CHAPTER 12
Vector-Valued Functions

Section 12.1 Vector-Valued Functions

1. $\mathbf{r}(t) = 5t\mathbf{i} - 4t\mathbf{j} - \frac{1}{t}\mathbf{k}$

Component functions: $f(t) = 5t$

$g(t) = -4t$

$h(t) = -\frac{1}{t}$

Domain: $(-\infty, 0) \cup (0, \infty)$

2. $\mathbf{r}(t) = \sqrt{4 - t^2}\mathbf{i} + t^2\mathbf{j} - 6t\mathbf{k}$

Component functions: $f(t) = \sqrt{4 - t^2}$

$g(t) = t^2$

$h(t) = -6t$

Domain: $[-2, 2]$

3. $\mathbf{r}(t) = \ln t\mathbf{i} - e^t\mathbf{j} - t\mathbf{k}$

Component functions: $f(t) = \ln t$

$g(t) = -e^t$

$h(t) = -t$

Domain: $(0, \infty)$

4. $\mathbf{r}(t) = \sin t\mathbf{i} + 4\cos t\mathbf{j} + t\mathbf{k}$

Component functions: $f(t) = \sin t$

$g(t) = 4\cos t$

$h(t) = t$

Domain: $(-\infty, \infty)$

5. $\mathbf{r}(t) = \mathbf{F}(t) + \mathbf{G}(t) = \left(\cos t\mathbf{i} - \sin t\mathbf{j} + \sqrt{t}\mathbf{k}\right) + (\cos t\mathbf{i} + \sin t\mathbf{j}) = 2\cos t\mathbf{i} + \sqrt{t}\mathbf{k}$

Domain: $[0, \infty)$

6.
$$\begin{aligned}\mathbf{r}(t) = \mathbf{F}(t) - \mathbf{G}(t) &= (\ln t\mathbf{i} + 5t\mathbf{j} - 3t^2\mathbf{k}) - (\mathbf{i} + 4t\mathbf{j} - 3t^2\mathbf{k})\\ &= (\ln t - 1)\mathbf{i} + (5t - 4t)\mathbf{j} + (-3t^2 + 3t^2)\mathbf{k}\\ &= (\ln t - 1)\mathbf{i} + t\mathbf{j}\end{aligned}$$

Domain: $(0, \infty)$

7. $\mathbf{r}(t) = \mathbf{F}(t) \times \mathbf{G}(t) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \sin t & \cos t & 0 \\ 0 & \sin t & \cos t \end{vmatrix} = \cos^2 t\mathbf{i} - \sin t\cos t\mathbf{j} + \sin^2 t\mathbf{k}$

Domain: $(-\infty, \infty)$

8. $\mathbf{r}(t) = \mathbf{F}(t) \times \mathbf{G}(t) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ t^3 & -t & t \\ \sqrt[3]{t} & \frac{1}{t+1} & t+2 \end{vmatrix} = \left(-t(t+2) - \frac{t}{t+1}\right)\mathbf{i} - \left(t^3(t+2) - t\sqrt[3]{t}\right)\mathbf{j} + \left(\frac{t^3}{t+1} + t\sqrt[3]{t}\right)\mathbf{k}$

Domain: $(-\infty, -1), (-1, \infty)$

9. $\mathbf{r}(t) = \frac{1}{2}t^2\mathbf{i} - (t - 1)\mathbf{j}$

(a) $\mathbf{r}(1) = \frac{1}{2}\mathbf{i}$

(b) $\mathbf{r}(0) = \mathbf{j}$

(c) $\mathbf{r}(s + 1) = \frac{1}{2}(s + 1)^2\mathbf{i} - (s + 1 - 1)\mathbf{j} = \frac{1}{2}(s + 1)^2\mathbf{i} - s\mathbf{j}$

(d) $$\begin{aligned}\mathbf{r}(2 + \Delta t) - \mathbf{r}(2) &= \tfrac{1}{2}(2 + \Delta t)^2\mathbf{i} - (2 + \Delta t - 1)\mathbf{j} - (2\mathbf{i} - \mathbf{j}) \\ &= \left(2 + 2\Delta t + \tfrac{1}{2}(\Delta t)^2\right)\mathbf{i} - (1 + \Delta t)\mathbf{j} - 2\mathbf{i} + \mathbf{j} \\ &= \left(2\Delta t + \tfrac{1}{2}(\Delta t)^2\right)\mathbf{i} - (\Delta t)\mathbf{j}\end{aligned}$$

