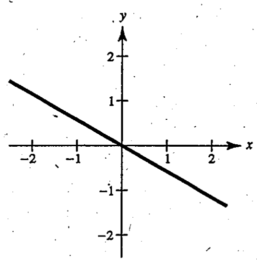
40. $\theta = \dfrac{5\pi}{6}$

$\tan\theta = \tan\dfrac{5\pi}{6}$

$\dfrac{y}{x} = -\dfrac{\sqrt{3}}{3}$

$y = -\dfrac{\sqrt{3}}{3}x$

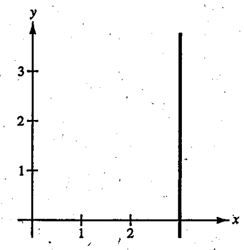


41. $r = 3\sec\theta$

$r\cos\theta = 3$

$x = 3$

$x - 3 = 0$

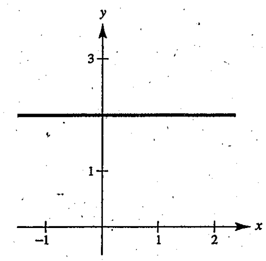


42. $r = 2\csc\theta$

$r\sin\theta = 2$

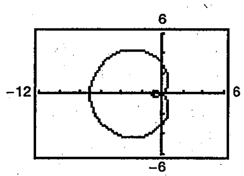
$y = 2$

$y - 2 = 0$



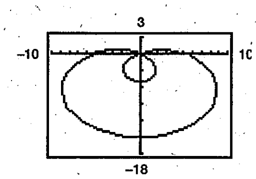
43. $r = 3 - 4\cos\theta$

$0 \le \theta < 2\pi$



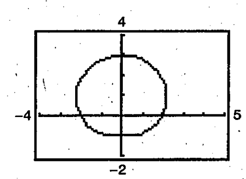
44. $r = 5(1 - 2\sin\theta)$

$0 \le \theta < 2\pi$



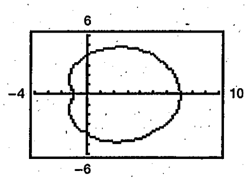
45. $r = 2 + \sin\theta$

$0 \le \theta < 2\pi$



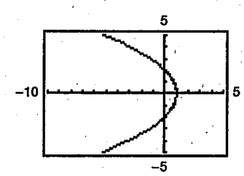
46. $r = 4 + 3\cos\theta$

$0 \le \theta < 2\pi$



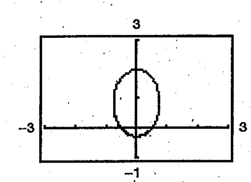
47. $r = \dfrac{2}{1 + \cos\theta}$

Traced out once on $-\pi < \theta < \pi$



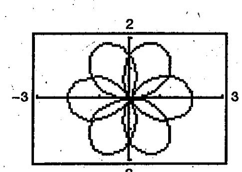
48. $r = \dfrac{2}{4 - 3\sin\theta}$

Traced out once on $0 \le \theta \le 2\pi$



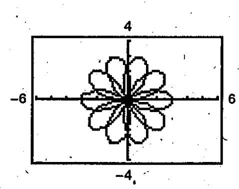
49. $r = 2\cos\left(\dfrac{3\theta}{2}\right)$

$0 \le \theta < 4\pi$



50. $r = 3\sin\left(\dfrac{5\theta}{2}\right)$

$0 \le \theta < 4\pi$

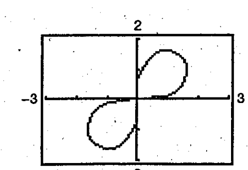


51. $r^2 = 4\sin 2\theta$

$r_1 = 2\sqrt{\sin 2\theta}$

$r_2 = -2\sqrt{\sin 2\theta}$

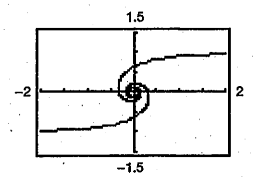
$0 \le \theta < \dfrac{\pi}{2}$



52. $r^2 = \dfrac{1}{\theta}$.

