17. $r = 1 + \cos\theta$

$r = 1 - \cos\theta$

Solving simultaneously,

$$1 + \cos\theta = 1 - \cos\theta$$
$$2\cos\theta = 0$$
$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}.$$

Replacing r by $-r$ and θ by $\theta + \pi$ in the first equation and solving, $-1 + \cos\theta = 1 - \cos\theta$, $\cos\theta = 1$, $\theta = 0$. Both curves pass through the pole, $(0, \pi)$, and $(0, 0)$, respectively.

Points of intersection: $\left(1, \frac{\pi}{2}\right), \left(1, \frac{3\pi}{2}\right)$, $(0, 0)$

18. $r = 3(1 + \sin\theta)$

$r = 3(1 - \sin\theta)$

Solving simultaneously,

$$3(1 + \sin\theta) = 3(1 - \sin\theta)$$
$$2\sin\theta = 0$$
$$\theta = 0, \pi.$$

Replacing r by $-r$ and θ by $\theta + \pi$ in the first equation and solving, $-3(1 - \sin\theta) = 3(1 - \sin\theta)$, $\sin\theta = 1$, $\theta = \pi/2$. Both curves pass through the pole, $(0, 3\pi/2)$, and $(0, \pi/2)$, respectively.

Points of intersection: $(3, 0)$, $(3, \pi)$, $(0, 0)$

19. $r = 1 + \cos\theta$

$r = 1 - \sin\theta$

Solving simultaneously,

$$1 + \cos\theta = 1 - \sin\theta$$
$$\cos\theta = -\sin\theta$$
$$\tan\theta = -1$$
$$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}.$$

Replacing r by $-r$ and θ by $\theta + \pi$ in the first equation and solving, $-1 + \cos\theta = 1 - \sin\theta$, $\sin\theta + \cos\theta = 2$, which has no solution. Both curves pass through the pole, $(0, \pi)$, and $(0, \pi/2)$, respectively.

Points of intersection: $\left(\frac{2 - \sqrt{2}}{2}, \frac{3\pi}{4}\right), \left(\frac{2 + \sqrt{2}}{2}, \frac{7\pi}{4}\right)$, $(0, 0)$

20. $r = 2 - 3\cos\theta$

$r = \cos\theta$

Solving simultaneously,

$$2 - 3\cos\theta = \cos\theta$$
$$\cos\theta = \frac{1}{2}$$
$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}.$$

Both curves pass through the pole, $(0, \arccos 2/3)$, and $(0, \pi/2)$, respectively.

Points of intersection: $\left(\frac{1}{2}, \frac{\pi}{3}\right), \left(\frac{1}{2}, \frac{5\pi}{3}\right)$, $(0, 0)$

21. $r = 4 - 5\sin\theta$

$r = 3\sin\theta$

Solving simultaneously,

$$4 - 5\sin\theta = 3\sin\theta$$
$$\sin\theta = \frac{1}{2}$$
$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}.$$

Both curves pass through the pole, $(0, \arcsin 4/5)$, and $(0, 0)$, respectively.

Points of intersection: $\left(\frac{3}{2}, \frac{\pi}{6}\right), \left(\frac{3}{2}, \frac{5\pi}{6}\right)$, $(0, 0)$

22. $r = 1 + \cos\theta$

$r = 3\cos\theta$

Solving simultaneously,

$$1 + \cos\theta = 3\cos\theta$$
$$\cos\theta = \frac{1}{2}$$
$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}.$$

Both curves pass through the pole, $(0, \pi)$, and $(0, \pi/2)$, respectively.