10. $\mathbf{r}(t) = \cos t\,\mathbf{i} + 2\sin t\,\mathbf{j}$

(a) $\mathbf{r}(0) = \mathbf{i}$

(b) $\mathbf{r}\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}\mathbf{i} + \sqrt{2}\,\mathbf{j}$

(c) $\mathbf{r}(\theta - \pi) = \cos(\theta - \pi)\mathbf{i} + 2\sin(\theta - \pi)\mathbf{j} = -\cos\theta\,\mathbf{i} - 2\sin\theta\,\mathbf{j}$

(d) $\mathbf{r}\left(\frac{\pi}{6} + \Delta t\right) - \mathbf{r}\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6} + \Delta t\right)\mathbf{i} + 2\sin\left(\frac{\pi}{6} + \Delta t\right)\mathbf{j} - \left(\cos\left(\frac{\pi}{6}\right)\mathbf{i} + 2\sin\frac{\pi}{6}\mathbf{j}\right)$

11. $\mathbf{r}(t) = \ln t\,\mathbf{i} + \frac{1}{t}\mathbf{j} + 3t\mathbf{k}$

(a) $\mathbf{r}(2) = \ln 2\,\mathbf{i} + \frac{1}{2}\mathbf{j} + 6\mathbf{k}$

(b) $\mathbf{r}(-3)$ is not defined. ($\ln(-3)$ does not exist.)

(c) $\mathbf{r}(t - 4) = \ln(t - 4)\mathbf{i} + \frac{1}{t - 4}\mathbf{j} + 3(t - 4)\mathbf{k}$

(d) $$\begin{aligned}\mathbf{r}(1 + \Delta t) - \mathbf{r}(1) &= \ln(1 + \Delta t)\mathbf{i} + \frac{1}{1 + \Delta t}\mathbf{j} + 3(1 + \Delta t)\mathbf{k} - (0\mathbf{i} + \mathbf{j} + 3\mathbf{k}) \\ &= \ln(1 + \Delta t)\mathbf{i} + \left(\frac{1}{1 + \Delta t} - 1\right)\mathbf{j} + (3\Delta t)\mathbf{k}\end{aligned}$$

12. $\mathbf{r}(t) = \sqrt{t}\,\mathbf{i} + t^{3/2}\mathbf{j} + e^{-t/4}\mathbf{k}$

(a) $\mathbf{r}(0) = \mathbf{k}$

(b) $\mathbf{r}(4) = 2\mathbf{i} + 8\mathbf{j} + e^{-1}\mathbf{k}$

(c) $\mathbf{r}(c + 2) = \sqrt{c + 2}\,\mathbf{i} + (c + 2)^{3/2}\mathbf{j} + e^{-[(c+2)/4]}\mathbf{k}$

(d) $$\begin{aligned}\mathbf{r}(9 + \Delta t) - \mathbf{r}(9) &= \left(\sqrt{9 + \Delta t}\right)\mathbf{i} + (9 + \Delta t)^{3/2}\mathbf{j} + e^{-[(9+\Delta t)/4]}\mathbf{k} - (3\mathbf{i} + 27\mathbf{j} + e^{-9/4}\mathbf{k}) \\ &= \left(\sqrt{9 + \Delta t} - 3\right)\mathbf{i} + \left((9 + \Delta t)^{3/2} - 27\right)\mathbf{j} + \left(e^{-[(9+\Delta t)/4]} - e^{-9/4}\right)\mathbf{k}\end{aligned}$$

13. $\mathbf{r}(t) = \sin 3t\mathbf{i} + \cos 3t\mathbf{j} + t\mathbf{k}$

$\|\mathbf{r}(t)\| = \sqrt{(\sin 3t)^2 + (\cos 3t)^2 + t^2} = \sqrt{1 + t^2}$

14. $\mathbf{r}(t) = \sqrt{t}\mathbf{i} + 3t\mathbf{j} - 4t\mathbf{k}$

$$\begin{aligned}\|\mathbf{r}(t)\| &= \sqrt{\left(\sqrt{t}\right)^2 + (3t)^2 + (-4t)^2} \\ &= \sqrt{t + 9t^2 + 16t^2} = \sqrt{t(1 + 25t)}\end{aligned}$$

15. $$\begin{aligned}\mathbf{r}(t) \cdot \mathbf{u}(t) &= (3t - 1)(t^2) + \left(\tfrac{1}{4}t^3\right)(-8) + 4(t^3) \\ &= 3t^3 - t^2 - 2t^3 + 4t^3 = 5t^3 - t^2, \text{ a scalar.}\end{aligned}$$

The dot product is a scalar-valued function.

