10.4 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Polar-to-Rectangular Conversion **In Exercises 1–10, plot the point in polar coordinates and find the corresponding rectangular coordinates for the point.**

1. $\left(8, \frac{\pi}{2}\right)$
2. $\left(-2, \frac{5\pi}{3}\right)$
3. $\left(-4, -\frac{3\pi}{4}\right)$
4. $\left(0, -\frac{7\pi}{6}\right)$
5. $\left(7, \frac{5\pi}{4}\right)$
6. $\left(-2, \frac{11\pi}{6}\right)$
7. $(\sqrt{2}, 2.36)$
8. $(-3, -1.57)$
9. $(-4.5, 3.5)$
10. $(9.25, 1.2)$

Rectangular-to-Polar Conversion **In Exercises 11–20, the rectangular coordinates of a point are given. Plot the point and find *two* sets of polar coordinates for the point for $0 \le \theta < 2\pi$.**

11. $(2, 2)$
12. $(0, -6)$
13. $(-3, 4)$
14. $(4, -2)$
15. $(-1, -\sqrt{3})$
16. $(3, -\sqrt{3})$
17. $(3, -2)$
18. $(3\sqrt{2}, 3\sqrt{2})$
19. $\left(\frac{7}{4}, \frac{5}{2}\right)$
20. $(0, -5)$

21. **Plotting a Point** Plot the point $(4, 3.5)$ when the point is given in
(a) rectangular coordinates.
(b) polar coordinates.

22. **Graphical Reasoning**
(a) Set the window format of a graphing utility to rectangular coordinates and locate the cursor at any position off the axes. Move the cursor horizontally and vertically. Describe any changes in the displayed coordinates of the points.
(b) Set the window format of a graphing utility to polar coordinates and locate the cursor at any position off the axes. Move the cursor horizontally and vertically. Describe any changes in the displayed coordinates of the points.
(c) Why are the results in parts (a) and (b) different?

Rectangular-to-Polar Conversion **In Exercises 23–32, convert the rectangular equation to polar form and sketch its graph.**

23. $x^2 + y^2 = 9$
24. $x^2 - y^2 = 9$
25. $x^2 + y^2 = a^2$
26. $x^2 + y^2 - 2ax = 0$
27. $y = 8$
28. $x = 12$
29. $3x - y + 2 = 0$
30. $xy = 4$
31. $y^2 = 9x$
32. $(x^2 + y^2)^2 - 9(x^2 - y^2) = 0$

Polar-to-Rectangular Conversion **In Exercises 33–42, convert the polar equation to rectangular form and sketch its graph.**

33. $r = 4$
34. $r = -5$
35. $r = 3 \sin \theta$
36. $r = 5 \cos \theta$
37. $r = \theta$
38. $\theta = \frac{5\pi}{6}$
39. $r = 3 \sec \theta$
40. $r = 2 \csc \theta$
41. $r = \sec \theta \tan \theta$
42. $r = \cot \theta \csc \theta$

Graphing a Polar Equation **In Exercises 43–52, use a graphing utility to graph the polar equation. Find an interval for θ over which the graph is traced *only once*.**

43. $r = 2 - 5 \cos \theta$
44. $r = 3(1 - 4 \cos \theta)$
45. $r = 2 + \sin \theta$
46. $r = 4 + 3 \cos \theta$
47. $r = \frac{2}{1 + \cos \theta}$
48. $r = \frac{2}{4 - 3 \sin \theta}$
49. $r = 2 \cos\left(\frac{3\theta}{2}\right)$
50. $r = 3 \sin\left(\frac{5\theta}{2}\right)$
51. $r^2 = 4 \sin 2\theta$
52. $r^2 = \frac{1}{\theta}$

53. **Verifying a Polar Equation** Convert the equation
$$r = 2(h \cos \theta + k \sin \theta)$$
to rectangular form and verify that it is the equation of a circle. Find the radius and the rectangular coordinates of the center of the circle.