Points of intersection: $\left(\frac{3}{2}, \frac{\pi}{3}\right), \left(\frac{3}{2}, \frac{5\pi}{3}\right)$, $(0, 0)$

23. $r = \dfrac{\theta}{2}$

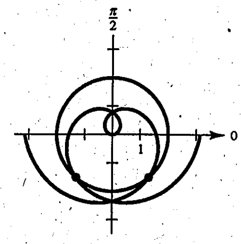
$r = 2$

Solving simultaneously, we have

$\theta/2 = 2, \theta = 4.$

Points of intersection:

$(2, 4), (-2, -4)$



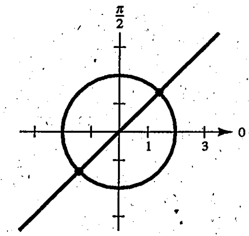
24. $\theta = \dfrac{\pi}{4}$

$r = 2$

Line of slope 1 passing through the pole and a circle of radius 2 centered at the pole.

Points of intersection:

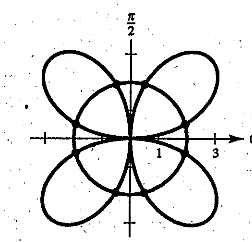
$\left(2, \dfrac{\pi}{4}\right), \left(-2, \dfrac{\pi}{4}\right)$



25. $r = 4 \sin 2\theta$

$r = 2$

$r = 4 \sin 2\theta$ is the equation of a rose curve with four petals and is symmetric to the polar axis, $\theta = \pi/2$, and the pole. Also, $r = 2$ is the equation of a circle of radius 2 centered at the pole. Solving simultaneously,



$$4 \sin 2\theta = 2$$

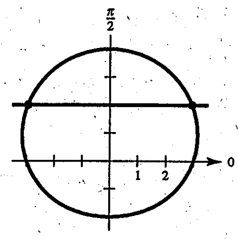
$$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\theta = \frac{\pi}{12}, \frac{5\pi}{12}.$$

Therefore, the points of intersection for one petal are $(2, \pi/12)$ and $(2, 5\pi/12)$. By symmetry, the other points of intersection are $(2, 7\pi/12)$, $(2, 11\pi/12)$, $(2, 13\pi/12)$, $(2, 17\pi/12)$, $(2, 19\pi/12)$, and $(2, 23\pi/12)$.

26. $r = 3 + \sin \theta$

$r = 2 \csc \theta$



The graph of $r = 3 + \sin \theta$ is a limaçon symmetric to $\theta = \pi/2$, and the graph of $r = 2 \csc \theta$ is the horizontal line $y = 2$. Therefore, there are two points of intersection. Solving simultaneously,

$$3 + \sin \theta = 2 \csc \theta$$

$$\sin^2 \theta + 3 \sin \theta - 2 = 0$$

$$\sin \theta = \frac{-3 \pm \sqrt{17}}{2}$$

$$\theta = \arcsin\left(\frac{\sqrt{17} - 3}{2}\right) \approx 0.596.$$

Points of intersection:

$\left(\dfrac{\sqrt{17} + 3}{2}, \arcsin\left(\dfrac{\sqrt{17} - 3}{2}\right)\right),$

$\left(\dfrac{\sqrt{17} + 3}{2}, \pi - \arcsin\left(\dfrac{\sqrt{17} - 3}{2}\right)\right),$

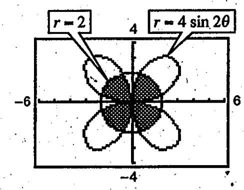
$(3.56, 0.596), (3.56, 2.545)$

31. From Exercise 25, the points of intersection for one petal are $(2, \pi/12)$ and $(2, 5\pi/12)$. The area within one petal is

$$A = \frac{1}{2}\int_0^{\pi/12} (4\sin 2\theta)^2\, d\theta + \frac{1}{2}\int_{\pi/12}^{5\pi/12} (2)^2\, d\theta + \frac{1}{2}\int_{5\pi/12}^{\pi/2} (4\sin 2\theta)^2\, d\theta$$

$$= 16\int_0^{\pi/12} \sin^2(2\theta)\, d\theta + 2\int_{\pi/12}^{5\pi/12} d\theta \text{ (by symmetry of the petal)}$$

$$= 8\left[\theta - \frac{1}{4}\sin 4\theta\right]_0^{\pi/12} + \left[2\theta\right]_{\pi/12}^{5\pi/12} = \frac{4\pi}{3} - \sqrt{3}.$$