16. $\mathbf{r}(t) \cdot \mathbf{u}(t) = (3\cos t)(4\sin t) + (2\sin t)(-6\cos t) + (t-2)(t^2) = t^3 - 2t^2$, a scalar.

The dot product is a scalar-valued function.

17. $\mathbf{r}(t) = t\mathbf{i} + 2t\mathbf{j} + t^2\mathbf{k},\ -2 \le t \le 2$

$x = t, y = 2t, z = t^2$

Thus, $z = x^2$. Matches (b)

18. $\mathbf{r}(t) = \cos(\pi t)\mathbf{i} + \sin(\pi t)\mathbf{j} + t^2\mathbf{k},\ -1 \le t \le 1$

$x = \cos(\pi t), y = \sin(\pi t), z = t^2$

Thus, $x^2 + y^2 = 1$. Matches (c)

19. $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + e^{0.75t}\mathbf{k},\ -2 \le t \le 2$

$x = t, y = t^2, z = e^{0.75t}$

Thus, $y = x^2$. Matches (d)

20. $\mathbf{r}(t) = t\mathbf{i} + \ln t\mathbf{j} + \frac{2t}{3}\mathbf{k},\ 0.1 \le t \le 5$

$x = t,\ y = \ln t,\ z = \frac{2t}{3}$

Thus, $z = \frac{2}{3}x$ and $y = \ln x$. Matches (a)

21. (a) View from the negative x-axis: $(-20, 0, 0)$

(b) View from above the first octant: $(10, 20, 10)$

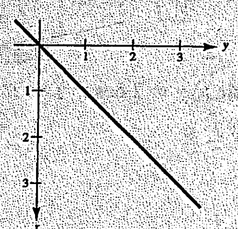
(c) View from the z-axis: $(0, 0, 20)$

(d) View from the positive x-axis: $(20, 0, 0)$

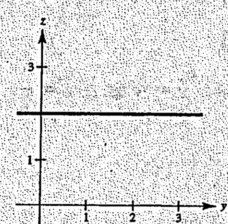
22. $\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + 2\mathbf{k}$

