \frac{1}{7} + 0 = \frac{1}{7}$

$\boxed{\text{Diverges by } n^{th} \text{ term test}}$

6. $\sum_{n=0}^{\infty} 5\left(\frac{5}{2}\right)^n$

$r = \frac{5}{2} > 1$ $\lim_{n\to\infty} 5\left(\frac{5}{2}\right)^n \neq 0$

Diverges by n^{th} term test or

Diverges by GST since $r > 1$

10.3 The nth Term Test for Divergence

Test Prep

7. The nth-Term Test can be used to determine divergence for which of the following series?

I. $\sum_{n=1}^{\infty} \sin 2n$ II. $\sum_{n=1}^{\infty}\left(2+\frac{3}{n}\right)$ III. $\sum_{n=1}^{\infty}\frac{n^3+1}{n^2}$

*If $\lim_{n\to\infty} a_n \neq 0$, Series diverge by n^{th} term test.

$\lim_{n\to\infty} a_n = dne$ Diverges

$\lim_{n\to\infty} a_n = 2$ Diverges

$\lim_{n\to\infty} a_n = \infty$ Diverges

(A) II only (B) III only (C) I and II only $\boxed{\text{(D) I, II, and III}}$

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8. The nth-Term Test can be used to determine divergence for which of the following series?

I. $\sum_{n=1}^{\infty} \ln\left(\frac{n-1}{n}\right)$ II. $\sum_{n=1}^{\infty}\frac{3n-2n^2}{5n^2}$ III. $\sum_{n=1}^{\infty} 3\left(\frac{5}{4}\right)^n$

$\lim_{n\to\infty} a_n = \ln(1) = 0$ (inconclusive by n^{th} term test)

$\lim_{n\to\infty} a_n = \frac{-2}{5} \neq 0$ (Diverges)

$\lim_{n\to\infty} a_n = \infty$ (Diverges)

(A) II only $\boxed{\text{(B) II and III only}}$ (C) I and II only (D) I, II, and III

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9. If $a_n = \cos\left(\frac{\pi}{2n}\right)$ for $n = 1,2,3,\dots$, which of the following about $\sum_{n=1}^{\infty} a_n$ must be true?

$\lim_{n\to\infty} \cos\left(\frac{\pi}{2n}\right) \to \cos(0) = 1$

(A) The series converges and $\lim_{n\to\infty} a_n = 0$.

(B) The series diverges and $\lim_{n\to\infty} a_n = 0$

$\boxed{\text{(C) The series diverges and } \lim_{n\to\infty} a_n \neq 0}$

(D) The series converges and $\lim_{n\to\infty} a_n \neq 0$

10.4 Integral Test for Convergence

Calculus

(* conditions are positive, continuous, decreasing)

Practice

If the Integral Test applies, use it to determine whether the series converges or diverges.

1. $\sum_{n=1}^{\infty} \frac{n}{e^n}$

$\int \frac{x}{e^x}dx \to \int x e^{-x} dx$

u	dv
+ x	e^{-x}
− 1	$-e^{-x}$
+ 0	e^{-x}

$-xe^{-x} - e^{-x}\Big]_1^b$

$\lim_{b\to\infty} \frac{-b}{e^b} - \frac{1}{e^b} - \left(\frac{-1}{e} - \frac{1}{e}\right)$

$0 - 0 - \left(-\frac{2}{e}\right) = \frac{2}{e}$

By Integral Test, series converges

(positive ✓
continuous ✓
decreasing ✓)

2. $\sum_{n=1}^{\infty} f(n) = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$ $\quad \frac{1}{n^2}$

pos. ✓
cont. ✓
dec. ✓

$\lim_{b\to\infty} \int_1^b \frac{1}{x^2} dx \to \int x^{-2} dx$

$\frac{x^{-1}}{-1}\Big]_1^b \to \lim_{b\to\infty} \frac{-1}{x^b} - \left(\frac{-1}{1}\right)$

$0 + 1 = 1$

Series Converges by Integral Test

3. $\sum_{n=1}^{\infty} f(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$ $\quad \frac{1}{n}$

pos, cont, dec. ✓

$\lim_{b\to\infty} \int_1^b \frac{1}{x} dx \to \ln|x|\Big]_1^b \to \lim_{b\to\infty} \ln|b| - \ln|1|$

$= \boxed{\infty}$

By Integral Test, series diverge.

