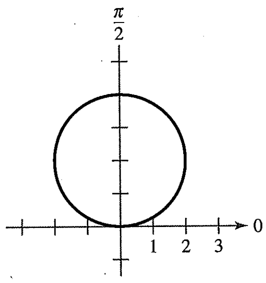
10.5 Exercises

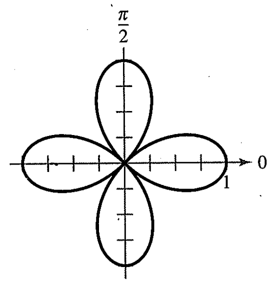
See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Area of a Polar Region In Exercises 1–4, write an integral that represents the area of the shaded region of the figure. Do not evaluate the integral.

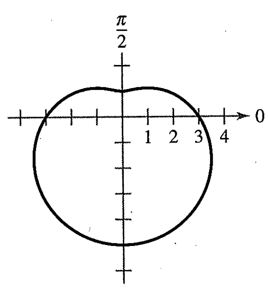
1. $r = 4\sin\theta$



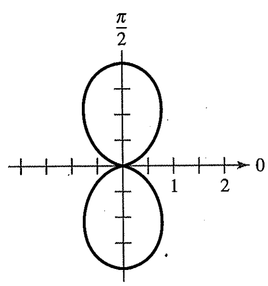
2. $r = \cos 2\theta$



3. $r = 3 - 2\sin\theta$



4. $r = 1 - \cos 2\theta$



Finding the Area of a Polar Region In Exercises 5–16, find the area of the region.

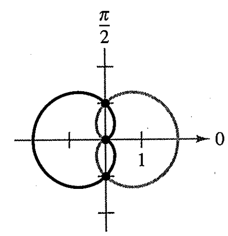
5. Interior of $r = 6\sin\theta$
6. Interior of $r = 3\cos\theta$
7. One petal of $r = 2\cos 3\theta$
8. One petal of $r = 4\sin 3\theta$
9. One petal of $r = \sin 2\theta$
10. One petal of $r = \cos 5\theta$
11. Interior of $r = 1 - \sin\theta$
12. Interior of $r = 1 - \sin\theta$ (above the polar axis)
13. Interior of $r = 5 + 2\sin\theta$
14. Interior of $r = 4 - 4\cos\theta$
15. Interior of $r^2 = 4\cos 2\theta$
16. Interior of $r^2 = 6\sin 2\theta$

Finding the Area of a Polar Region In Exercises 17–24, use a graphing utility to graph the polar equation. Find the area of the given region analytically.

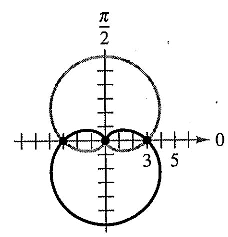
17. Inner loop of $r = 1 + 2\cos\theta$
18. Inner loop of $r = 2 - 4\cos\theta$
19. Inner loop of $r = 1 + 2\sin\theta$
20. Inner loop of $r = 4 - 6\sin\theta$
21. Between the loops of $r = 1 + 2\cos\theta$
22. Between the loops of $r = 2(1 + 2\sin\theta)$
23. Between the loops of $r = 3 - 6\sin\theta$
24. Between the loops of $r = \frac{1}{2} + \cos\theta$

Finding Points of Intersection In Exercises 25–32, find the points of intersection of the graphs of the equations.