$x = t, y = t, z = 2 \implies x = y$

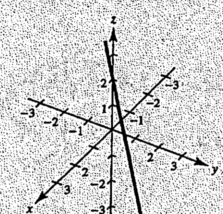
(a) $(0, 0, 20)$



(b) $(10, 0, 0)$



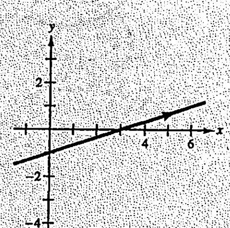
(c) $(5, 5, 5)$



23. $x = 3t$

$y = t - 1$

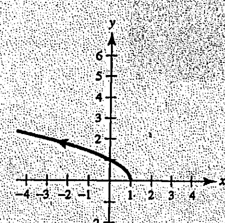
$y = \frac{x}{3} - 1$



24. $x = 1 - t, y = \sqrt{t}$

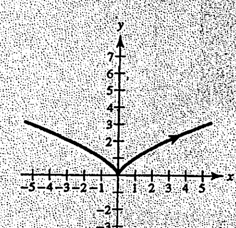
$y = \sqrt{1-x}$

Domain: $t \ge 0$

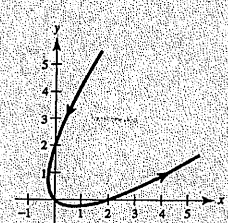


25. $x = t^3, y = t^2$

$y = x^{2/3}$

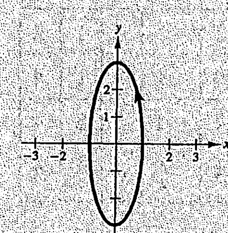


26. $x = t^2 + t, y = t^2 - t$



27. $x = \cos\theta, y = 3\sin\theta$

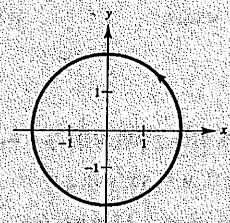
$x^2 + \frac{y^2}{9} = 1$ Ellipse



28. $x = 2\cos t$

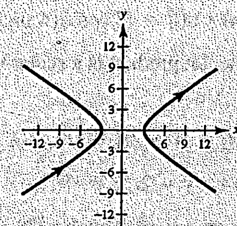
$y = 2\sin t$

$x^2 + y^2 = 4$



29. $x = 3\sec\theta,\ y = 2\tan\theta$

$\dfrac{x^2}{9} = \dfrac{y^2}{4} + 1$ Hyperbola

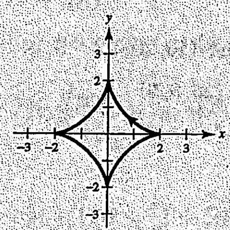


30. $x = 2\cos^3 t,\ y = 2\sin^3 t$

$$\left(\frac{x}{2}\right)^{2/3} + \left(\frac{y}{2}\right)^{2/3} = \cos^2 t + \sin^2 t$$

$$= 1$$

$$x^{2/3} + y^{2/3} = 2^{2/3}$$



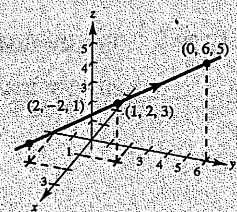
31. $x = -t + 1$

$y = 4t + 2$

$z = 2t + 3$

Line passing through the points:

$(0, 6, 5),\ (1, 2, 3)$



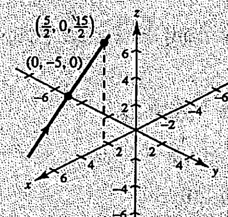
32. $x = t$

$y = 2t - 5$

$y = 3t$

Line passing through the points:

$(0, -5, 0), \left(\frac{5}{2}, 0, \frac{15}{2}\right)$

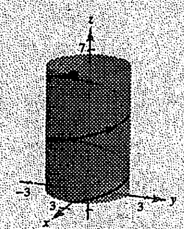


33. $x = 2\cos t,\ y = 2\sin t,\ z = t$

$\dfrac{x^2}{4} + \dfrac{y^2}{4} = 1$

$z = t$

Circular helix

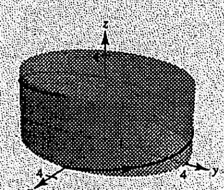


34. $x = 3\cos t,\ y = 4\sin t,\ z = \dfrac{t}{2}$

$\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$

$z = \dfrac{t}{2}$

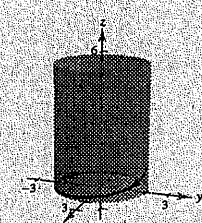
Elliptic helix



35. $x = 2\sin t,\ y = 2\cos t,\ z = e^{-t}$

$x^2 + y^2 = 4$

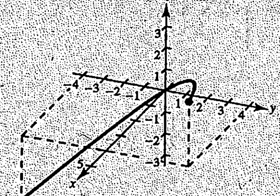
$z = e^{-t}$



36. $x = t^2, y = 2t, z = \dfrac{3}{2}t$

$x = \dfrac{y^2}{4}, z = \dfrac{3}{4}y$

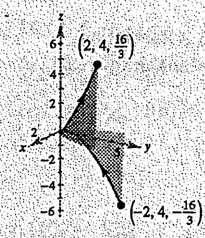
t	-2	-1	0	1	2
x	4	1	0	1	4
y	-4	-2	0	2	4
z	-3	$-\frac{3}{2}$	0	$\frac{3}{2}$	3



37. $x = t,\ y = t^2,\ z = \frac{2}{3}t^3$

$y = x^2,\ z = \frac{2}{3}x^3$

t	-2	-1	0	1	2
x	-2	-1	0	1	2
y	4	1	0	1	4
z	$-\frac{16}{3}$	$-\frac{2}{3}$	0	$\frac{2}{3}$	$\frac{16}{3}$