4. $\sum_{n=1}^{\infty} \frac{1}{n^2} \sin\frac{\pi}{n}$

pos, cont, dec. ✓

$\lim_{b\to\infty} \int_1^b \sin\left(\frac{\pi}{x}\right) \cdot \frac{1}{x^2} dx$

$u = \frac{\pi}{x} = \pi x^{-1}$

$\frac{du}{dx} = -\pi x^{-2} = \frac{-\pi}{x^2}$

$dx = -\frac{x^2}{\pi} du$

$\lim_{b\to\infty} \int \sin(u) \cdot \cancel{\frac{1}{x^2}} \cdot \frac{-\cancel{x^2}}{\pi} du$

$-\frac{1}{\pi} \cdot -\cos\left(\frac{\pi}{x}\right)\Big]_1^b$

$\lim_{b\to\infty} \frac{1}{\pi}\cos\left(\frac{\pi}{b}\right) - \frac{1}{\pi}\cos\left(\frac{\pi}{1}\right)$

$\frac{1}{\pi}\cos(0) - \frac{1}{\pi}(\cos(\pi)) = \frac{1}{\pi} + \frac{1}{\pi} = \frac{2}{\pi}$

Converges by Integral Test.

pos, cont, dec. (p.c.d.)

5. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

$\lim_{b\to\infty} \int_1^b \frac{x}{x^2+1}\,dx$ $u = x^2+1$ $\frac{du}{dx} = 2x$ $dx = \frac{du}{2x}$

$\int_1^b \frac{\cancel{x}}{u} \cdot \frac{du}{2\cancel{x}}$

$\frac{1}{2}\int \frac{1}{u}\,du \to \lim_{b\to\infty} \frac{1}{2}\ln|x^2+1| \Big]_1^b$

$\lim_{b\to\infty} \frac{1}{2}\ln|b^2+1| - \frac{1}{2}\ln|1+1| = \infty$

Diverges by Integral Test.

6. $\sum_{n=1}^{\infty} \frac{n^{k-1}}{n^k+2}$, where k is a positive integer. Assume the series meets the criteria for the Integral Test.

p.c.d. $\lim_{b\to\infty} \int_1^b \frac{x^{k-1}}{x^k+2}\,dx$ $u = x^k+2$ $\frac{du}{dx} = kx^{k-1}$ $dx = \frac{du}{kx^{k-1}}$

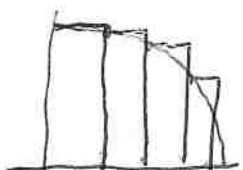
$\int \frac{\cancel{x^{k-1}}}{u} \cdot \frac{du}{k\cancel{x^{k-1}}}$

$\lim_{b\to\infty} \frac{1}{k}\int_1^b \frac{1}{u}\,du \to \frac{1}{k}\ln|x^k+2| \Big]_1^b$

$\lim_{b\to\infty} \frac{1}{k}\ln|b^k+2| - \frac{1}{k}|1^k+2| = \infty$

Diverges by Integral Test

7. Let f be a positive, continuous, and decreasing function. If $\int_1^{\infty} f(x)\,dx = 4$, which of the following statements about the series $\sum_{n=1}^{\infty} f(n)$ must be true?



Left Rectangle Approximation

A. $\sum_{n=1}^{\infty} f(n) = 0$

B. $\boxed{\sum_{n=1}^{\infty} f(n) \text{ converges, and } \sum_{n=1}^{\infty} f(n) > 4}$ overapproximation

C. $\sum_{n=1}^{\infty} f(n)$ converges, and $\sum_{n=1}^{\infty} f(n) < 4$

D. $\sum_{n=1}^{\infty} f(n)$ diverges, and $\sum_{n=1}^{\infty} f(n) = 0$

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8. Explain why the Integral Test does not apply for the series $\sum_{x=1}^{\infty} e^x \sin x$. oscillating function

f(x) is positive, but not positive and decreasing for $x \geq 1$

9. Show that the series $\sum_{x=1}^{\infty} \frac{\tan^{-1} x}{x^2+1}$ meets the criteria to apply the Integral Test for convergence.