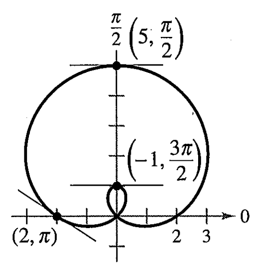
54. **Distance Formula**
(a) Verify that the Distance Formula for the distance between the two points (r_1, θ_1) and (r_2, θ_2) in polar coordinates is
$$d = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos(\theta_1 - \theta_2)}.$$
(b) Describe the positions of the points relative to each other for $\theta_1 = \theta_2$. Simplify the Distance Formula for this case. Is the simplification what you expected? Explain.
(c) Simplify the Distance Formula for $\theta_1 - \theta_2 = 90°$. Is the simplification what you expected? Explain.
(d) Choose two points on the polar coordinate system and find the distance between them. Then choose different polar representations of the same two points and apply the Distance Formula again. Discuss the result.

Distance Formula **In Exercises 55–58, use the result of Exercise 54 to approximate the distance between the two points in polar coordinates.**

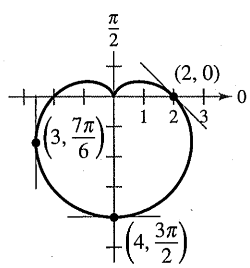
55. $\left(1, \frac{5\pi}{6}\right), \left(4, \frac{\pi}{3}\right)$
56. $\left(8, \frac{7\pi}{4}\right), (5, \pi)$
57. $(2, 0.5), (7, 1.2)$
58. $(4, 2.5), (12, 1)$

Finding Slopes of Tangent Lines In Exercises 59 and 60, find dy/dx and the slopes of the tangent lines shown on the graph of the polar equation.

59. $r = 2 + 3 \sin \theta$



60. $r = 2(1 - \sin \theta)$



Finding Slopes of Tangent Lines In Exercises 61–64, use a graphing utility to (a) graph the polar equation, (b) draw the tangent line at the given value of θ, and (c) find dy/dx at the given value of θ. (*Hint:* Let the increment between the values of θ equal $\pi/24$.)

61. $r = 3(1 - \cos \theta),\ \theta = \frac{\pi}{2}$

62. $r = 3 - 2 \cos \theta,\ \theta = 0$

63. $r = 3 \sin \theta,\ \theta = \frac{\pi}{3}$

64. $r = 4,\ \theta = \frac{\pi}{4}$

Horizontal and Vertical Tangency In Exercises 65 and 66, find the points of horizontal and vertical tangency (if any) to the polar curve.

65. $r = 1 - \sin \theta$

66. $r = a \sin \theta$

Horizontal Tangency In Exercises 67 and 68, find the points of horizontal tangency (if any) to the polar curve.

67. $r = 2 \csc \theta + 3$

68. $r = a \sin \theta \cos^2 \theta$

Tangent Lines at the Pole In Exercises 69–76, sketch a graph of the polar equation and find the tangents at the pole.

69. $r = 5 \sin \theta$

70. $r = 5 \cos \theta$

71. $r = 2(1 - \sin \theta)$

72. $r = 3(1 - \cos \theta)$

73. $r = 4 \cos 3\theta$

74. $r = -\sin 5\theta$

75. $r = 3 \sin 2\theta$

76. $r = 3 \cos 2\theta$

Sketching a Polar Graph In Exercises 77–88, sketch a graph of the polar equation.

77. $r = 8$

78. $r = 1$

79. $r = 4(1 + \cos \theta)$

80. $r = 1 + \sin \theta$

81. $r = 3 - 2 \cos \theta$

82. $r = 5 - 4 \sin \theta$

83. $r = 3 \csc \theta$

84. $r = \dfrac{6}{2 \sin \theta - 3 \cos \theta}$

85. $r = 2\theta$

86. $r = \dfrac{1}{\theta}$

87. $r^2 = 4 \cos 2\theta$

88. $r^2 = 4 \sin \theta$

Asymptote In Exercises 89–92, use a graphing utility to graph the equation and show that the given line is an asymptote of the graph.