38. $x = \cos t + t\sin t$

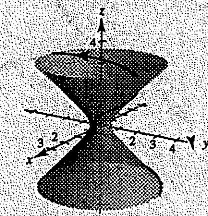
$y = \sin t - t\cos t$

$z = t$

$x^2 + y^2 = 1 + t^2 = 1 + z^2$ or $x^2 + y^2 - z^2 = 1$

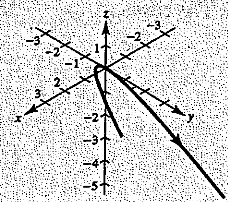
$z = t$

Helix along a hyperboloid of one sheet



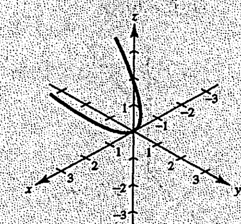
39. $\mathbf{r}(t) = -\frac{1}{2}t^2\mathbf{i} + t\mathbf{j} - \frac{\sqrt{3}}{2}t^2\mathbf{k}$

Parabola



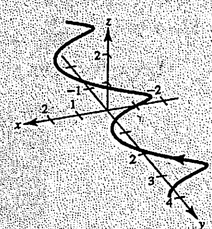
40. $\mathbf{r}(t) = t\mathbf{i} - \frac{\sqrt{3}}{2}t^2\mathbf{j} + \frac{1}{2}t^2\mathbf{k}$

Parabola



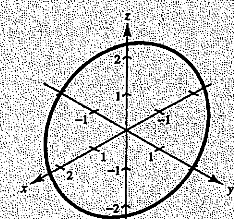
41. $\mathbf{r}(t) = \sin t\mathbf{i} + \left(\frac{\sqrt{3}}{2}\cos t - \frac{1}{2}t\right)\mathbf{j} + \left(\frac{1}{2}\cos t + \frac{\sqrt{3}}{2}\right)\mathbf{k}$

Helix

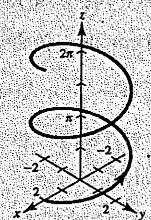


42. $\mathbf{r}(t) = -\sqrt{2}\sin t\mathbf{i} + 2\cos t\mathbf{j} + \sqrt{2}\sin t\mathbf{k}$

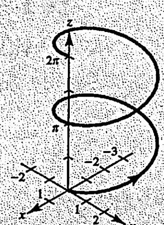
Ellipse



43.

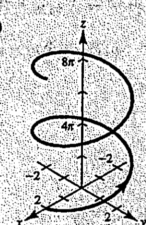


(a)



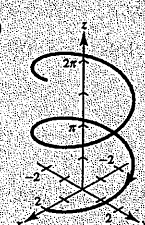
The helix is translated 2 units back on the x-axis.

(b)



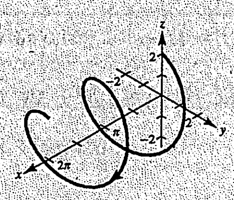
The height of the helix increases at a faster rate.

(c)



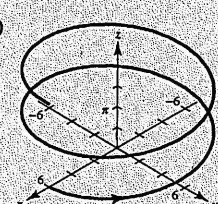
The orientation of the helix is reversed.

(d)



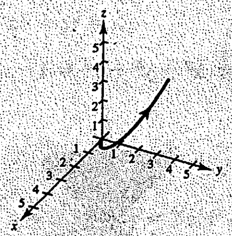
The axis of the helix is the x-axis.

(e)

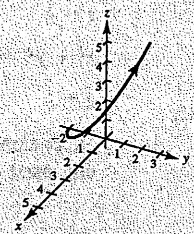


The radius of the helix is increased from 2 to 6.

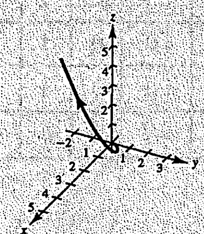
44. $r(t) = t\mathbf{i} + t^2\mathbf{j} + \frac{1}{2}t^3\mathbf{h}$



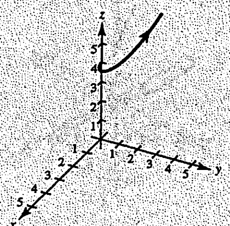
(a) $\mathbf{u}(t) = \mathbf{r}(t) - 2\mathbf{j}$ is a translation 2 units to the left along the y-axis.