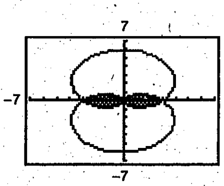


$$\text{Total area} = 4\left(\frac{4\pi}{3} - \sqrt{3}\right) = \frac{16\pi}{3} - 4\sqrt{3} = \frac{4}{3}(4\pi - 3\sqrt{3})$$

32. $A = 4\left[\frac{1}{2}\int_0^{\pi/2} 9(1 - \sin\theta)^2\, d\theta\right]$

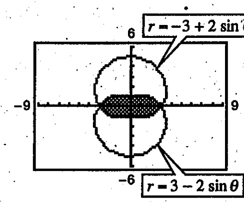
$$= 18\int_0^{\pi/2} (1 - \sin\theta)^2\, d\theta = \frac{9}{2}(3\pi - 8)$$

(from Exercise 14)



33. $A = 4\left[\frac{1}{2}\int_0^{\pi/2} (3 - 2\sin\theta)^2\, d\theta\right]$

$$= 2\left[11\theta + 12\cos\theta - \sin(2\theta)\right]_0^{\pi/2} = 11\pi - 24$$



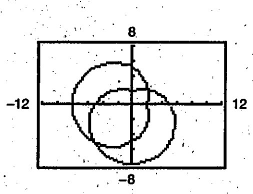
34. $r = 5 - 3\sin\theta$ and $r = 5 - 3\cos\theta$ intersect at $\theta = \pi/4$ and $\pi = 5\pi/4$.

$$A = 2\left[\frac{1}{2}\int_{\pi/4}^{5\pi/4} (5 - 3\sin\theta)^2\, d\theta\right]$$

$$= \left[\frac{59}{2}\theta + 30\cos\theta - \frac{9}{4}\sin 2\theta\right]_{\pi/4}^{5\pi/4}$$

$$= \left(\frac{59}{2}\left(\frac{5\pi}{4}\right) - 30\frac{\sqrt{2}}{2} - \frac{9}{4}\right) - \left(\frac{59}{2}\left(\frac{\pi}{4}\right) + 30\frac{\sqrt{2}}{2} - \frac{9}{4}\right)$$

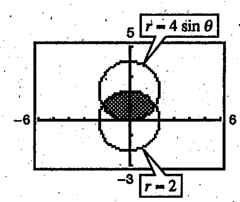
$$= \frac{59\pi}{2} - 30\sqrt{2} \approx 50.251$$



35. $A = 2\left[\frac{1}{2}\int_0^{\pi/6} (4\sin\theta)^2\, d\theta + \frac{1}{2}\int_{\pi/6}^{\pi/2} (2)^2\, d\theta\right]$

$$= 16\left[\frac{1}{2}\theta - \frac{1}{4}\sin(2\theta)\right]_0^{\pi/6} + \left[4\theta\right]_{\pi/6}^{\pi/2}$$

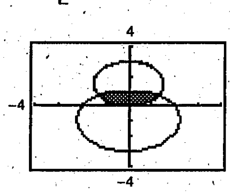
$$= \frac{8\pi}{3} - 2\sqrt{3} = \frac{2}{3}(4\pi - 3\sqrt{3})$$



36. $A = 2\left[\frac{1}{2}\int_{\pi/6}^{\pi/2} (3\sin\theta)^2\, d\theta - \frac{1}{2}\int_{\pi/6}^{\pi/2} (2 - \sin\theta)^2\, d\theta\right]$

$$= \int_{\pi/6}^{\pi/2} (-4\cos 2\theta + 4\sin\theta)\, d\theta$$

$$= \left[-2\sin(2\theta) - 4\cos\theta\right]_{\pi/6}^{\pi/2} = 3\sqrt{3}$$



37. $A = 2\left[\frac{1}{2}\int_0^{\pi} [a(1 + \cos\theta)]^2\, d\theta\right] - \frac{a^2\pi}{4}$