Graph as $r_1 = \dfrac{1}{\sqrt{\theta}}, r_2 = -\dfrac{1}{\sqrt{\theta}}$.

It is traced out once on $[0, \infty)$.



53.

$$\begin{aligned} r &= 2(h\cos\theta + k\sin\theta) \\ r^2 &= 2r(h\cos\theta + k\sin\theta) \\ r^2 &= 2[h(r\cos\theta) + k(r\sin\theta)] \\ x^2 + y^2 &= 2(hx + ky) \\ x^2 + y^2 - 2hx - 2ky &= 0 \\ (x^2 - 2hx + h^2) + (y^2 - 2ky + k^2) &= 0 + h^2 + k^2 \\ (x-h)^2 + (y-k)^2 &= h^2 + k^2 \end{aligned}$$

Radius: $\sqrt{h^2 + k^2}$

Center: (h, k)

54. (a) The rectangular coordinates of (r_1, θ_1) are $(r_1\cos\theta_1, r_1\sin\theta_1)$. The rectangular coordinates of (r_2, θ_2) are $(r_2\cos\theta_2, r_2\sin\theta_2)$.

$$\begin{aligned} d^2 &= (x_2 - x_1)^2 + (y_2 - y_1)^2 \\ &= (r_2\cos\theta_2 - r_1\cos\theta_1)^2 + (r_2\sin\theta_2 - r_1\sin\theta_1)^2 \\ &= r_2^2\cos^2\theta_2 - 2r_1r_2\cos\theta_1\cos\theta_2 + r_1^2\cos^2\theta_1 + r_2^2\sin^2\theta_2^2 - 2r_1r_2\sin\theta_1\sin\theta_2 + r_1^2\sin^2\theta_1 \\ &= r_2^2(\cos^2\theta_2 + \sin^2\theta_2) + r_1^2(\cos^2\theta_1 + \sin^2\theta_1) - 2r_1r_2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) \\ &= r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2) \\ d &= \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)} \end{aligned}$$

(b) If $\theta_1 = \theta_2$, the points lie on the same line passing through the origin. In this case,

$$\begin{aligned} d &= \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(0)} \\ &= \sqrt{(r_1 - r_2)^2} = |r_1 - r_2|. \end{aligned}$$

(c) If $\theta_1 - \theta_2 = 90°$, then $\cos(\theta_1 - \theta_2) = 0$ and $d = \sqrt{r_1^2 + r_2^2}$, the Pythagorean Theorem!

(d) Many answers are possible. For example, consider the two points $(r_1, \theta_1) = (1, 0)$ and $(r_2, \theta_2) = (2, \pi/2)$.

$$d = \sqrt{1 + 2^2 - 2(1)(2)\cos\left(0 - \frac{\pi}{2}\right)} = \sqrt{5}$$

Using $(r_1, \theta_1) = (-1, \pi)$ and $(r_2, \theta_2) = [2, (5\pi/2)]$, $d = \sqrt{(-1)^2 + (2)^2 - 2(-1)(2)\cos\left(\pi - \frac{5\pi}{2}\right)} = \sqrt{5}$.

You always obtain the same distance.

55. $\left(4, \frac{2\pi}{3}\right), \left(2, \frac{\pi}{6}\right)$

$$d = \sqrt{4^2 + 2^2 - 2(4)(2)\cos\left(\frac{2\pi}{3} - \frac{\pi}{6}\right)}$$
$$= \sqrt{20 - 16\cos\frac{\pi}{2}} = 2\sqrt{5} \approx 4.5$$

56. $\left(10, \frac{7\pi}{6}\right), (3, \pi)$

$$d = \sqrt{10^2 + 3^2 - 2(10)(3)\cos\left(\frac{7\pi}{6} - \pi\right)}$$
$$= \sqrt{109 - 60\cos\frac{\pi}{6}} = \sqrt{109 - 30\sqrt{3}} \approx 7.6$$