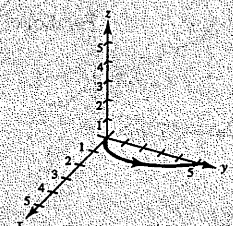
(b) $\mathbf{u}(t) = t^2\mathbf{i} + t\mathbf{j} + \frac{1}{2}t^3\mathbf{k}$ has the roles of x and y interchanged. The graph is a reflection in the plane $x = y$.



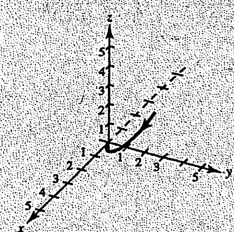
(c) $\mathbf{u}(t) = \mathbf{r}(t) + 4\mathbf{k}$ is an upward shift 4 units.



(d) $\mathbf{u}(t) = t\mathbf{i} + t^2\mathbf{j} + \frac{1}{8}t^3\mathbf{k}$ shrinks the z-value by a factor of 4. The curve rises more slowly.



(e) $\mathbf{u}(t) = \mathbf{r}(-t)$ reverses the orientation.



45. $y = 4 - x$

Let $x = t$, then $y = 4 - t$.

$\mathbf{r}(t) = t\mathbf{i} + (4 - t)\mathbf{j}$

46. $2x - 3y + 5 = 0$

Let $x = t$, then $y = \frac{1}{3}(2t + 5)$.

$\mathbf{r}(t) = t\mathbf{i} + \frac{1}{3}(2\mathbf{t} + 5)\mathbf{j}$

47. $y = (x - 2)^2$

Let $x = t$, then $y = (t - 2)^2$.

$\mathbf{r}(t) = t\mathbf{i} + (t - 2)^2\mathbf{j}$

48. $y = 4 - x^2$

Let $x = t$, then $y = 4 - t^2$.

$\mathbf{r}(t) = t\mathbf{i} + (4 - t^2)\mathbf{j}$

49. $x^2 + y^2 = 25$

Let $x = 5\cos t$, then $y = 5\sin t$.

$\mathbf{r}(t) = 5\cos t\mathbf{i} + 5\sin t\mathbf{j}$

50. $(x - 2)^2 + y^2 = 4$

Let $x - 2 = 2\cos t$, $y = 2\sin t$.

$\mathbf{r}(t) = (2 + 2\cos t)\mathbf{i} + 2\sin t\mathbf{j}$

51. $\frac{x^2}{16} - \frac{y^2}{4} = 1$

Let $x = 4\sec t$, $y = 2\tan t$.

$\mathbf{r}(t) = 4\sec t\mathbf{i} + 2\tan t\mathbf{j}$

52. $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Let $x = 4\cos t$, $y = 3\sin t$.

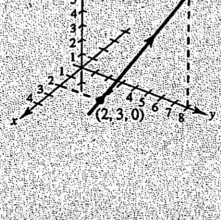
$\mathbf{r}(t) = 4\cos t\mathbf{i} + 3\sin t\mathbf{j}$

53. The parametric equations for the line are

$$x = 2 - 2t,\ y = 3 + 5t,\ z = 8t.$$

One possible answer is

$$\mathbf{r}(t) = (2 - 2t)\mathbf{i} + (3 + 5t)\mathbf{j} + 8t\mathbf{k}.$$



54. One possible answer is

$$\mathbf{r}(t) = 1.5\cos t\mathbf{i} + 1.5\sin t\mathbf{j} + \frac{1}{\pi}t\mathbf{k},\ 0 \le t \le 2\pi$$

Note that $\mathbf{r}(2\pi) = 1.5\mathbf{i} + 2\mathbf{k}$.

55. $\mathbf{r}_1(t) = t\mathbf{i}, \quad 0 \le t \le 4 \quad (\mathbf{r}_1(0) = \mathbf{0}, \mathbf{r}_1(4) = 4\mathbf{i})$

$\mathbf{r}_2(t) = (4 - 4t)\mathbf{i} + 6t\mathbf{j}, \quad 0 \le t \le 1 \quad (\mathbf{r}_2(0) = 4\mathbf{i}, \mathbf{r}_2(1) = 6\mathbf{j})$

$\mathbf{r}_3(t) = (6 - t)\mathbf{j}, \quad 0 \le t \le 6 \quad (\mathbf{r}_3(0) = 6\mathbf{j}, \mathbf{r}_3(6) = \mathbf{0})$