continuous, positive, decreasing

$u = \tan^{-1}(x)$ $dx = x^2+1\,du$

$\frac{du}{dx} = \frac{1}{x^2+1}$

$\int \frac{u}{\cancel{x^2+1}} \cdot \cancel{x^2+1}\,du$

$\frac{u^2}{2} \to \frac{1}{2}[\arctan(x)]^2 \Big]_1^b$

$\lim_{b\to\infty} \frac{1}{2}[\arctan(b)]^2 - \frac{1}{2}[\arctan(1)]^2$

$\frac{1}{2}\left[\frac{\pi}{2}\right]^2 - \frac{1}{2}\left[\frac{\pi}{4}\right]^2 = \frac{\pi^2}{8} - \frac{\pi^2}{32} = \boxed{\frac{3\pi^2}{32}}$

Converges by Integral Test

10. Let f be positive, continuous, and decreasing on $[1, \infty)$, such that $a_n = f(n)$. If $\sum_{n=1}^{\infty} a_n = 7$, which of the following must be true?

A. $\lim_{n\to\infty} a_n = 7$

B. $\int_1^{\infty} f(x)\,dx = 7$

C. $\int_1^{\infty} f(x)\,dx$ diverges

D. $\boxed{\int_1^{\infty} f(x)\,dx \text{ converges}}$

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11. Which of the following can be used to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n+3}{n+4}$?

I. Properties of Geometric Series (not geometric)
II. $\boxed{n\text{th-Term Test}}$ $\dashrightarrow$ $\lim_{n\to\infty} \frac{n+3}{n+4} = 1 \neq 0$
III. Integral Test → not decreasing

A. I only

B. $\boxed{\text{II only}}$

C. III only

D. II and III only

E. I, II, and III

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12. Which of the following can be used to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{1}{2^n}$? → $\left(\frac{1}{2}\right)^n$

✓ I. Properties of Geometric Series
✗ II. nth-Term Test → $\lim_{n\to\infty} a_n = 0$ (inconclusive)
✓ III. Integral Test
(positive, continuous, decreasing)

A. I only

B. II only

C. III only

D. I and II only

E. $\boxed{\text{I and III only}}$

10.4 Integral Test for Convergence

Test Prep

13. Consider the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^3}$. The integral test can be used to determine convergence or divergence of the series because $f(x) = \frac{1}{x^3}$ is positive, continuous, and decreasing on $[1, \infty)$. Which of the following is true?

Left Approximation overapproximates

A. $$1 + \int_1^{\infty} \frac{1}{x^3}dx < \sum_{n=1}^{\infty} \frac{1}{n^3} < \int_1^{\infty} \frac{1}{x^3}dx$$

B. $$\boxed{\int_1^{\infty} \frac{1}{x^3}dx < \sum_{n=1}^{\infty} \frac{1}{n^3} < 1 + \int_1^{\infty} \frac{1}{x^3}dx}$$

C. $$\sum_{n=1}^{\infty} \frac{1}{n^3} < \int_1^{\infty} \frac{1}{x^3}dx < 1 + \int_1^{\infty} \frac{1}{x^3}dx$$

D. $$\int_1^{\infty} \frac{1}{x^3}dx < 1 + \int_1^{\infty} \frac{1}{x^3}dx < \sum_{n=1}^{\infty} \frac{1}{n^3}$$

10.5 Harmonic Series and p-series

Calculus

Practice

Determine the convergence or divergence of the following p-series.

1. $\sum_{n=1}^{\infty} n^{-\frac{3}{2}}$ $\rightarrow \frac{1}{n^{3/2}}$ $p=\frac{3}{2}$

$p>1$ so series converge

2. $\sum_{n=1}^{\infty} \frac{1}{n^{0.13}}$ $p=0.13$

$0<p<1$

Diverges

3. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$ $\frac{1}{n \cdot n^{1/2}} \rightarrow \frac{1}{n^{3/2}}$

$p=\frac{3}{2}>1$ converges

What are all the values of p for which…

4. $\sum_{n=1}^{\infty} \frac{2n}{n^p+2}$ converges?

$\frac{\cancel{n} \cdot 2}{\cancel{n} \cdot (n^{p-1} + \frac{2}{n})}$

$p-1>1$

$\boxed{p>2}$

5. $\sum_{n=1}^{\infty} \frac{1}{n^{3p}}$ diverges?