$$= a^2\left[\frac{3}{2}\theta + 2\sin\theta + \frac{\sin 2\theta}{4}\right]_0^{\pi} - \frac{a^2\pi}{4}$$

$$= \frac{3a^2\pi}{2} - \frac{a^2\pi}{4} = \frac{5a^2\pi}{4}$$

38. Area = Area of $r = 2a\cos\theta$ − Area of sector − twice area between $r = 2a\cos\theta$ and the lines

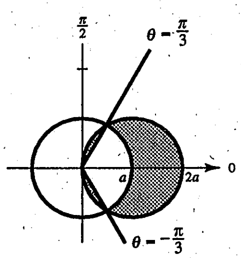
$\theta = \frac{\pi}{3}, \theta = \frac{\pi}{2}.$

$$A = \pi a^2 - \left(\frac{\pi}{3}\right)a^2 - 2\left[\frac{1}{2}\int_{\pi/3}^{\pi/2} (2a\cos\theta)^2\,d\theta\right]$$

$$= \frac{2\pi a^2}{3} - 2a^2\int_{\pi/3}^{\pi/2} (1 + \cos 2\theta)\,d\theta$$

$$= \frac{2\pi a^2}{3} - 2a^2\left[\theta + \frac{\sin 2\theta}{2}\right]_{\pi/3}^{\pi/2}$$

$$= \frac{2\pi a^2}{3} - 2a^2\left[\frac{\pi}{2} - \frac{\pi}{3} - \frac{\sqrt{3}}{4}\right] = \frac{2\pi a^2 + 3\sqrt{3}a^2}{6}$$

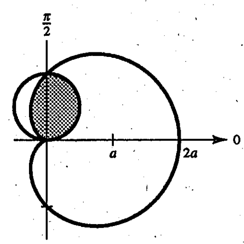


39. $A = \frac{\pi a^2}{8} + \frac{1}{2}\int_{\pi/2}^{\pi} [a(1 + \cos\theta)]^2\,d\theta$

$$= \frac{\pi a^2}{8} + \frac{a^2}{2}\int_{\pi/2}^{\pi}\left(\frac{3}{2} + 2\cos\theta + \frac{\cos 2\theta}{2}\right)d\theta$$

$$= \frac{\pi a^2}{8} + \frac{a^2}{2}\left[\frac{3}{2}\theta + 2\sin\theta + \frac{\sin 2\theta}{4}\right]_{\pi/2}^{\pi}$$

$$= \frac{\pi a^2}{8} + \frac{a^2}{2}\left[\frac{3\pi}{2} - \frac{3\pi}{4} - 2\right] = \frac{a^2}{2}[\pi - 2]$$



40. $r = a\cos\theta, r = a\sin\theta$

$\tan\theta = 1, \theta = \pi/4$

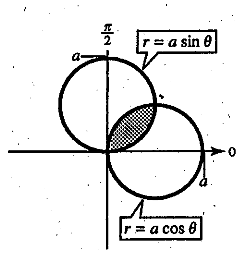
$$A = 2\left[\frac{1}{2}\int_0^{\pi/4} (a\sin\theta)^2\,d\theta\right]$$

$$= a^2\int_0^{\pi/4} \frac{1 - \cos 2\theta}{2}\,d\theta$$

$$= \frac{1}{2}a^2\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/4}$$

$$= \frac{1}{2}a^2\left[\frac{\pi}{4} - \frac{1}{2}\right]$$

$$= \frac{1}{8}a^2\pi - \frac{1}{4}a^2$$

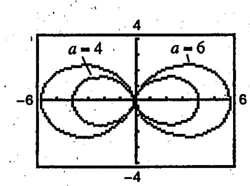


41. (a) $r = a\cos^2\theta$

$r^3 = ar^2\cos^2\theta$

$(x^2 + y^2)^{3/2} = ax^2$

(b)