(Other answers possible)

56. $\mathbf{r}_1(t) = t\mathbf{i}, \quad 0 \le t \le 10 \quad (\mathbf{r}_1(0) = \mathbf{0}, \mathbf{r}_1(10) = 10\mathbf{i})$

$\mathbf{r}_2(t) = 10(\cos t\mathbf{i} + \sin t\mathbf{j}), \quad 0 \le t \le \frac{\pi}{4} \quad \left(\mathbf{r}_2(0) = 10\mathbf{i}, \mathbf{r}_2\left(\frac{\pi}{4}\right) = 5\sqrt{2}\mathbf{i} + 5\sqrt{2}\mathbf{j}\right)$

$\mathbf{r}_3(t) = 5\sqrt{2}(1 - t)\mathbf{i} + 5\sqrt{2}(1 - t)\mathbf{j}, \quad 0 \le t \le 1 \quad \left(\mathbf{r}_3(0) = 5\sqrt{2}\mathbf{i} + 5\sqrt{2}\mathbf{j}, \mathbf{r}_3(1)\right) = \mathbf{0}$

(Other answers possible)

57. $\mathbf{r}_1(t) = t\mathbf{i} + t^2\mathbf{j}, \quad 0 \le t \le 2 \quad (y = x^2)$

$\mathbf{r}_2(t) = (2 - t)\mathbf{i} + 4\mathbf{j}, \quad 0 \le t \le 2$

$\mathbf{r}_3(t) = (4 - t)\mathbf{j}, \quad 0 \le t \le 4$

(Other answers possible)

58. $\mathbf{r}_1(t) = t\mathbf{i} + \sqrt{t}\mathbf{j}, \quad 0 \le t \le 1 \quad \left(y = \sqrt{x}\right)$

$\mathbf{r}_2(t) = (1 - t)\mathbf{i} + (1 - t)\mathbf{j}, \quad 0 \le t \le 1 \quad (y = x)$

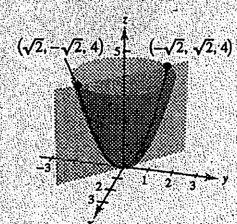
(Other answers possible)

59. $z = x^2 + y^2, \; x + y = 0$

Let $x = t$, then $y = -x = -t$ and $z = x^2 + y^2 = 2t^2$. Therefore,

$x = t, \; y = -t, \; z = 2t^2.$

$\mathbf{r}(t) = t\mathbf{i} - t\mathbf{j} + 2t^2\mathbf{k}$

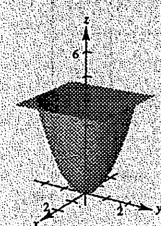


60. $z = x^2 + y^2, \; z = 4$

Therefore, $x^2 + y^2 = 4$ or

$x = 2\cos t, y = 2\sin t, z = 4.$

$\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j} + 4\mathbf{k}$



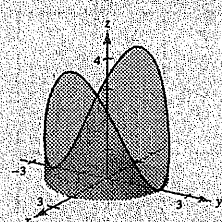
61. $x^2 + y^2 = 4, \; z = x^2$

$x = 2\sin t, \; y = 2\cos t$

$z = x^2 = 4\sin^2 t$

t	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
x	0	1	$\sqrt{2}$	2	$\sqrt{2}$	0
y	2	$\sqrt{3}$	$\sqrt{2}$	0	$-\sqrt{2}$	-2
z	0	1	2	4	2	0

$\mathbf{r}(t) = 2\sin t\mathbf{i} + 2\cos t\mathbf{j} + 4\sin^2 t\mathbf{k}$



89. True

90. False. The graph of $x = y = z = t^3$ represents a line.

91. False. Although $\mathbf{r}(4) = \langle 4, 16\rangle = \mathbf{u}(2)$, they do not collide. Their paths cross this point at different times.