57. $(2, 0.5), (7, 1.2)$

$$d = \sqrt{2^2 + 7^2 - 2(2)(7)\cos(0.5 - 1.2)}$$
$$= \sqrt{53 - 28\cos(-0.7)} \approx 5.6$$

58. $(4, 2.5), (12, 1)$

$$d = \sqrt{4^2 + 12^2 - 2(4)(12)\cos(2.5 - 1)}$$
$$= \sqrt{160 - 96\cos 1.5} \approx 12.3$$

59. $r = 2 + 3\sin\theta$

$$\frac{dy}{dx} = \frac{3\cos\theta\sin\theta + \cos\theta(2 + 3\sin\theta)}{3\cos\theta\cos\theta - \sin\theta(2 + 3\sin\theta)}$$
$$= \frac{2\cos\theta(3\sin\theta + 1)}{3\cos 2\theta - 2\sin\theta} = \frac{2\cos\theta(3\sin\theta + 1)}{6\cos^2\theta - 2\sin\theta - 3}$$

At $\left(5, \frac{\pi}{2}\right)$, $\frac{dy}{dx} = 0$.

At $(2, \pi)$, $\frac{dy}{dx} = -\frac{2}{3}$.

At $\left(-1, \frac{3\pi}{2}\right)$, $\frac{dy}{dx} = 0$.

60. $r = 2(1 - \sin\theta)$

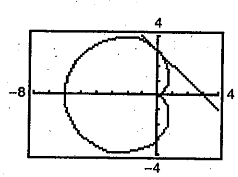
$$\frac{dy}{dx} = \frac{-2\cos\theta\sin\theta + 2\cos\theta(1 - \sin\theta)}{-2\cos\theta\cos\theta - 2\sin\theta(1 - \sin\theta)}$$

At $(2, 0)$, $\frac{dy}{dx} = -1$.

At $\left(3, \frac{7\pi}{6}\right)$, $\frac{dy}{dx}$ is undefined.

At $\left(4, \frac{3\pi}{2}\right)$, $\frac{dy}{dx} = 0$.

61. (a), (b) $r = 3(1 - \cos\theta)$



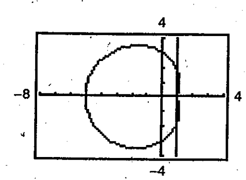
$(r, \theta) = \left(3, \frac{\pi}{2}\right) \Rightarrow (x, y) = (0, 3)$

Tangent line: $y - 3 = -1(x - 0)$

$y = -x + 3$

(c) At $\theta = \frac{\pi}{2}, \frac{dy}{dx} = -1.0$.

62. (a), (b) $r = 3 - 2\cos\theta$

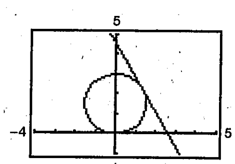


$(r, \theta) = (1, 0) \Rightarrow (x, y) = (1, 0)$

Tangent line: $x = 1$

(c) At $\theta = 0$, $\frac{dy}{dx}$ does not exist (vertical tangent).

63. (a), (b) $r = 3\sin\theta$



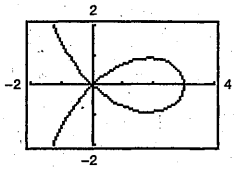
$(r, \theta) = \left(\frac{3\sqrt{3}}{2}, \frac{\pi}{3}\right) \Rightarrow (x, y) = \left(\frac{3\sqrt{3}}{4}, \frac{9}{4}\right)$

Tangent line: $y - \frac{9}{4} = -\sqrt{3}\left(x - \frac{3\sqrt{3}}{4}\right)$

$y = -\sqrt{3}x + \frac{9}{2}$

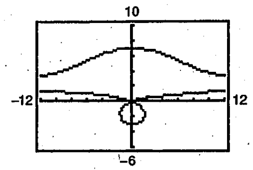
(c) At $\theta = \frac{\pi}{3}, \frac{dy}{dx} = -\sqrt{3} \approx -1.732$.