(c) $A = 4\left(\frac{1}{2}\right)\int_0^{\pi/2} [(6\cos^2\theta)^2 - (4\cos^2\theta)^2]\,d\theta$

$$= 40\int_0^{\pi/2} \cos^4\theta\,d\theta$$

$$= 10\int_0^{\pi/2} (1 + \cos 2\theta)^2\,d\theta$$

$$= 10\int_0^{\pi/2}\left(1 + 2\cos 2\theta + \frac{1 - \cos 4\theta}{2}\right)d\theta$$

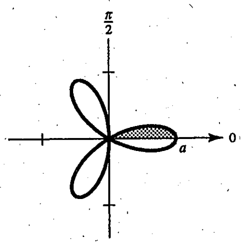
$$= 10\left[\frac{3}{2}\theta + \sin 2\theta + \frac{1}{8}\sin 4\theta\right]_0^{\pi/2} = \frac{15\pi}{2}$$

43. —CONTINUED—

For $n = 3$:

$r = a\cos 3\theta$

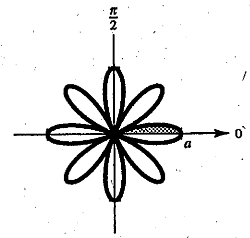
$$A = 6\left(\frac{1}{2}\right)\int_0^{\pi/6} (a\cos 3\theta)^2\,d\theta = \frac{\pi a^2}{4}$$



For $n = 4$:

$r = a\cos 4\theta$

$$A = 16\left(\frac{1}{2}\right)\int_0^{\pi/8} (a\cos 4\theta)^2\,d\theta = \frac{\pi a^2}{2}$$



In general, the area of the region enclosed by $r = a\cos(n\theta)$ for $n = 1, 2, 3, \ldots$ is $(\pi a^2)/4$ if n is odd and is $(\pi a^2)/2$ if n is even.

44. $r = \sec\theta - 2\cos\theta,\ -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

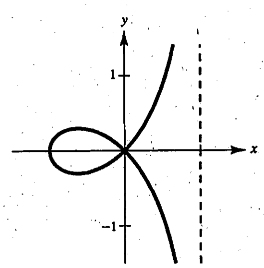
$r\cos\theta = 1 - 2\cos^2\theta$

$$x = 1 - 2\frac{r^2\cos^2\theta}{r^2} = 1 - 2\left(\frac{x^2}{x^2+y^2}\right)$$

$(x^2 + y^2)x = x^2 + y^2 - 2x^2$

$y^2(x - 1) = -x^2 - x^3$

$$y^2 = \frac{x^2(1 + x)}{1 - x}$$



$$A = 2\left(\frac{1}{2}\right)\int_0^{\pi/4} (\sec\theta - 2\cos\theta)^2\,d\theta$$

$$= \int_0^{\pi/4} (\sec^2\theta - 4 + 4\cos^2\theta)\,d\theta = \int_0^{\pi/4} (\sec^2\theta - 4 + 2(1 + \cos 2\theta))\,d\theta = \left[\tan\theta - 2\theta + \sin 2\theta\right]_0^{\pi/4} = 2 - \frac{\pi}{2}$$

45. $r = a$

$r' = 0$

$$s = \int_0^{2\pi} \sqrt{a^2 + 0^2}\,d\theta = \left[a\theta\right]_0^{2\pi} = 2\pi a$$

(circumference of circle of radius a)

46. $r = 2a\cos\theta$

$r' = -2a\sin\theta$

$$s = \int_{-\pi/2}^{\pi/2} \sqrt{(2a\cos\theta)^2 + (-2a\sin\theta)^2}\,d\theta$$

$$= \int_{-\pi/2}^{\pi/2} 2a\,d\theta = \left[2a\theta\right]_{-\pi/2}^{\pi/2} = 2\pi a$$