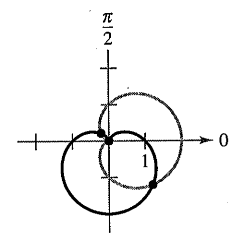
25. $r = 1 + \cos\theta$
$r = 1 - \cos\theta$



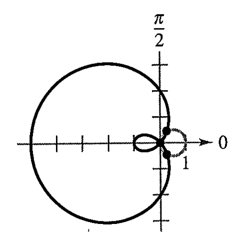
26. $r = 3(1 + \sin\theta)$
$r = 3(1 - \sin\theta)$



27. $r = 1 + \cos\theta$
$r = 1 - \sin\theta$



28. $r = 2 - 3\cos\theta$
$r = \cos\theta$



29. $r = 4 - 5\sin\theta$
$r = 3\sin\theta$

30. $r = 3 + \sin\theta$
$r = 2\csc\theta$

31. $r = \dfrac{\theta}{2}$
$r = 2$

32. $\theta = \dfrac{\pi}{4}$
$r = 2$

Writing In Exercises 33 and 34, use a graphing utility to graph the polar equations and approximate the points of intersection of the graphs. Watch the graphs as they are traced in the viewing window. Explain why the pole is not a point of intersection obtained by solving the equations simultaneously.

33. $r = \cos\theta$
$r = 2 - 3\sin\theta$

34. $r = 4\sin\theta$
$r = 2(1 + \sin\theta)$

Finding the Area of a Polar Region Between Two Curves In Exercises 35–42, use a graphing utility to graph the polar equations. Find the area of the given region analytically.

35. Common interior of $r = 4\sin 2\theta$ and $r = 2$
36. Common interior of $r = 2(1 + \cos\theta)$ and $r = 2(1 - \cos\theta)$
37. Common interior of $r = 3 - 2\sin\theta$ and $r = -3 + 2\sin\theta$
38. Common interior of $r = 5 - 3\sin\theta$ and $r = 5 - 3\cos\theta$
39. Common interior of $r = 4\sin\theta$ and $r = 2$
40. Common interior of $r = 2\cos\theta$ and $r = 2\sin\theta$
41. Inside $r = 2\cos\theta$ and outside $r = 1$
42. Inside $r = 3\sin\theta$ and outside $r = 1 + \sin\theta$

Finding the Area of a Polar Region Between Two Curves **In Exercises 43–46, find the area of the region.**

43. Inside $r = a(1 + \cos\theta)$ and outside $r = a\cos\theta$

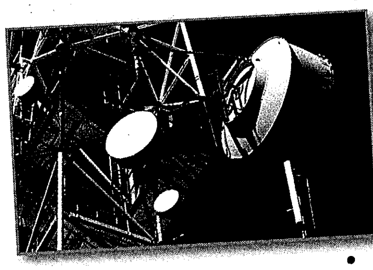
44. Inside $r = 2a\cos\theta$ and outside $r = a$

45. Common interior of $r = a(1 + \cos\theta)$ and $r = a\sin\theta$

46. Common interior of $r = a\cos\theta$ and $r = a\sin\theta$, where $a > 0$

47. Antenna Radiation

The radiation from a transmitting antenna is not uniform in all directions. The intensity from a particular antenna is modeled by $r = a\cos^2\theta$.



(a) Convert the polar equation to rectangular form.

(b) Use a graphing utility to graph the model for $a = 4$ and $a = 6$.

(c) Find the area of the geographical region between the two curves in part (b).

48. **Area** The area inside one or more of the three interlocking circles

$$r = 2a\cos\theta, \quad r = 2a\sin\theta, \quad \text{and} \quad r = a$$

is divided into seven regions. Find the area of each region.

49. **Conjecture** Find the area of the region enclosed by

$$r = a\cos(n\theta)$$

for $n = 1, 2, 3, \ldots$. Use the results to make a conjecture about the area enclosed by the function when n is even and when n is odd.

50. **Area** Sketch the strophoid

$$r = \sec\theta - 2\cos\theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}.$$

Convert this equation to rectangular coordinates. Find the area enclosed by the loop.

