Ch. 12.1 Vector-valued functions

HW p.837 #2-22 D2S1, 23-37 odd

Find domain of vector-valued function.

2) $r(t) = \sqrt{4-t^2}\,i + t^2 j - 6tK$

use most restrictive domain.

Component functions: $f(t) = \sqrt{4-t^2} \rightarrow [-2,2]$

$g(t) = t^2$

$h(t) = -6t$

look for shared/common domain

$\boxed{\text{Domain: } [-2,2]}$

6) $r(t) = F(t) - G(t)$

$= \ln t\, i + 5t j - 3t^2 K - (i + 4t j - 3t^2 K)$

$= (\ln t - 1) i + (5t - 4t) j + (-3t^2 + 3t^2) K$

$= (\ln t - 1) i + 1t j + 0K$

$\boxed{\text{Domain: } (0,\infty)}$

* think determinant

$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

8) $r(t) = F(t) \times G(t)$

$\begin{vmatrix} i & j & k \\ t^3 & -t & t \\ \sqrt[3]{t} & \frac{1}{t+1} & t+2 \end{vmatrix}$

$\left[-t(t+2) - \frac{t}{t+1}\right] i + \left[t^3(t+2) - t\sqrt[3]{t}\right] j + \left[\frac{t^3}{t+1} - t\sqrt[3]{t}\right] k$

$\boxed{\text{Domain: } (-\infty,-1) \cup (-1,\infty)}$

12) Evaluate vector-valued function at given value t

$r(t) = \sqrt{t}\,i + t^{3/2} j + e^{-t/4} k$

(a) $r(0) = k$

(b) $r(4) = 2i + 8j + e^{-1} k$

(c) $r(c+2) = \sqrt{c+2}\,i + (c+2)^{3/2} j + e^{-[(c+2)/4]} k$

(d) $r(9+\Delta t) - r(9) = \sqrt{9+\Delta t}\,i + (9+\Delta t)^{3/2} j + e^{-[(9+\Delta t)/4]} k -$
$(3i + 27j + e^{-9/4} k)$

$= (\sqrt{9+\Delta t} - 3) i + [(9+\Delta t)^{3/2} - 27] j + [e^{-(\frac{9+\Delta t}{4})} - e^{-9/4}] k$

14) Find $\|r(t)\|$ $\quad r(t) = \sqrt{t}\,i + 3tj - 4tk = \langle \sqrt{t}, 3t, -4t \rangle$

length $|v|$ for $\langle v_1, v_2, v_3 \rangle$ is $\sqrt{v_1^2 + v_2^2 + v_3^2}$

$\|r(t)\| = \sqrt{(\sqrt{t})^2 + (3t)^2 + (-4t)^2} = \boxed{\sqrt{t + 9t^2 + 16t^2}}$

$= \boxed{\sqrt{t(1+25t)}}$

15) $r(t) \cdot u(t)$ yields a scalar, not a vector

$u \cdot v = u_1 v_1 + u_2 v_2$

$r(t) \cdot u(t) = (3t-1)(t^2) + (\frac{1}{4}t^3)(-8) + 4(t^3)$

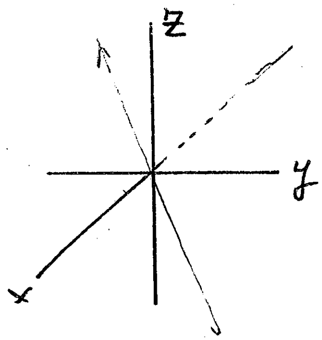
$= 3t^3 - t^2 - 2t^3 + 4t^3 = \boxed{5t^3 - t^2}$

18) $r(t) = \cos(\pi t)i + \sin(\pi t)j + t^2 k \qquad -1 \le t \le 1$

$x = \cos(\pi t) \qquad y = \sin(\pi t) \qquad z = t^2$

$x^2 + y^2 = 1$ matches (c)

22) $r(t) = ti + tj + 2k \qquad x = t,\ y = t,\ z = 2 \qquad x = y$



23) $r(t) = 3ti + (t-1)j$

$x = 3t \qquad y = t - 1$

$\boxed{y = \frac{1}{3}x - 1}$

25) $r(t) = t^3 i + t^2 j$

$x = t^3 \qquad y = t^2$

$t = \sqrt[3]{x} = x^{1/3} \qquad y = (x^{1/3})^2 = x^{2/3}$

27) $r(\theta) = \cos\theta\, i + 3\sin\theta\, j$

$x = \cos\theta \qquad y = 3\sin\theta$

$x^2 + \frac{y^2}{9} = 1$

29) $r(\theta) = 3\sec\theta i + 2\tan\theta j$

$x = 3\sec\theta \quad y = 2\tan\theta$

$\frac{x^2}{9} = \frac{y^2}{4} + 1$

31) $r(t) = (-t+1)i + (4t+2)j + (2t+3)k$

$x = -t + 1$

$y = 4t + 2$ Line passing $(0,6,5)$ and $(1,2,3)$

$z = 2t + 3$

33) $r(t) = 2\cos t i + 2\sin t j + tk$

$x = 2\cos t \quad y = 2\sin t \quad z = t$

$\frac{x^2}{4} + \frac{y^2}{4} = 1 \quad z = t$ circular helix

35) $r(t) = 2\sin t i + 2\cos t j + e^{-t}k$

$x = 2\sin t \quad y = 2\cos t \quad z = e^{-t}$

$x^2 + y^2 = 4 \quad z = e^{-t}$

37) $r(t) = \langle t, t^2, \frac{2}{3}t^3 \rangle$

$x = t \quad y = t^2 \quad z = \frac{2}{3}t^3$

$y = x^2 \quad z = \frac{2}{3}x^3$