47. $r = 1 + \sin\theta$

$r' = \cos\theta$

$$s = 2\int_{\pi/2}^{3\pi/2} \sqrt{(1 + \sin\theta)^2 + (\cos\theta)^2}\,d\theta$$

$$= 2\sqrt{2}\int_{\pi/2}^{3\pi/2} \sqrt{1 + \sin\theta}\,d\theta$$

$$= 2\sqrt{2}\int_{\pi/2}^{3\pi/2} \frac{-\cos\theta}{\sqrt{1 - \sin\theta}}\,d\theta$$

$$= \left[4\sqrt{2}\sqrt{1 - \sin\theta}\right]_{\pi/2}^{3\pi/2}$$

$$= 4\sqrt{2}\left(\sqrt{2} - 0\right) = 8$$

48. $r = 8(1 + \cos\theta), 0 \le \theta \le 2\pi$

$r' = -8\sin\theta$

$$s = 2\int_0^{\pi} \sqrt{[8(1+\cos\theta)]^2 + (-8\sin\theta]^2}\, d\theta$$

$$= 16\int_0^{\pi} \sqrt{1 + 2\cos\theta + \cos^2\theta + \sin^2\theta}\, d\theta$$

$$= 16\sqrt{2}\int_0^{\pi} \sqrt{1+\cos\theta}\, d\theta$$

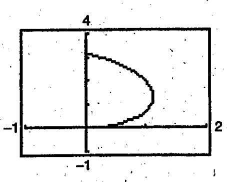
$$= 16\sqrt{2}\int_0^{\pi} \sqrt{1+\cos\theta}\cdot\left(\frac{\sqrt{1-\cos\theta}}{\sqrt{1-\cos\theta}}\right) d\theta$$

$$= 16\sqrt{2}\int_0^{\pi} \frac{\sin\theta}{\sqrt{1-\cos\theta}}\, d\theta$$

$$= \left[32\sqrt{2}\sqrt{1-\cos\theta}\right]_0^{\pi}$$

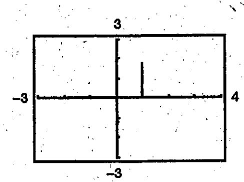
$$= 64$$

49. $r = 2\theta, 0 \le \theta \le \dfrac{\pi}{2}$



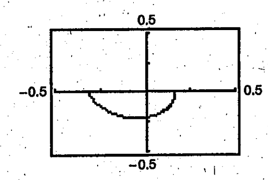
Length ≈ 4.16

50. $r = \sec\theta, 0 \le \theta \le \dfrac{\pi}{3}$



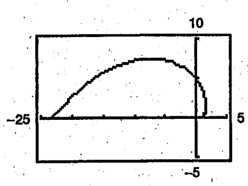
Length ≈ 1.73 (exact $\sqrt{3}$)

51. $r = \dfrac{1}{\theta}, \pi \le \theta \le 2\pi$



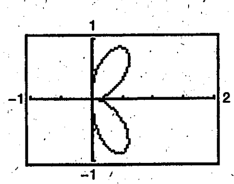
Length ≈ 0.71

52. $r = e^{\theta}, 0 \le \theta \le \pi$



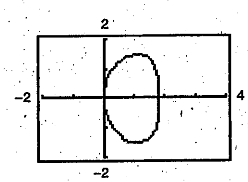
Length ≈ 31.31

53. $r = \sin(3\cos\theta), 0 \le \theta \le \pi$



Length ≈ 4.39

54. $r = 2\sin(2\cos\theta), 0 \le \theta \le \pi$



Length ≈ 7.78

55. $r = 6\cos\theta$

$r' = -6\sin\theta$

$$S = 2\pi\int_0^{\pi/2} 6\cos\theta\sin\theta\sqrt{36\cos^2\theta + 36\sin^2\theta}\, d\theta$$

$$= 72\pi\int_0^{\pi/2} \sin\theta\cos\theta\, d\theta$$

$$= \left[36\pi\sin^2\theta\right]_0^{\pi/2}$$

$$= 36\pi$$

56. $r = a\cos\theta$

$r' = -a\sin\theta$

$$S = 2\pi\int_0^{\pi/2} a\cos\theta(\cos\theta)\sqrt{a^2\cos\theta + a^2\sin^2\theta}\, d\theta$$

$$= 2\pi a^2\int_0^{\pi/2} \cos^2\theta\, d\theta = \pi a^2\int_0^{\pi/2} (1+\cos 2\theta)\, d\theta$$

$$= \left[\pi a^2\left(\theta + \frac{\sin 2\theta}{2}\right)\right]_0^{\pi/2} = \frac{\pi^2 a^2}{2}$$