Finding the Arc Length of a Polar Curve **In Exercises 51–56, find the length of the curve over the given interval.**

	Polar Equation	Interval
51.	$r = 8$	$0 \le \theta \le 2\pi$
52.	$r = a$	$0 \le \theta \le 2\pi$
53.	$r = 4\sin\theta$	$0 \le \theta \le \pi$
54.	$r = 2a\cos\theta$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
55.	$r = 1 + \sin\theta$	$0 \le \theta \le 2\pi$
56.	$r = 8(1 + \cos\theta)$	$0 \le \theta \le 2\pi$

Finding the Arc Length of a Polar Curve **In Exercises 57–62, use a graphing utility to graph the polar equation over the given interval. Use the integration capabilities of the graphing utility to approximate the length of the curve accurate to two decimal places.**

	Polar Equation	Interval
57.	$r = 2\theta$	$0 \le \theta \le \frac{\pi}{2}$
58.	$r = \sec\theta$	$0 \le \theta \le \frac{\pi}{3}$
59.	$r = \frac{1}{\theta}$	$\pi \le \theta \le 2\pi$
60.	$r = e^{\theta}$	$0 \le \theta \le \pi$
61.	$r = \sin(3\cos\theta)$	$0 \le \theta \le \pi$
62.	$r = 2\sin(2\cos\theta)$	$0 \le \theta \le \pi$

Finding the Area of a Surface of Revolution **In Exercises 63–66, find the area of the surface formed by revolving the curve about the given line.**

	Polar Equation	Interval	Axis of Revolution
63.	$r = 6\cos\theta$	$0 \le \theta \le \frac{\pi}{2}$	Polar axis
64.	$r = a\cos\theta$	$0 \le \theta \le \frac{\pi}{2}$	$\theta = \frac{\pi}{2}$
65.	$r = e^{a\theta}$	$0 \le \theta \le \frac{\pi}{2}$	$\theta = \frac{\pi}{2}$
66.	$r = a(1 + \cos\theta)$	$0 \le \theta \le \pi$	Polar axis

Finding the Area of a Surface of Revolution **In Exercises 67 and 68, use the integration capabilities of a graphing utility to approximate, to two decimal places, the area of the surface formed by revolving the curve about the polar axis.**

67. $r = 4\cos 2\theta, \quad 0 \le \theta \le \frac{\pi}{4}$

68. $r = \theta, \quad 0 \le \theta \le \pi$

WRITING ABOUT CONCEPTS

69. **Points of Intersection** Explain why finding points of intersection of polar graphs may require further analysis beyond solving two equations simultaneously.

70. **Area of a Surface of Revolution** Give the integral formulas for the area of the surface of revolution formed when the graph of $r = f(\theta)$ is revolved about

(a) the polar axis.

(b) the line $\theta = \pi/2$.

71. **Area of a Region** For each polar equation, sketch its graph, determine the interval that traces the graph only once, and find the area of the region bounded by the graph using a geometric formula and integration.

(a) $r = 10\cos\theta$ (b) $r = 5\sin\theta$

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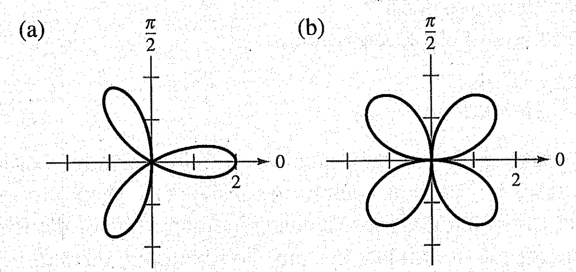
72. HOW DO YOU SEE IT? Which graph, traced out only once, has a larger arc length? Explain your reasoning.



73. Surface Area of a Torus Find the surface area of the torus generated by revolving the circle given by $r = 2$ about the line $r = 5 \sec \theta$.

74. Surface Area of a Torus Find the surface area of the torus generated by revolving the circle given by $r = a$ about the line $r = b \sec \theta$, where $0 < a < b$.

75. Approximating Area Consider the circle $r = 8 \cos \theta$.

(a) Find the area of the circle.

(b) Complete the table giving the areas A of the sectors of the circle between $\theta = 0$ and the values of θ in the table.

