10.3b p.726 # 44-57 D2S2, 63-88 D2S2

Arc Length: $s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

44) Write integral representing length of arc.

$x = \ln t$, $y = t+1$ $\quad 1 \le t \le 6$

$\frac{dx}{dt} = \frac{1}{t} \quad \frac{dy}{dt} = 1 \qquad s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$$\boxed{s = \int_1^6 \sqrt{\frac{1}{t^2} + 1}\, dt}$$

48) $x = t^2 + 1 \quad y = 4t^3 + 3 \quad -1 \le t \le 0$

$$s = \int_{-1}^{0} \sqrt{(2t)^2 + (12t^2)^2}\, dt = \frac{-1}{54}\left(1 - 37^{3/2}\right) \approx 4.149$$

52) $x = t \quad y = \frac{t^5}{10} + \frac{1}{6t^3} \quad 1 \le t \le 2$

$x'(t) = 1 \quad y'(t) = \frac{1}{2}t^4 - \frac{1}{2t^4}$

$$s = \int_1^2 \sqrt{1 + \left(\frac{t^4}{2} - \frac{1}{2t^4}\right)^2} = \sqrt{\underline{1} + \left(\frac{t^4}{2}\right)^2 - 2\left(\frac{t^4}{2}\right)\left(\frac{1}{2t^4}\right) + \left(\frac{1}{2t^4}\right)^2}$$

$-\frac{1}{2}$

$$= \sqrt{\left(\frac{t^4}{2}\right)^2 + 2\left(\frac{t^4}{2}\right)\left(\frac{1}{2t^4}\right) + \left(\frac{1}{2t^4}\right)^2}$$

$$= \sqrt{\left(\frac{t^4}{2} + \frac{1}{2t^4}\right)^2}$$

$$= \int_1^2 \frac{t^4}{2} + \frac{1}{2t^4}\, dt = \left.\frac{t^5}{10} - \frac{1}{6t^3}\right]_1^2 = \boxed{\frac{779}{240}}$$

56) Find arc length on $[0, 2\pi]$

$x = \cos\theta + \theta\sin\theta \quad y = \sin\theta - \theta\cos\theta$

$x'(\theta) = -\sin\theta + \sin\theta + \theta\cos\theta = \theta\cos\theta$

$y'(\theta) = \cos\theta + (-1)\cos\theta + (-\theta)(-\sin\theta) = \theta\sin\theta$

$$S = \int_0^{2\pi} \sqrt{(\theta\cos\theta)^2 + (\theta\sin\theta)^2}\, d\theta = \sqrt{\theta^2(\cos^2\theta + \sin^2\theta)}\, d\theta$$

$$S = \int_0^{2\pi} \theta\, d\theta = \frac{\theta^2}{2}\Big]_0^{2\pi} = \boxed{2\pi^2}$$

64) Surface Area

a) About y-axis $S_y = 2\pi \int_a^b x\sqrt{x'(t)^2 + y'(t)^2}\, dt$

b) About x-axis $S_x = 2\pi \int_a^b y\sqrt{x'(t)^2 + y'(t)^2}\, dt$

$x = \frac{1}{4}t^2,\ y = t+2$ Interval $0 \le t \le 2$

$x'(t) = \frac{1}{2}t \quad y'(t) = 1$

$$S_x = 2\pi \int_0^2 t+2\sqrt{x'(t)^2 + y'(t)^2}$$

$$S_x = 2\pi \int_0^2 t+2\sqrt{t^2/4 + 1}\, dt \approx 159.6264$$

68) $x = t,\ y = 4-2t \quad 0 \le t \le 2$

$\frac{dx}{dt} = 1 \quad \frac{dy}{dt} = -2$

a) $S_x = 2\pi \int_0^2 (4-2t)\sqrt{1^2 + 2^2}\, dt$

$= 2\pi\sqrt{5}(4t - t^2)\Big]_0^2 = 8\pi\sqrt{5}$

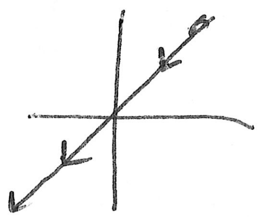
b) $S_y = 2\pi \int_0^2 t\sqrt{5}\, dt = \frac{2\pi\sqrt{5}t^2}{2}\Big]_0^2 = 4\pi\sqrt{5}$

72) $x = a\cos\theta \quad y = b\sin\theta \quad 0 \le \theta \le 2\pi$

$\frac{dx}{d\theta} = -a\sin\theta \quad \frac{dy}{d\theta} = b\cos\theta$

a) $S_x = \quad S =$

76) Sketch graph $x = g(t) \quad y = f(t)$, $\frac{dx}{dt} < 0$, $\frac{dy}{dt} < 0$



$x = -t$

$y = -t$

80)