	Name of Graph	Polar Equation	Asymptote
89.	Conchoid	$r = 2 - \sec \theta$	$x = -1$
90.	Conchoid	$r = 2 + \csc \theta$	$y = 1$
91.	Hyperbolic spiral	$r = 2/\theta$	$y = 2$
92.	Strophoid	$r = 2 \cos 2\theta \sec \theta$	$x = -2$

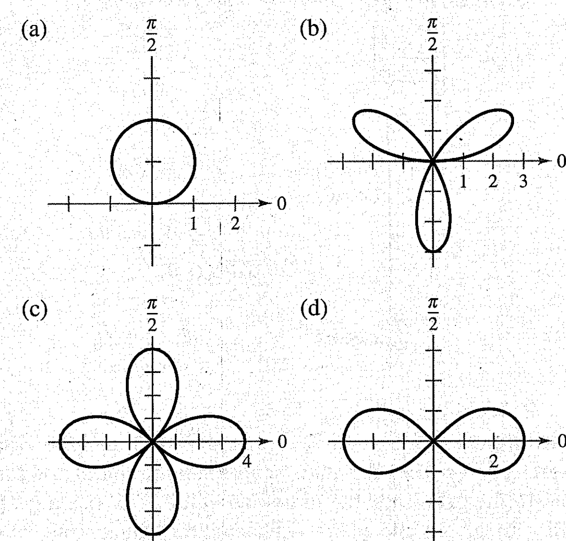
WRITING ABOUT CONCEPTS

93. Comparing Coordinate Systems Describe the differences between the rectangular coordinate system and the polar coordinate system.

94. Coordinate Conversion Give the equations for the coordinate conversion from rectangular to polar coordinates and vice versa.

95. Tangent Lines How are the slopes of tangent lines determined in polar coordinates? What are tangent lines at the pole and how are they determined?

96. HOW DO YOU SEE IT? Identify each special polar graph and write its equation.



97. Sketching a Graph Sketch the graph of $r = 4 \sin \theta$ over each interval.

(a) $0 \le \theta \le \frac{\pi}{2}$ (b) $\frac{\pi}{2} \le \theta \le \pi$ (c) $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

98. Think About It Use a graphing utility to graph the polar equation $r = 6[1 + \cos(\theta - \phi)]$ for (a) $\phi = 0$, (b) $\phi = \pi/4$, and (c) $\phi = \pi/2$. Use the graphs to describe the effect of the angle ϕ. Write the equation as a function of $\sin \theta$ for part (c).

99. Rotated Curve Verify that if the curve whose polar equation is $r = f(\theta)$ is rotated about the pole through an angle ϕ, then an equation for the rotated curve is $r = f(\theta - \phi)$.

100. Rotated Curve The polar form of an equation of a curve is $r = f(\sin \theta)$. Show that the form becomes

(a) $r = f(-\cos \theta)$ if the curve is rotated counterclockwise $\pi/2$ radians about the pole.

(b) $r = f(-\sin \theta)$ if the curve is rotated counterclockwise π radians about the pole.

(c) $r = f(\cos \theta)$ if the curve is rotated counterclockwise $3\pi/2$ radians about the pole.

Rotated Curve In Exercises 101–104, use the results of Exercises 99 and 100.

101. Write an equation for the limaçon $r = 2 - \sin \theta$ after it has been rotated by the given amount. Use a graphing utility to graph the rotated limaçon for (a) $\theta = \pi/4$, (b) $\theta = \pi/2$, (c) $\theta = \pi$, and (d) $\theta = 3\pi/2$.

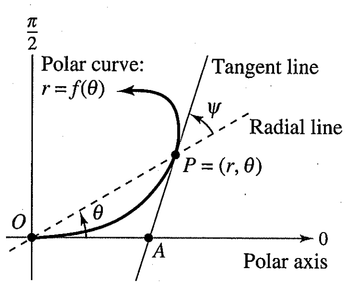
102. Write an equation for the rose curve $r = 2 \sin 2\theta$ after it has been rotated by the given amount. Verify the results by using a graphing utility to graph the rotated rose curve for (a) $\theta = \pi/6$, (b) $\theta = \pi/2$, (c) $\theta = 2\pi/3$, and (d) $\theta = \pi$.