θ	0.2	0.4	0.6	0.8	1.0	1.2	1.4
A							

(c) Use the table in part (b) to approximate the values of θ for which the sector of the circle composes $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of the total area of the circle.

(d) Use a graphing utility to approximate, to two decimal places, the angles θ for which the sector of the circle composes $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of the total area of the circle.

(e) Do the results of part (d) depend on the radius of the circle? Explain.

76. Approximating Area Consider the circle $r = 3 \sin \theta$.

(a) Find the area of the circle.

(b) Complete the table giving the areas A of the sectors of the circle between $\theta = 0$ and the values of θ in the table.

θ	0.2	0.4	0.6	0.8	1.0	1.2	1.4
A							

(c) Use the table in part (b) to approximate the values of θ for which the sector of the circle composes $\frac{1}{8}$, $\frac{1}{4}$, and $\frac{1}{2}$ of the total area of the circle.

(d) Use a graphing utility to approximate, to two decimal places, the angles θ for which the sector of the circle composes $\frac{1}{8}$, $\frac{1}{4}$, and $\frac{1}{2}$ of the total area of the circle.

77. Conic What conic section does the polar equation $r = a \sin \theta + b \cos \theta$ represent?

78. Area Find the area of the circle given by

$$r = \sin \theta + \cos \theta.$$

Check your result by converting the polar equation to rectangular form, then using the formula for the area of a circle.

79. Spiral of Archimedes The curve represented by the equation $r = a\theta$, where a is a constant, is called the spiral of Archimedes.

(a) Use a graphing utility to graph $r = \theta$, where $\theta \geq 0$. What happens to the graph of $r = a\theta$ as a increases? What happens if $\theta \leq 0$?

(b) Determine the points on the spiral $r = a\theta$ $(a > 0, \theta \geq 0)$, where the curve crosses the polar axis.

(c) Find the length of $r = \theta$ over the interval $0 \leq \theta \leq 2\pi$.

(d) Find the area under the curve $r = \theta$ for $0 \leq \theta \leq 2\pi$.

80. Logarithmic Spiral The curve represented by the equation $r = ae^{b\theta}$, where a and b are constants, is called a **logarithmic spiral.** The figure shows the graph of $r = e^{\theta/6}$, $-2\pi \leq \theta \leq 2\pi$. Find the area of the shaded region.

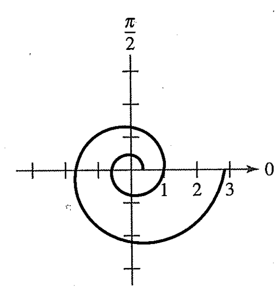


Figure for 80

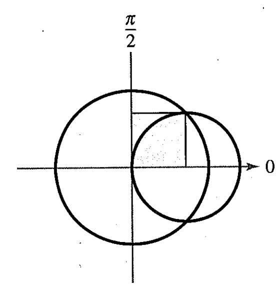


Figure for 81

81. Area The larger circle in the figure is the graph of $r = 1$. Find the polar equation of the smaller circle such that the shaded regions are equal.

82. Folium of Descartes A curve called the **folium of Descartes** can be represented by the parametric equations

$$x = \frac{3t}{1 + t^3} \quad \text{and} \quad y = \frac{3t^2}{1 + t^3}.$$

(a) Convert the parametric equations to polar form.

(b) Sketch the graph of the polar equation from part (a).

(c) Use a graphing utility to approximate the area enclosed by the loop of the curve.

True or False? **In Exercises 83 and 84, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

83. If $f(\theta) > 0$ for all θ and $g(\theta) < 0$ for all θ, then the graphs of $r = f(\theta)$ and $r = g(\theta)$ do not intersect.

84. If $f(\theta) = g(\theta)$ for $\theta = 0$, $\pi/2$, and $3\pi/2$, then the graphs of $r = f(\theta)$ and $r = g(\theta)$ have at least four points of intersection.

85. Arc Length in Polar Form Use the formula for the arc length of a curve in parametric form to derive the formula for the arc length of a polar curve.