92. True. $y^2 + z^2 = t^2 \sin^2 t + t^2 \cos^2 t = t^2 = x$

Section 12.2 Differentiation and Integration of Vector-Valued Functions

1. $\mathbf{r}(t) = t^2\mathbf{i} + t\mathbf{j},\ t_0 = 2$

$x(t) = t^2,\ y(t) = t$

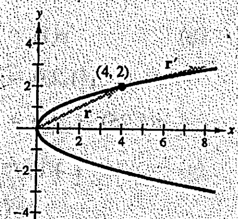
$x = y^2$

$\mathbf{r}(2) = 4\mathbf{i} + 2\mathbf{j}$

$\mathbf{r}'(t) = 2t\mathbf{i} + \mathbf{j}$

$\mathbf{r}'(2) = 4\mathbf{i} + \mathbf{j}$

$\mathbf{r}'(t_0)$ is tangent to the curve.



2. $\mathbf{r}(t) = t\mathbf{i} + t^3\mathbf{j},\ t_0 = 1$

$x(t) = t,\ y(t) = t^3$

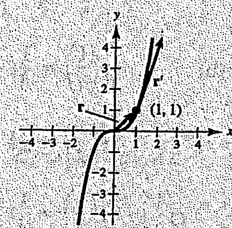
$y = x^3$

$\mathbf{r}(1) = \mathbf{i} + \mathbf{j}$

$\mathbf{r}'(t) = \mathbf{i} + 3t^2\mathbf{j}$

$\mathbf{r}'(1) = \mathbf{i} + 3\mathbf{j}$

$\mathbf{r}'(t_0)$ is tangent to the curve.



3. $\mathbf{r}(t) = t^2\mathbf{i} + \frac{1}{t}\mathbf{j},\ t_0 = 2$

$x(t) = t^2,\ y(t) = \frac{1}{t}$

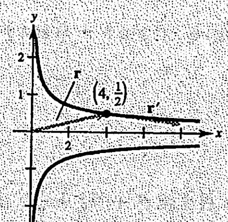
$x = \frac{1}{y^2}$

$\mathbf{r}(2) = 4\mathbf{i} + \frac{1}{2}\mathbf{j}$

$\mathbf{r}'(t) = 2t\mathbf{i} - \frac{1}{t^2}\mathbf{j}$

$\mathbf{r}'(2) = 4\mathbf{i} - \frac{1}{4}\mathbf{j}$

$\mathbf{r}'(t_0)$ is tangent to the curve.



4. (a) $\mathbf{r}(t) = (1 + t)\mathbf{i} + t^3\mathbf{j},\ t_0 = 1$

$x = 1 + t$

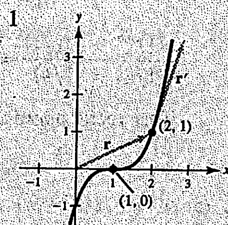
$y = t^3 = (x - 1)^3$

(b) $\mathbf{r}(1) = 2\mathbf{i} + \mathbf{j}$

$\mathbf{r}'(t) = \mathbf{i} + 3t^2\mathbf{j}$

$\mathbf{r}'(1) = \mathbf{i} + 3\mathbf{j}$

$\mathbf{r}'(1)$ is tangent to the curve.



5. $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j},\ t_0 = \frac{\pi}{2}$

$x(t) = \cos t,\ y(t) = \sin t$

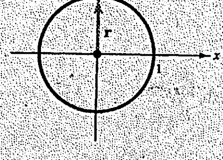
$x^2 + y^2 = 1$

$\mathbf{r}\left(\frac{\pi}{2}\right) = \mathbf{j}$

$\mathbf{r}'(t) = -\sin t\mathbf{i} + \cos t\mathbf{j}$

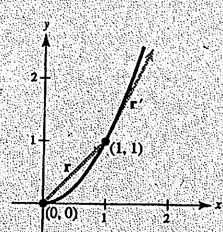
$\mathbf{r}'\left(\frac{\pi}{2}\right) = -\mathbf{i}$

$\mathbf{r}'(t_0)$ is tangent to the curve.



6. (a) $\mathbf{r}(t) = e^t\mathbf{i} + e^{2t}\mathbf{j},\ t_0 = 0$

$x = e^t, y = e^{2t} = x^2 \Rightarrow y = x^2, x > 0$



(b) $\mathbf{r}(0) = \mathbf{i} + \mathbf{j}$

$\mathbf{r}'(t) = e^t\mathbf{i} + 2e^{2t}\mathbf{j}$

$\mathbf{r}'(0) = \mathbf{i} + 2\mathbf{j}$

$\mathbf{r}'(0)$ is tangent to the curve.