70. $r = 3\cos 2\theta \sec \theta$



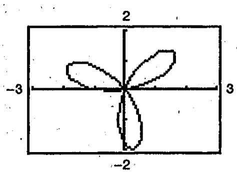
Horizontal tangents: $(r, \theta) = (2.061, \pm 0.452)$

71. $r = 2\csc \theta + 5$



Horizontal tangents: $(r, \theta) = \left(7, \frac{\pi}{2}\right), \left(3, \frac{3\pi}{2}\right)$

72. $r = 2\cos(3\theta - 2)$



Horizontal tangents:

$(r, \theta) = (1.894, 0.776), (1.755, 2.594), (1.998, -1.442), (-0.423, 0.072)$

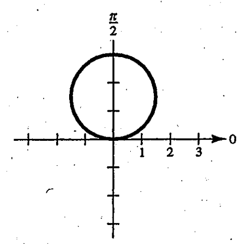
73.

$$r = 3\sin\theta$$
$$r^2 = 3r\sin\theta$$
$$x^2 + y^2 = 3y$$
$$x^2 + \left(y - \frac{3}{2}\right)^2 = \frac{9}{4}$$

Circle $r = \frac{3}{2}$

Center: $\left(0, \frac{3}{2}\right)$

Tangent at the pole: $\theta = 0$



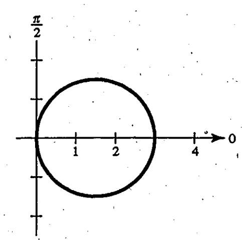
74.

$$r = 3\cos\theta$$
$$r^2 = 3r\cos\theta$$
$$x^2 + y^2 = 3x$$
$$\left(x - \frac{3}{2}\right)^2 + y^2 = \frac{9}{4}$$

Circle: $r = \frac{3}{2}$

Center: $\left(\frac{3}{2}, 0\right)$

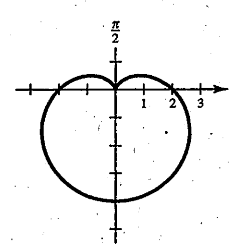
Tangent at pole: $\theta = \frac{\pi}{2}$



75. $r = 2(1 - \sin\theta)$

Cardioid

Symmetric to y-axis, $\theta = \frac{\pi}{2}$

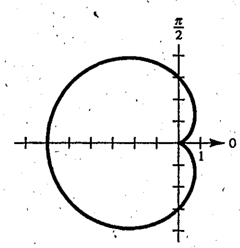


76. $r = 3(1 - \cos\theta)$

Cardioid

Symmetric to polar axis since r is a function of $\cos\theta$.

θ	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π
r	0	$\frac{3}{2}$	3	$\frac{9}{2}$	6



77. $r = 2\cos(3\theta)$

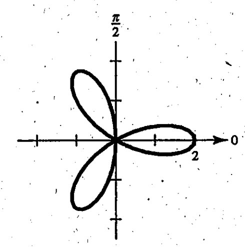
Rose curve with three petals

Symmetric to the polar axis

Relative extrema: $(2, 0), \left(-2, \frac{\pi}{3}\right), \left(2, \frac{2\pi}{3}\right)$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
r	2	0	$-\sqrt{2}$	-2	0	2	0	-2

Tangents at the pole: $\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$



78. $r = -\sin(5\theta)$

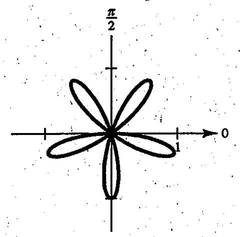
Rose curve with five petals

Symmetric to $\theta = \frac{\pi}{2}$

Relative extrema occur when

$$\frac{dr}{d\theta} = -5\cos(5\theta) = 0 \text{ at } \theta = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{5\pi}{10}, \frac{7\pi}{10}, \frac{9\pi}{10}.$$

Tangents at the pole: $\theta = 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}$



79. $r = 3\sin 2\theta$

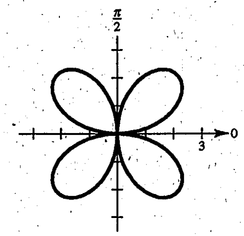
Rose curve with four petals

Symmetric to the polar axis, $\theta = \frac{\pi}{2}$, and pole

Relative extrema: $\left(\pm 3, \frac{\pi}{4}\right), \left(\pm 3, \frac{5\pi}{4}\right)$

Tangents at the pole: $\theta = 0, \frac{\pi}{2}$

($\theta = \pi$, $3\pi/2$ give the same tangents.)