103. Sketch the graph of each equation.

(a) $r = 1 - \sin \theta$ (b) $r = 1 - \sin\left(\theta - \frac{\pi}{4}\right)$

104. Prove that the tangent of the angle $\psi \ (0 \le \psi \le \pi/2)$ between the radial line and the tangent line at the point (r, θ) on the graph of $r = f(\theta)$ (see figure) is given by

$$\tan \psi = \left|\frac{r}{dr/d\theta}\right|.$$



Finding an Angle In Exercises 105–110, use the result of Exercise 104 to find the angle ψ between the radial and tangent lines to the graph for the indicated value of θ. Use a graphing utility to graph the polar equation, the radial line, and the tangent line for the indicated value of θ. Identify the angle ψ.

	Polar Equation	Value of θ
105.	$r = 2(1 - \cos \theta)$	$\theta = \pi$
106.	$r = 3(1 - \cos \theta)$	$\theta = \frac{3\pi}{4}$
107.	$r = 2 \cos 3\theta$	$\theta = \frac{\pi}{4}$
108.	$r = 4 \sin 2\theta$	$\theta = \frac{\pi}{6}$
109.	$r = \frac{6}{1 - \cos \theta}$	$\theta = \frac{2\pi}{3}$
110.	$r = 5$	$\theta = \frac{\pi}{6}$

True or False? In Exercises 111–114, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

111. If (r_1, θ_1) and (r_2, θ_2) represent the same point on the polar coordinate system, then $|r_1| = |r_2|$.

112. If (r, θ_1) and (r, θ_2) represent the same point on the polar coordinate system, then $\theta_1 = \theta_2 + 2\pi n$ for some integer n.

113. If $x > 0$, then the point (x, y) on the rectangular coordinate system can be represented by (r, θ) on the polar coordinate system, where $r = \sqrt{x^2 + y^2}$ and $\theta = \arctan(y/x)$.

114. The polar equations $r = \sin 2\theta$, $r = -\sin 2\theta$, and $r = \sin(-2\theta)$ all have the same graph.

SECTION PROJECT

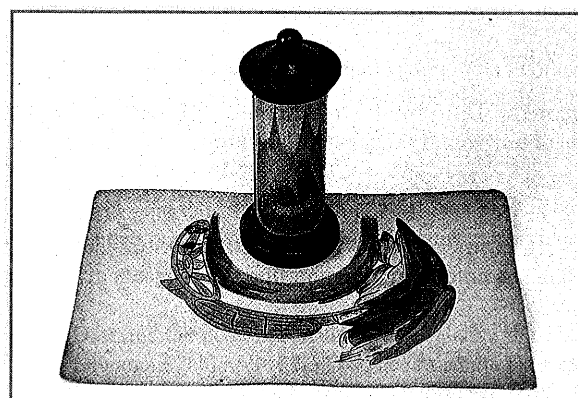
Anamorphic Art

Anamorphic art appears distorted, but when the art is viewed from a particular point or is viewed with a device such as a mirror, it appears to be normal. Use the anamorphic transformations

$$r = y + 16 \quad \text{and} \quad \theta = -\frac{\pi}{8}x, \quad -\frac{3\pi}{4} \le \theta \le \frac{3\pi}{4}$$

to sketch the transformed polar image of the rectangular graph. When the reflection (in a cylindrical mirror centered at the pole) of each polar image is viewed from the polar axis, the viewer will see the original rectangular image.

(a) $y = 3$ (b) $x = 2$ (c) $y = x + 5$ (d) $x^2 + (y - 5)^2 = 5^2$



This example of anamorphic art is from the Millington-Barnard Collection at the University of Mississippi. When the reflection of the transformed "polar painting" is viewed in the mirror, the viewer sees the distorted art in its proper proportions.

■ **FOR FURTHER INFORMATION** For more information on anamorphic art, see the article "Anamorphisms" by Philip Hickin in the *Mathematical Gazette.*