57. $r = e^{a\theta}$

$r' = ae^{a\theta}$

$$S = 2\pi \int_0^{\pi/2} e^{a\theta} \cos\theta \sqrt{(e^{a\theta})^2 + (ae^{a\theta})^2}\, d\theta$$

$$= 2\pi\sqrt{1 + a^2} \int_0^{\pi/2} e^{2a\theta} \cos\theta\, d\theta$$

$$= 2\pi\sqrt{1 + a^2} \left[\frac{e^{2a\theta}}{4a^2 + 1}(2a\cos\theta + \sin\theta)\right]_0^{\pi/2}$$

$$= \frac{2\pi\sqrt{1 + a^2}}{4a^2 + 1}(e^{\pi a} - 2a)$$

58. $r = a(1 + \cos\theta)$

$r' = -a\sin\theta$

$$S = 2\pi \int_0^{\pi} a(1 + \cos\theta)\sin\theta\sqrt{a^2(1 + \cos\theta)^2 + a^2\sin^2\theta}\, d\theta = 2\pi a^2 \int_0^{\pi} \sin\theta(1 + \cos\theta)\sqrt{2 + 2\cos\theta}\, d\theta$$

$$= -2\sqrt{2}\pi a^2 \int_0^{\pi} (1 + \cos\theta)^{3/2}(-\sin\theta)\, d\theta = -\frac{4\sqrt{2}\pi a^2}{5}\left[(1 + \cos\theta)^{5/2}\right]_0^{\pi} = \frac{32\pi a^2}{5}$$

59. $r = 4\cos 2\theta$

$r' = -8\sin 2\theta$

$$S = 2\pi \int_0^{\pi/4} 4\cos 2\theta \sin\theta \sqrt{16\cos^2 2\theta + 64\sin^2 \theta 2\theta}\, d\theta$$

$$= 32\pi \int_0^{\pi/4} \cos 2\theta \sin\theta\sqrt{\cos^2 2\theta + 4\sin^2 2\theta}\, d\theta \approx 21.87$$

60. $r = \theta$

$r' = 1$

$$S = 2\pi \int_0^{\pi} \theta\sin\theta\sqrt{\theta^2 + 1}\, d\theta \approx 42.32$$

61. $$\text{Area} = \frac{1}{2}\int_\alpha^\beta [f(\theta)]^2\, d\theta = \frac{1}{2}\int_\alpha^\beta r^2\, d\theta$$

$$\text{Arc length} = \int_\alpha^\beta \sqrt{f(\theta)^2 + f'(\theta)^2}\, d\theta = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

62. The curves might intersect for different values of θ: See page 741.

63. (a) is correct: $s \approx 33.124$.

64. (a) $S = 2\pi \int_\alpha^\beta f(\theta)\sin\theta\sqrt{f(\theta)^2 + f'(\theta)^2}\, d\theta$

(b) $S = 2\pi \int_\alpha^\beta f(\theta)\cos\theta\sqrt{f(\theta)^2 + f'(\theta)^2}\, d\theta$

65. Revolve $r = 2$ about the line $r = 5\sec\theta$.

$f(\theta) = 2, f'(\theta) = 0$

$$S = 2\pi \int_0^{2\pi} (5 - 2\cos\theta)\sqrt{2^2 + 0^2}\, d\theta$$

$$= 4\pi \int_0^{2\pi} (5 - 2\cos\theta)\, d\theta$$

$$= 4\pi\Big[5\theta - 2\sin\theta\Big]_0^{2\pi}$$

$$= 40\pi^2$$