$3p \leq 1$

$\boxed{p \leq \frac{1}{3}}$

6. Both series $\sum_{n=1}^{\infty} n^{-5p}$ and $\sum_{n=1}^{\infty} \left(\frac{p}{5}\right)^n$ converge?

geometric series $|r|<1$

$\frac{1}{n^{5p}}$

$5p>1$

$p>\frac{1}{5}$

$\frac{p}{5}<1$

$p<5$

$\boxed{\frac{1}{5}<p<5}$

7. $\int_1^{\infty} \frac{1}{x^{3p+4}}\,dx$ converges?

positive, continuous, decreasing

$3p+4>1$

$3p>3$

$\boxed{p>-1}$

Find the positive values of p for which the infinite series converge?

8. $\sum_{n=1}^{\infty} \left(\frac{4}{p}\right)^n$

$\frac{4}{p}<1$

$\boxed{p>4}$

9. $\sum_{n=1}^{\infty} \frac{n}{(n^2+1)^p}$

$u=n^2+1$

$\frac{du}{dn}=2n$

$\int \frac{\cancel{n}}{u^p} \cdot \frac{du}{2\cancel{n}}$

$\boxed{p>1}$

10. $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$

$2p>1$

$\boxed{p>1/2}$

10.5 Harmonic Series and p-series

Test Prep

11. Which of the following infinite series converge?

I. $\sum_{n=1}^{\infty} n^{-\frac{1}{2}}$ $\frac{1}{n^{1/2}}$ diverges $(p<1)$

✓ II. $\sum_{n=1}^{\infty} \left(\frac{e}{2}\right)^{-n}$ $\left(\frac{2}{e}\right)^n$ converges $r=\frac{2}{e}<1$

✓ III. $\sum_{n=1}^{\infty} \frac{1}{n^e}$ $\frac{1}{n^e}$ $e>1$ $p=e>1$ converges

A. None

B. II only

C. III only

D. I and II only

E. II and III only

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12. Which of the following infinite series converge?

✓ I. $\sum_{n=1}^{\infty} 3^{-n}$ $\left(\frac{1}{3}\right)^n$ converges by GST $r=\frac{1}{3}<1$

✓ II. $\sum_{n=1}^{\infty} \frac{1}{(3n+1)^3}$ $p=3$ (converges)

III. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$ $\frac{1}{n^{1/5}}$ $p=1/5$ $0<p<1$ diverges

A. I only

B. II only

C. III only

D. I and II only

E. I and III only

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13. Which of the following infinite series is a divergent p-series?

A. $\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n$ converges by GST

B. $\sum_{n=1}^{\infty} n^{-\frac{1}{2}}$ $\frac{1}{n^{1/2}}$ $p=1/2$ $0<p<1$ (Diverges)

C. $\sum_{n=1}^{\infty} n^{-\frac{3}{2}}$ $\frac{1}{n^{3/2}}$ $p=3/2>1$ converges

D. $\sum_{n=1}^{\infty} n^{\frac{3}{2}}$ diverges by n^{th} term test

14. Which of the following is not a p-series?

Geometric Series (converges)

(converges) A. $\sum_{n=1}^{\infty} n^{-3}$ $\frac{1}{n^3}$ $p=3$

(diverges) B. $\sum_{n=1}^{\infty} \frac{1}{n}$ $p=1$

(Converges) C. $\sum_{n=1}^{25} \frac{1}{n^{\pi}}$ $\left(\frac{1}{n}\right)^{\pi}$ $p=\pi$

$\boxed{\text{D. } \sum_{n=1}^{\infty} \frac{1}{\pi^n}}$ $\left(\frac{1}{\pi}\right)^n$

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15. Which of the following is a harmonic series?

A. $\sum_{n=1}^{\infty} \frac{1}{3n}$

$\boxed{\text{B. } \sum_{n=1}^{\infty} \frac{1}{n}}$

C. $\sum_{n=1}^{1000} \frac{1}{n}$

D. $\sum_{n=1}^{\infty} \frac{3n^2}{4n^2+1}$

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16. Find the positive values of k for which the series $\sum_{n=3}^{\infty} \frac{1}{(n \ln n)(\ln(\ln n))^k}$ converges.

$u = \ln(\ln n)$

$\frac{du}{dn} = \frac{\frac{1}{n}}{\ln(n)} = \frac{1}{n \ln n}$

$dn = du(n \ln(n))$

$dn = (n \ln(n))du$

$\int_3^{\infty} \frac{1}{\cancel{n \ln(n)} \cdot u^k} \cdot \cancel{n \ln(n)}\, du$

$\boxed{k > 1}$