80. $r = 3\cos 2\theta$

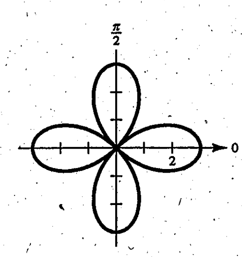
Rose curve with four petals

Symmetric to the polar axis, $\theta = \frac{\pi}{2}$, and pole

Relative extrema: $(3, 0), \left(-3, \frac{\pi}{2}\right), (3, \pi), \left(-3, \frac{3\pi}{2}\right)$

Tangents at the pole: $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

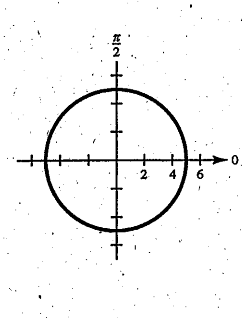
$\theta = \frac{5\pi}{4}$ and $\frac{7\pi}{4}$ given the same tangents.



81. $r = 5$

Circle radius: 5

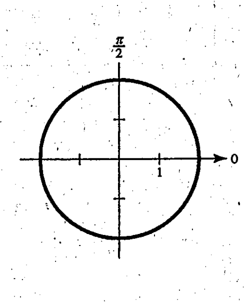
$x^2 + y^2 = 25$



82. $r = 2$

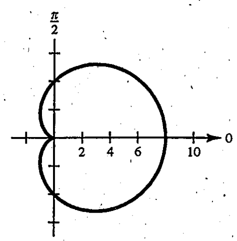
Circle radius: 2

$x^2 + y^2 = 4$



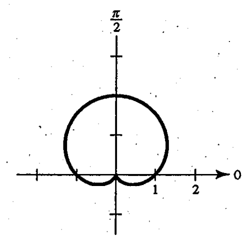
83. $r = 4(1 + \cos\theta)$

Cardioid



84. $r = 1 + \sin\theta$

Cardioid

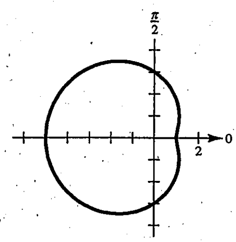


85. $r = 3 - 2\cos\theta$

Limaçon

Symmetric to polar axis

θ	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π
r	1	2	3	4	5

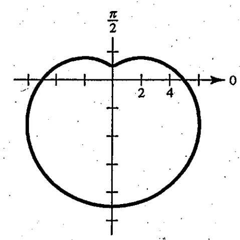


86. $r = 5 - 4\sin\theta$

Limaçon

Symmetric to $\theta = \frac{\pi}{2}$

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$
r	9	7	5	3	1

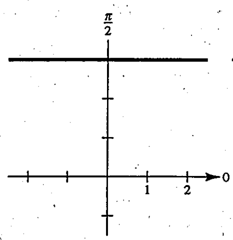


87. $r = 3\csc\theta$

$r\sin\theta = 3$

$y = 3$

Horizontal line

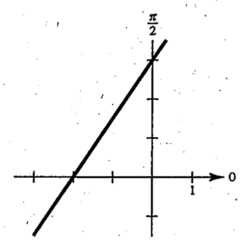


88. $r = \dfrac{6}{2\sin\theta - 3\cos\theta}$

$2r\sin\theta - 3r\cos\theta = 6$

$2y - 3x = 6$

Line



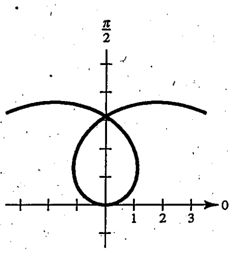
89. $r = 2\theta$

Spiral of Archimedes

Symmetric to $\theta = \frac{\pi}{2}$

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$
r	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π

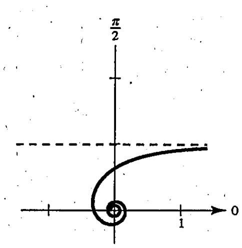
Tangent at the pole: $\theta = 0$



90. $r = \frac{1}{\theta}$

Hyperbolic spiral

θ	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$
r	$\frac{4}{\pi}$	$\frac{2}{\pi}$	$\frac{4}{3\pi}$	$\frac{1}{\pi}$	$\frac{4}{5\pi}$	$\frac{2}{3\pi}$



91. $r^2 = 4\cos(2\theta)$

$r = 2\sqrt{\cos 2\theta}, \quad 0 \le \theta \le 2\pi$

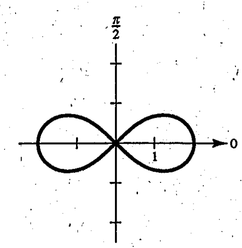
Lemniscate

Symmetric to the polar axis, $\theta = \dfrac{\pi}{2}$, and pole

Relative extrema: $(\pm 2, 0)$

θ	0	$\dfrac{\pi}{6}$	$\dfrac{\pi}{4}$
r	± 2	$\pm\sqrt{2}$	0

Tangents at the pole: $\theta = \dfrac{\pi}{4}, \dfrac{3\pi}{4}$



92. $r^2 = 4\sin\theta$

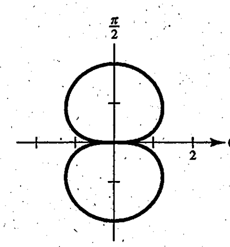
Lemniscate

Symmetric to the polar axis, $\theta = \dfrac{\pi}{2}$, and pole

Relative extrema: $\left(\pm 2, \dfrac{\pi}{2}\right)$

θ	0	$\dfrac{\pi}{6}$	$\dfrac{\pi}{2}$	$\dfrac{5\pi}{6}$	π
r	0	$\pm\sqrt{2}$	± 2	$\pm\sqrt{2}$	0

Tangent at the pole: $\theta = 0$



93. Since

$$r = 2 - \sec\theta = 2 - \frac{1}{\cos\theta},$$

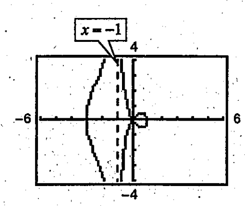
the graph has polar axis symmetry and the tangents at the pole are

$$\theta = \frac{\pi}{3}, \frac{-\pi}{3}.$$

Furthermore,

$$r \Longrightarrow -\infty \text{ as } \theta \Longrightarrow \frac{\pi}{2^-}$$

$$r \Longrightarrow \infty \text{ as } \theta \Longrightarrow -\frac{\pi}{2^+}.$$



Also, $r = 2 - \dfrac{1}{\cos\theta} = 2 - \dfrac{r}{r\cos\theta} = 2 - \dfrac{r}{x}$

$$rx = 2x - r$$

$$r = \frac{2x}{1 + x}.$$

Thus, $r \Longrightarrow \pm\infty$ as $x \Longrightarrow -1$.

94. Since

$$r = 2 + \csc\theta = 2 + \frac{1}{\sin\theta},$$

the graphs has symmetry with respect to $\theta = \pi/2$. Furthermore,

$$r \Longrightarrow \infty \text{ as } \theta \Longrightarrow 0^+$$

$$r \Longrightarrow \infty \text{ as } \theta \Longrightarrow \pi^-.$$

Also, $r = 2 + \dfrac{1}{\sin\theta} = 2 + \dfrac{r}{\sin\theta} = 2 + \dfrac{r}{y}$

$$ry = 2y + r$$

$$r = \frac{2y}{y - 1}.$$

Thus, $r \Longrightarrow \pm\infty$ as $y \Longrightarrow 1$.